

Monday 12 May 2025 – Afternoon

AS Level Further Mathematics B (MEI)

Y410/01 Core Pure

Time allowed: 1 hour 15 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

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INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- This document has **4** pages.

ADVICE

- Read each question carefully before you start your answer.

1 The matrices \mathbf{A} and \mathbf{B} are given by

$$\mathbf{A} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}.$$

(a) Use \mathbf{A} and \mathbf{B} to show that matrix multiplication is **not**, in general, commutative. [2]

(b) Verify that \mathbf{A} and \mathbf{B} satisfy $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$. [3]

2 **In this question you must show detailed reasoning.**

Find the acute angle between the vector $3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and the normal vector to the plane $2x + 3y + z = 6$. [4]

3 The matrices \mathbf{M} and \mathbf{N} are given by

$$\mathbf{M} = \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \text{ and } \mathbf{N} = \begin{pmatrix} b & -a \\ a & b \end{pmatrix} \text{ where } a \text{ and } b \text{ are positive constants.}$$

(a) Given that $\mathbf{M}^2 = \mathbf{N}$, determine the exact values of a and b . [4]

(b) Hence state the transformations of the plane associated with matrices \mathbf{M} and \mathbf{N} . [3]

- 4 (a) The transformation T is represented by the matrix $\mathbf{M} = \begin{pmatrix} 1 & -2 & 2 \\ 2 & 1 & 0 \\ 1 & 2 & -1 \end{pmatrix}$.

A shape S_1 is mapped to a shape S_2 by the transformation T .

Show that volume of S_1 is the same as the volume of S_2 . [2]

- (b) Three planes have equations

$$\begin{aligned} x - 2y + 2z &= \lambda, \\ 2x + y &= 2, \\ x + 2y - z &= 0, \end{aligned}$$

where λ is a constant.

(i) Explain why the three planes intersect at a point for any value of λ . [2]

(ii) Use a matrix method to determine, in terms of λ , the coordinates of this point. [4]

5 In this question you must show detailed reasoning.

The complex number w is given by $w = -4\sqrt{2} + (4\sqrt{2})i$.

(a) (i) Find $|w|$. [2]

(ii) Find $\arg(w)$. [2]

The complex numbers z_1 and z_2 are given by $z_1 = a + i$ and $z_2 = 4(\cos \theta + i \sin \theta)$, where a is a positive real constant and $-\pi < \theta \leq \pi$.

(b) You are given that $z_1 z_2 = w$.

(i) Find the exact value of a . [3]

(ii) Find the value of θ . Give your answer as an exact multiple of π . [3]

6 (a) Express $\frac{1}{(r-1)^2} - \frac{1}{(r+1)^2}$ as a single simplified fraction. [2]

(b) Hence determine the limit which $\sum_{r=2}^n \frac{r}{(r-1)^2(r+1)^2}$ converges to as $n \rightarrow \infty$. [5]

7 The region R of the Argand diagram consists of the set of points representing complex numbers z which satisfy the following inequalities.

$$\operatorname{Im}(z) \geq 0 \qquad \arg(z+2) \leq \frac{1}{4}\pi \qquad |z| \leq |z-4-2i|$$

(a) Sketch and clearly label the region R on an Argand diagram. [4]

(b) **In this question you must show detailed reasoning.**

Find the largest value of $\operatorname{Im}(z)$ in the region R. [7]

8 The three distinct roots of the equation $z^3 - 4z^2 + pz + q = 0$, where p and q are real, are drawn on an Argand diagram. The three points which represent these roots do not lie on a straight line but instead form a triangle T.

(a) Show that T is isosceles. [3]

(b) **In this question you must show detailed reasoning.**

You are given the following information.

- The area of T is 10 square units.
- One of the roots of the equation $z^3 - 4z^2 + pz + q = 0$ is $z = -2$.

Find the other roots of the equation. [5]

END OF QUESTION PAPER

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