Question 1a

The curve C has equation $y = x^2 - 3x + 2$.

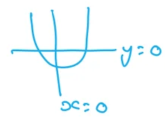
(a) Find the coordinates of any points where C intersects the coordinate axes.

[3]

(b) Sketch the graph of C, showing clearly all points of intersection with the coordinate axes.

[3]

a) $x = 0$ y intercept



$y = x^2 - 3x + 2$

$y = 2$

$\underline{(0, 2)}$

x intercept $y = 0$

$x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0$

FACTORISE + SOLVE

$x - 1 = 0 \qquad x - 2 = 0$

$x = 1 \qquad x = 2$

$$\boxed{(1, 0) \qquad (2, 0) \qquad (0, 2)}$$

Question 1b

The curve C has equation $y = x^2 - 3x + 2$.

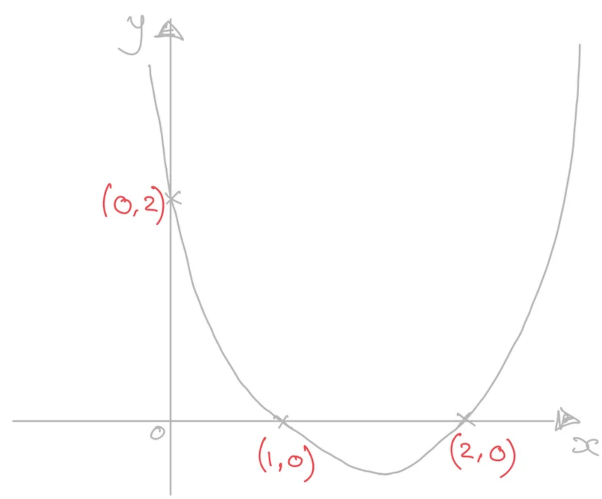
(a) Find the coordinates of any points where C intersects the coordinate axes.

[3]

(b) Sketch the graph of C, showing clearly all points of intersection with the coordinate axes.

[3]

b) $(1, 0) \quad (2, 0) \quad (0, 2)$



SMOOTH CURVE

Question 2a

(a) Write the quadratic function $y = x^2 + 8x - 9$ in the form $y = a(x + b)^2 + c$ where a, b and c are integers to be found.

COMPLETING THE SQUARE

[2]

(b) Write down the minimum point on the graph of $y = x^2 + 8x - 9$.

[1]

(c) Sketch the graph of $y = x^2 + 8x - 9$, clearly labelling the minimum point and any point where the graph intersects the coordinate axes.

[3]

a) $x^2 + \underline{8}x - 9$ $\qquad (x+b)^2 + c$

$(x+4)^2 - 4^2 - 9$

$(x+4)^2 - 16 - 9$

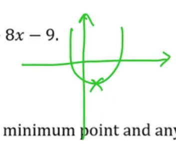
$\boxed{(x+4)^2 - 25}$

Question 2b

(a) Write the quadratic function $y = x^2 + 8x - 9$ in the form $y = a(x + b)^2 + c$ where a, b and c are integers to be found.

[2]

(b) Write down the minimum point on the graph of $y = \underline{x^2} + 8x - 9$.



[1]

(c) Sketch the graph of $y = x^2 + 8x - 9$, clearly labelling the minimum point and any point where the graph intersects the coordinate axes.

[3]

b) $(x+4)^2 - 25$ $\qquad (x+b)^2 + c$

$\boxed{(-4, -25)}$ $\qquad (-b, c)$

Question 2c

(a) Write the quadratic function $y = x^2 + 8x - 9$ in the form $y = a(x + b)^2 + c$ where a, b and c are integers to be found.

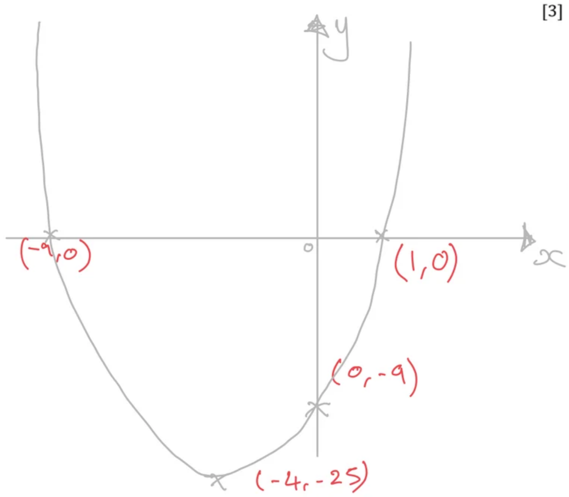
[2]

(b) Write down the minimum point on the graph of $y = x^2 + 8x - 9$.

[1]

(c) Sketch the graph of $y = x^2 + 8x - 9$, clearly labelling the minimum point and any point where the graph intersects the coordinate axes.

[3]



c) MINIMUM $\boxed{(-4, -25)}$

Y INTERCEPT $x = 0$

$y = x^2 + 8x - 9 = -9$ $\boxed{(0, -9)}$

X INTERCEPT $y = 0$

$x^2 + 8x - 9 = 0$

$(x - 1)(x + 9) = 0$

FACTORISE + SOLVE

$x = 1 \quad x = -9$

$\boxed{(1, 0)\ (-9, 0)}$

Question 3a

(a) Solve the equation $2x^2 + x - 6 = 0$.

FACTORISE
COMPLETE SQUARE
QUADRATIC FORMULA

[2]

(b) Find the coordinates of the turning point on the graph of $y = 2x^2 + x - 6$.

[3]

(c) Sketch the graph of $y = 2x^2 + x - 6$, labelling the turning point and any points where the graph crosses the coordinate axes.

[2]

a) $2x^2 + x - 6 = 0$

FACTORISE

$(2x - 3)(x + 2) = 0$

$2x - 3 = 0 \qquad x + 2 = 0$ SOLVE

$\boxed{x = \frac{3}{2} \qquad x = -2}$

Question 3b

(a) Solve the equation $2x^2 + x - 6 = 0$. [2]

(b) Find the coordinates of the turning point on the graph of $y = 2x^2 + x - 6$. [3]

(c) Sketch the graph of $y = 2x^2 + x - 6$, labelling the turning point and any points where the graph crosses the coordinate axes. [2]

b) $2x^2 + x - 6$

COMPLETING THE SQUARE

$a(x+b)^2 + c$

$2\left[x^2 + \frac{1}{2}x\right] - 6$

$2\left[\left(x+\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2\right] - 6$

$2\left[\left(x+\frac{1}{4}\right)^2 - \frac{1}{16}\right] - 6$

$2\left(x+\frac{1}{4}\right)^2 - \frac{1}{8} - 6$

$2\left(x+\frac{1}{4}\right)^2 - \frac{49}{8}$

TURNING POINT

$(-b, c)$

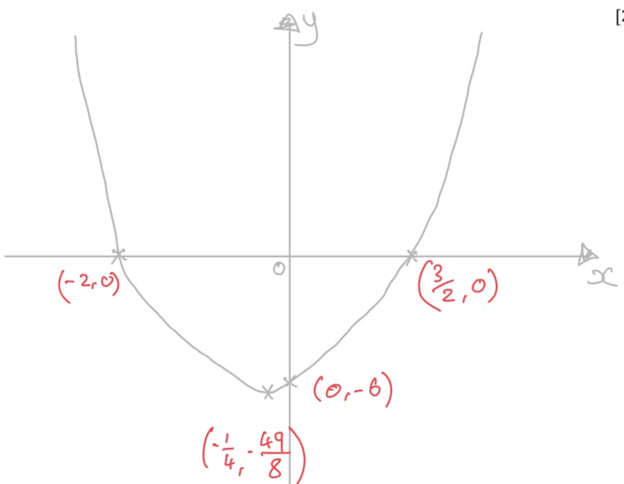
$\boxed{\left(-\frac{1}{4}, -\frac{49}{8}\right)}$

Question 3c

(a) Solve the equation $2x^2 + x - 6 = 0$. [2]

(b) Find the coordinates of the turning point on the graph of $y = 2x^2 + x - 6$. [3]

(c) Sketch the graph of $y = 2x^2 + x - 6$, labelling the turning point and any points where the graph crosses the coordinate axes. [2]



c) TURNING POINT $\boxed{\left(-\frac{1}{4}, -\frac{49}{8}\right)}$

Y INTERCEPT $x = 0$

$y = 2x^2 + x - 6 = -6$ $\boxed{(0, -6)}$

X INTERCEPT $y = 0$

$x = \frac{3}{2}$ $x = -2$

$\boxed{\left(\frac{3}{2}, 0\right) \quad (-2, 0)}$

Question 4a

(a) Find the minimum value of the function $f(x) = x^2 + 4x + 5$.

[3]

(b) Hence, or otherwise, prove that the function $f(x) = x^2 + 4x + 5$ has no real roots.

[2]

a) $x^2 + 4x + 5$

COMPLETING THE SQUARE

$(x+2)^2 - 2^2 + 5$

$a(x+b)^2 + c$

$(x+2)^2 + 1$

MINIMUM POINT

$(-b, c)$

$\boxed{(-2, 1)}$

Question 4b

(a) Find the minimum value of the function $f(x) = x^2 + 4x + 5$.

[3]

(b) Hence, or otherwise, prove that the function $f(x) = x^2 + 4x + 5$ has no real roots.

[2]

DISCRIMINANT

$b^2 - 4ac < 0$

NO REAL ROOTS

b) $x^2 + 4x + 5$

$a=1 \quad b=4 \quad c=5$

$b^2 - 4ac < 0$

$4^2 - 4 \times 1 \times 5 = 16 - 20$

$\boxed{-4 < 0 \quad \therefore \text{NO REAL ROOTS}}$

OR

$(x+2)^2 + 1$

COMPLETED SQUARE

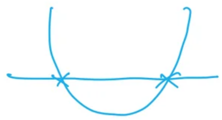
$(x+2)^2 \geq 0$ FOR ALL VALUES OF x

$\boxed{(x+2)^2 + 1 \geq 0 \quad \therefore \text{NO REAL ROOTS}}$

Question 5

The function $f(x) = kx^2 + 2kx - 3$ has two distinct real roots.
Show that $k < -3$ or $k > 0$.

[3]



DISCRIMINANT

$kx^2 + 2kx - 3$

$a = k \quad b = 2k \quad c = -3$

$b^2 - 4ac \geq 0$

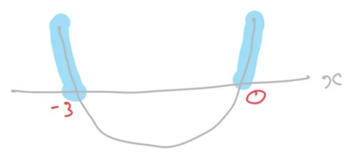
2 DISTINCT REAL ROOTS

$(2k)^2 - 4(k)(-3) \geq 0$

$4k^2 + 12k \geq 0$

$4k(k+3) \geq 0$

$k = 0 \quad k = -3$



$\boxed{k < -3 \quad k > 0}$

Question 6

The equation $2x^2 - 4x + 3 - 2k = 0$ has real roots.
Find the possible values of k.

[3]

$2x^2 - 4x + 3 - 2k = 0$ DISCRIMINANT

$a = 2 \quad b = -4 \quad c = 3 - 2k \quad b^2 - 4ac \geq 0$

$(-4)^2 - 4(2)(3 - 2k) \geq 0$

EXPAND

$16 - 4(6 - 4k) \geq 0$

$16 - 24 + 16k \geq 0$

SOLVE

$-8 + 16k \geq 0$

$16k \geq 8$

$k \geq \frac{8}{16} = \frac{1}{2}$

$\boxed{k \geq \frac{1}{2}}$

Question 7

The equation $y = x^2 + px + q$ has no real roots. Show that $p^2 < 4q$.

[2]

$x^2 + px + q$

$a = 1 \quad b = p \quad c = q$

DISCRIMINANT

$b^2 - 4ac < 0$

NO REAL ROOTS

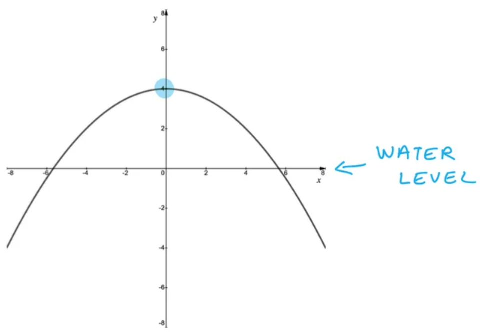
$p^2 - 4q < 0$

$$\boxed{p^2 < 4q}$$

Question 8a

The graph below shows the curve $f(x) = 4 - \frac{x^2}{8}$.

The curve is to be used as the model for the arch on a bridge where the water level under the bridge is represented by the x-axis. All measurements are in meters.



(a) Write down the maximum height of the bridge above the water.

[1]

a) $\boxed{4 \text{ m}}$

(b) Show that the bridge is wide enough to span a river of width 11m.

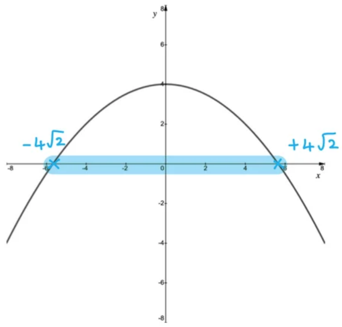
[3]

EXAM PAPERS PRACTICE

Question 8b

The graph below shows the curve $f(x) = 4 - \frac{x^2}{8}$.

The curve is to be used as the model for the arch on a bridge where the water level under the bridge is represented by the x-axis. All measurements are in meters.



(a) Write down the maximum height of the bridge above the water.

[1]

(b) Show that the bridge is wide enough to span a river of width 11m.

[3]

b) FIND WIDTH BY FINDING SOLUTIONS FOR x

x AXIS SOLUTIONS WHERE $y = 0$

SOLVE $4 - \frac{x^2}{8} = 0$

$4 = \frac{x^2}{8}$ OR USE CALCULATOR

$\frac{x^2}{8} = 4$

$\times 8$ $x^2 = 32$

$\sqrt{\ }$ $x = \pm\sqrt{32} = \pm 4\sqrt{2}$

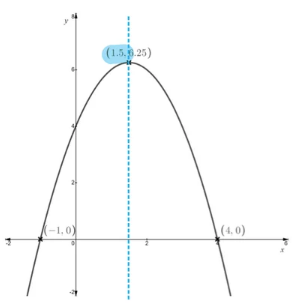
WIDTH $= 2 \times 4\sqrt{2} = 8\sqrt{2} = 11.3137...$

$8\sqrt{2} > 11$

BRIDGE SPANS RIVER

Question 9a

The diagram below shows the graph of $y = f(x)$, where $f(x)$ is a quadratic function. The intercepts with the x-axis and the turning point have been labelled.



(a) Write down the equation of the axis of symmetry for the graph of $y = f(x)$.

[1]

(b) The function $f(x)$ can be written in the form of $f(x) = a(x-h)^2 + k$.
Find the values of a, h and k.

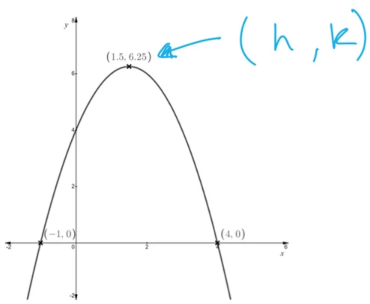
[3]

a) AXIS OF SYMMETRY AT x-COORDINATE OF TURNING POINT $(1.5, 6.25)$

$x = 1.5$

Question 9b

The diagram below shows the graph of $y = f(x)$, where $f(x)$ is a quadratic function.
The intercepts with the x-axis and the turning point have been labelled.



(a) Write down the equation of the axis of symmetry for the graph of $y = f(x)$.

[1]

(b) The function $f(x)$ can be written in the form of $f(x) = a(x - h)^2 + k$.
Find the values of a, h and k.

VERTEX FORM

[3]

b) USING $a(x-h)^2+k$ WHERE (h,k) = TURNING POINT

$h = 1.5 \quad k = 6.25$

$a(x-1.5)^2 + 6.25 = 0$

USE $x = -1$ OR $x = 4$

$a(4-1.5)^2 = -6.25$

$6.25a = -6.25$

$a = -1$

$$\boxed{a = -1, \; h = 1.5, \; k = 6.25}$$

Question 10

Solve the equation $x^4 - 13x^2 + 36 = 0$.

$ax^2 + bx + c = 0$

[3]

$f(x) = x^2$

$x^4 - 13x^2 + 36$

$(x^2)^2 - 13(x^2) + 36 = 0$

let $y = x^2$

$y^2 - 13y + 36 = 0$

FACTORISE $(y-9)(y-4) = 0$

SOLVE $y = 9 \quad y = 4$

$x^2 = 9 \quad x^2 = 4$

$\sqrt{}$

$x = \pm 3 \quad x = \pm 2$

$$\boxed{x = -3, -2, 2, 3}$$

Question 11

Solve $x^{\frac{2}{5}} + x^{\frac{1}{5}} = 6$.

[4]

$ax^2+bx+c=0$

$x^{\frac{2}{5}} + x^{\frac{1}{5}} - 6 = 0$

$f(x) = x^{1/5}$

$(x^{1/5})^2 + (x^{1/5}) - 6 = 0$

let $y = x^{1/5}$

$y^2 + y - 6 = 0$

FACTORISE $(y-2)(y+3) = 0$

$y = 2 \quad y = -3$

$x^{\frac{1}{5}} = 2 \quad x^{1/5} = -3$

$x = 2^5 = 32 \quad x = (-3)^5 = -243$

$x^{\frac{1}{2}} = \sqrt{}$

$x^{1/5} = \sqrt[5]{}$

$\boxed{x = 32, -243}$

Question 12a

Let $f(x) = 2px^2 + (2p-5)x + p - \frac{5}{2}$, for $x \in \mathbb{R}$, where $p \in \mathbb{Q}$.

REAL RATIONAL

(a) Show that the discriminant of f is $-4p^2 + 25$.

[3]

(b) Find the values of p so that the function $f(x)$ has two **distinct** roots.

[3]

a) DISCRIMINANT $b^2 - 4ac$

$a = 2p \quad b = (2p-5) \quad c = p - \frac{5}{2}$

SUB IN AND SIMPLIFY

$(2p-5)^2 - 4(2p)(p - \frac{5}{2})$

$(2p-5)(2p-5) - 4(2p^2 - 5p)$

$4p^2 - \cancel{20p} + 25 - 8p^2 + \cancel{20p}$

$-4p^2 + 25$ AS REQUIRED

DISCRIMINANT $= -4p^2 + 25$

EXAM PAPERS PRACTICE

Question 12b

Let $f(x) = 2px^2 + (2p-5)x + p - \frac{5}{2}$, for $x \in \mathbb{R}$, where $p \in \mathbb{Q}$.

(a) Show that the discriminant of f is $-4p^2 + 25$. [3]

(b) Find the values of p so that the function $f(x)$ has two **distinct** roots. [3]

b) TWO DISTINCT ROOTS WHEN DISCRIMINANT

$b^2 - 4ac > 0$

$-4p^2 + 25 > 0$

$+4p^2$ TO AVOID NEED TO SWITCH SIGN IF $\div(-p)$

$25 > 4p^2$

$\div 4$

$\frac{25}{4} > p^2$

$\sqrt{}$

$\pm\frac{5}{2} > p$

$$\boxed{-\frac{5}{2} < p < \frac{5}{2}}$$