EXAM PAPERS PRACTICE

Quadratic Equations

Model Answers

Question 1

Solve the equation $3x^2 - 11x + 4 = 0$.
Show all your working and give your answers correct to 2 decimal places.

Answer:

We can solve the equation using the quadratic formula, which is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substituting the values of a, b, and c into the formula, we get:

$$x = \frac{11 \pm \sqrt{(-11)^2 - 4x3x4}}{2(3)}$$

So, the solutions are:

$$x = \frac{11 \pm \sqrt{73}}{6}$$

$$x_1 = \frac{-(-11) + \sqrt{73}}{2(3)} \approx 3.26$$

$$x_2 = \frac{-(-11) - \sqrt{73}}{2(3)} \approx 0.41$$

Question 2

Solve the equation $5x^2 - 6x - 3 = 0$.
Show all your working and give your answers correct to 2 decimal places.

Answer:

First, we need to use the quadratic formula to solve this equation. The quadratic formula is given by: x = [-b ± sqrt(b² - 4ac)] / (2a) In this equation, a = 5, b = -6, and c = -3.

*First, calculate the value under the square root, which is b² - 4ac: (-6)² - 4*5*(-3) = 36 + 60 = 96 Then, calculate the values of x: x = [6 ± sqrt(96)] / (2*5) x = [6 ± 9.8] / 10 So, the solutions are: x = (6 + 9.8) / 10 = 1.58 and x = (6 - 9.8) / 10 = -0.38 So, the solutions to the equation 5x² - 6x - 3 = 0 are x = 1.58 and x = -0.38, correct to two decimal places.*

Question 3

Solve the equation $3x^2 + 4x – 5 = 0$.
Show all your working and give your answers correct to 2 decimal places

Answer:

This gives us two solutions: x = (-4 + 8.72) / 6 = 0.79 (rounded to two decimal places) x = (-4 - 8.72) / 6 = -2.12 (rounded to two decimal places)

So, the solutions to the equation $3x^2 + 4x – 5 = 0$ are x = 0.79 and x = -2.12, to two decimal places.

Question 4

$$f(x) = x^2 + 4x - 6$$

(a) f(x) can be written in the form $(x + m)^2 + n$.
Find the value of m and the value of n.

Answer:

The general form for a quadratic equation is $ax^2 + bx + c$. In this case, a = 1, b = 4, and c = -6. To complete the square, we take half of the coefficient of x (which is b/2a = 4/2 = 2) and square it (which gives us $2^2 = 4$). We then add and subtract this value inside the equation to get:

$f(x) = (x^2 + 4x + 4) - 4 - 6$ This simplifies to: $f(x) = (x + 2)^2 - 10$ So, m = 2 and n = -10.

(b) Use your answer to part (a) to find the positive solution to $x^2 + 4x – 6 = 0$.

Answer:

The positive solution is $x = -2 + \sqrt{10} \approx 1.16$.

Question 5

Solve the equation.

$$2x^2 + x - 2 = 0$$

Show your working and give your answers correct to 2 decimal places.

Answer:

Calculate the value under the square root, which is $b^2 - 4ac = (1)^2 - 4*2*(-2) = 1 + 16 = 17$. Then, we substitute these values into the quadratic formula to get $x = [-1 \pm \sqrt{17}] / 4$.

So, the solutions are $x = [-1 + \sqrt{17}] / 4$ and $x = [-1 - \sqrt{17}] / 4$. Using a calculator, we find that $\sqrt{17}$ is approximately 4.12. Substituting this value into the equations, we get $x = [-1 + 4.12] / 4 = 0.78$ and $x = [-1 - 4.12] / 4 = -1.28$. So, the solutions to the equation $2x^2 + x - 2 = 0$ are $x = 0.78$ and $x = -1.28$, correct to two decimal places.

Question 6

Use the quadratic equation formula to solve

$$2x^2 + 7x - 3 = 0 .$$

Show all your working and give your answers correct to 2 decimal places.

Answer:

Identify the coefficients in the equation. In this case, $a = 2$, $b = 7$, and $c = -3$.

Use the quadratic formula, which is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute the coefficients into the formula

So, the solutions to the equation $2x^2 + 7x - 3 = 0$ are $x = 0.39$ and $x = -3.89$.

Question 7

Solve the equation $2x^2+6x-3=0$

Show your working and give your answers correct to 2 decimal places

Answer:

First, we can simplify the equation by dividing all terms by 2. This gives us:

$$x^2 + 3x - 1.5 = 0$$

Next, we can use the quadratic formula to solve for x

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

.

So the solutions are: x = [-3 + sqrt(15)] / 2 = 0.58 (rounded to two decimal places) x = [-3 - sqrt(15)] / 2 = -2.58 (rounded to two decimal places) So, the solutions to the equation $2x^2+6x-3=0$ are x = 0.58 and x = -2.58, correct to two decimal places.

Question 8

Solve the equations

(a) $\frac{2x}{3} - 9 = 0,$

Answer:

First, we can add 9 to both sides of the equation to isolate the fraction on one side. This gives us:

$$\frac{2x}{3} = 9$$

Next, we can multiply both sides of the equation by 3 to get rid of the denominator on the left side. This gives us: 2x = 27

Finally, we can divide both sides of the equation by 2 to solve for x:

$$x = \frac{27}{2} = 13.5$$

(b) $x^2 - 3x - 4 = 0$

Answer:

This is a quadratic equation, which has the general form $ax^2 + bx + c = 0$. In this case, $a = 1$, $b = -3$, and $c = -4$. The quadratic formula is used to solve this type of equation, and it is given by: $x = [-b \pm sqrt(b^2 - 4ac)] / 2a$ Substituting the values of a, b, and c into the formula, we get: $x = [3 \pm sqrt((-3)^2 - 4*1*(-4))] / 2*1$ $x = [3 \pm sqrt(9 + 16)] / 2$ $x = [3 \pm sqrt(25)] / 2$ $x = [3 \pm 5] / 2$

This gives us two solutions: $x = (3 + 5) / 2 = 4$ $x = (3 - 5) / 2 = -1$

So, the solutions to the equation $x^2 - 3x - 4 = 0$ are $x = 4$ and $x = -1$.

Question 9

Solve the equation

$$x^2 + 4x - 22 = 0$$

Give your answers correct to 2 decimal places.
Show all your working.

Answer:

The solutions to the equation $x^2 + 4x - 22 = 0$ are $x = 3.10$ and $x = -7.10$, correct to 2 decimal places.

Question 10

Solve the equation $5x^2 + 10x + 2 = 0$.
You must show all your working and give your answers correct to 2 decimal places.

Answer:

The solutions to the equation $5x^2 + 10x + 2 = 0$ are $x = -0.23$ and $x = -1.77$, correct to two decimal places.

Question 11

Solve the equation $2x^2 + 3x - 3 = 0$.
Show all your working and give your answers correct to 2 decimal places.

Answer:

The solutions to the equation $2x^2 + 3x - 3 = 0$ are $x \approx 0.42$ and $x \approx -3.42$, correct to two decimal places.

Question 12

Solve $(x – 7)(x + 4) = 0$

Answer:

First, we set each factor equal to zero and solve for x. $x - 7 = 0$ $x = 7$
$x + 4 = 0$ $x = -4$

So, the solutions to the equation $(x – 7)(x + 4) = 0$ are $x = 7$ and $x = -4$.

Question 13

a. Factorise $3x^2 + 2x – 8$

Answer:

First, we need to find two numbers that multiply to $(3*-8) = -24$ and add to 2. Those numbers are 4 and -6.
Next, we rewrite the middle term of the quadratic as the sum of the terms 4x and -6x. So, $3x^2 + 2x – 8$ becomes $3x^2 + 4x - 6x - 8$. Now, we can factor by grouping. The first two terms have a common factor of x, and the last two terms have a common factor of -2. So, $3x^2 + 4x - 6x - 8$ becomes $x(3x + 4) - 2(3x + 4)$

Finally, we can factor out the common binomial term $(3x + 4)$ to get the final factored form of the quadratic: $(3x + 4)(x - 2)$.

(b) Solve the equation 3x2 + 2x – 8 =0.

Answer:

*First, we need to find the discriminant (D) of the quadratic equation. The discriminant is found using the formula $D = b^2 - 4ac$, where a, b, and c are the coefficients of the quadratic equation. In this case, a = 3, b = 2, and c = -8. So, $D = (2)^2 - 4*(3)*(-8) = 4 + 96 = 100$. Since the discriminant is positive, we have two distinct real roots for the equation. The roots can be found using the formula x = [-b ± sqrt(D)] / 2a. So, the roots are x = [-2 ± sqrt(100)] / 2*3 = [-2 ± 10] / 6.*

Therefore, the roots of the equation $3x^2 + 2x - 8 = 0$ are x = 8/6 = 4/3 and x = -12/6 = -2.

Question 14

The solutions of the equation $x^2 – 6x + d = 0$ are both integers.

d is a prime number.
Find d.

Answer:

The solutions of the equation $x^2 – 6x + d = 0$ are given by the quadratic formula x = [6 ± sqrt(36 - 4d)] / 2. For the solutions to be integers, the discriminant (the part under the square root) must be a perfect square. So, $36 - 4d = n^2$ for some integer n. Rearranging, we get $4d = 36 - n^2$. Since d is a prime number, it must be odd (except for 2).

Therefore, $36 - n^2$ must be a multiple of 4. The only perfect squares less than 36 that are multiples of 4 are 0, 4, 16, and 36. If $n^2 = 0$, then d = 9, which is not prime. If $n^2 = 4$, then d = 8, which is not prime. If $n^2 = 16$, then d = 5, which is prime. If $n^2 = 36$, then d = 0, which is not prime. So, the only possible value for d is 5.

Question 15

Solve the equation $2x^2 + 3x – 6 = 0$
Show all your working and give your answers correct to 2 decimal places.

Answer:

*First, we can use the quadratic formula to solve this equation. The quadratic formula is given by: x = [-b ± sqrt(b² - 4ac)] / 2a where a, b, and c are coefficients in the quadratic equation ax² + bx + c = 0. In this case, a = 2, b = 3, and c = -6. So, we can substitute these values into the quadratic formula: x = [-3 ± sqrt((3)² - 4*2*(-6))] / 2*2*

x = [-3 ± sqrt(9 + 48)] / 4 x = [-3 ± sqrt(57)] / 4 Now, we can calculate the two possible values for x: x_1 = (-3 + sqrt(57)) / 4 ≈ 1.18 x_2 = (-3 - sqrt(57)) / 4 ≈ -2.68 So, the solutions to the equation 2x² + 3x – 6 = 0 are x ≈ 1.18 and x ≈ -2.68, to two decimal places.

Question 16

Solve the equation.

$x^2 – 8x + 6 = 0$

Show all your working and give your answers correct to 2 decimal places.

Answer:

*First, we can rewrite the equation in the form of a quadratic equation ax² + bx + c = 0, where a = 1, b = -8, and c = 6. The solutions to a quadratic equation are given by the formula x = [-b ± sqrt(b² - 4ac)] / 2a. Substituting the values of a, b, and c into the formula, we get: x = [8 ± sqrt((-8)² - 4*1*6)] / 2*1 x = [8 ± sqrt(64 - 24)] / 2 x = [8 ± sqrt(40)] / 2 x = [8 ± 2*sqrt(10)] / 2 x = 4 ± sqrt(10) So, the solutions to the equation are x = 4 + sqrt(10) and x = 4 - sqrt(10). To get the solutions to 2 decimal places, we can use a calculator to find the approximate values of these expressions. sqrt(10) is approximately 3.16. So, x = 4 + 3.16 = 7.16 and x = 4 - 3.16 = 0.84.*

Therefore, the solutions to the equation x² – 8x + 6 = 0 to 2 decimal places are x = 7.16 and x = 0.84.