

Please write clearly in block capitals.

Centre number

Candidate number

Surname

Forename(s)

Candidate signature

I declare this is my own work.

INTERNATIONAL AS

FURTHER MATHEMATICS

(9665/FM01) Unit FP1 Pure Mathematics

Wednesday 7 January 2026

07:00 UK Time

Time allowed: 1 hour 30 minutes

Materials

- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

| For Examiner's Use |      |
|--------------------|------|
| Question           | Mark |
| 1                  |      |
| 2                  |      |
| 3                  |      |
| 4                  |      |
| 5                  |      |
| 6                  |      |
| 7                  |      |
| 8                  |      |
| 9                  |      |
| 10                 |      |
| TOTAL              |      |

Answer **all** questions in the spaces provided.

**1** The complex number  $z$  is given by

$$z = -\sqrt{3} - i$$

**1 (a) (i)** Express  $z$  in the form  $r(\cos\theta + i\sin\theta)$  where  $r \geq 0$  and  $-\pi < \theta \leq \pi$

**[3 marks]**

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Answer \_\_\_\_\_

**1 (a) (ii)** Hence write down  $z^*$  in the form  $r(\cos\theta + i\sin\theta)$  where  $r \geq 0$  and  $-\pi < \theta \leq \pi$

**[1 mark]**

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Answer \_\_\_\_\_



- 1 (b)** On an Argand diagram, the point  $P$  represents  $z$  and the point  $Q$  represents  $z^*$

Find the acute angle  $POQ$  where  $O$  is the origin.

**[2 marks]**

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Answer \_\_\_\_\_

6

**Turn over for the next question**

**Turn over ►**



**2**

$$y = 8 - 3x - 6x^2$$

**2**

Find the gradient of  $L$  in the form  $p + qh$  where  $p$  and  $q$  are constants.

**[3 marks]**

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Answer



**2 (b)** Use your answer to **part (a)** to find the gradient of  $C$  at the point where  $x = 7$

Show the limiting process used.

**[2 marks]**

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Answer

      
**5**

**Turn over for the next question**

**Turn over ►**



**3 (a)** Show that

$$\frac{1}{3r-1} - \frac{1}{3r+2} = \frac{3}{9r^2+3r-2}$$

**[1 mark]**

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**3 (b)** Hence use the method of differences to show that

$$\sum_{r=1}^n \frac{3}{9r^2+3r-2}$$

can be written in the form  $\frac{3n}{cn+d}$  where  $c$  and  $d$  are integers.

**[4 marks]**

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- 3 (c) Find the value of  $p$  such that

$$\sum_{r=p}^{\infty} \frac{3}{9r^2 + 3r - 2} = \frac{1}{1001}$$

[3 marks]

$p =$  \_\_\_\_\_

Turn over ►



**4** The integral  $I_k$  is defined by

$$I_k = \int_k^5 \left( \frac{1}{x^2} \right) dx$$

where  $k < 5$

**4 (a)** Write down the range of values of  $k$  for which  $I_k$  is an improper integral.

**[1 mark]**

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Answer 

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**4 (b)** Explain why  $I_{-3}$  does not have a finite value.

**[1 mark]**

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$$z^2 = 4 - 4\sqrt{15}i$$

**[5 marks]**

[illegible]

Answer

5

**[1 mark]**

Answer

**[4 marks]**



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Answer \_\_\_\_\_

- 6 (c)** Use your answer to **part (b)** to find the sum of the cubes of all positive odd numbers less than 100

**[2 marks]**

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Answer \_\_\_\_\_

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Turn over ►



7 The equation

$$\cos\left(3x + \frac{\pi}{4}\right) = \frac{1}{2}$$

has the general solution

$$x = \frac{\pi}{36} (24n \pm a + b)$$

where  $a$  and  $b$  are **positive** integers.

7 (a) A student claims that  $n$  can be any real number. This student's claim is incorrect.

Write down a correct statement about the values  $n$  can take.

[1 mark]

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7 (b) In the case when  $a = 4$  find the least possible value of  $b$

[3 marks]

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$b =$  \_\_\_\_\_

7 (c) In the case when  $b = 9$  find the least possible value of  $a$

[3 marks]

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$a =$  \_\_\_\_\_



- 7 (d) Find the number of solutions of the equation

$$\cos\left(3x + \frac{\pi}{4}\right) = \frac{1}{2}$$

which lie in the interval  $-\pi < x \leq \pi$

[2 marks]

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Answer \_\_\_\_\_

- 7 (e) Hence, or otherwise, find the number of solutions of the equation

$$\cos\left(3x + \frac{\pi}{4}\right) = \frac{1}{2}$$

which lie in the interval  $\theta < x \leq \theta + 80\pi$  where  $\theta$  is a constant.

[2 marks]

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Answer \_\_\_\_\_



8

The line  $L$  has equation  $x = \frac{2}{k}$

The locus of a point  $R$  is such that the distance from  $R$  to  $S$  is equal to  $k$  times the distance from  $R$  to  $L$

The locus of  $R$  is the hyperbola  $H$

**8 (a)**

Give your answer in the form

$$px^2 - y^2 = q$$

where  $p$  and  $q$  are constants in terms of  $k$

**[4 marks]**

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Answer \_\_\_\_\_

**8 (b)** The asymptotes of  $H$  are perpendicular to each other.

**8 (b) (i)** Find the value of  $k$

**[3 marks]**

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$k =$  \_\_\_\_\_

**Question 8 continues on the next page**

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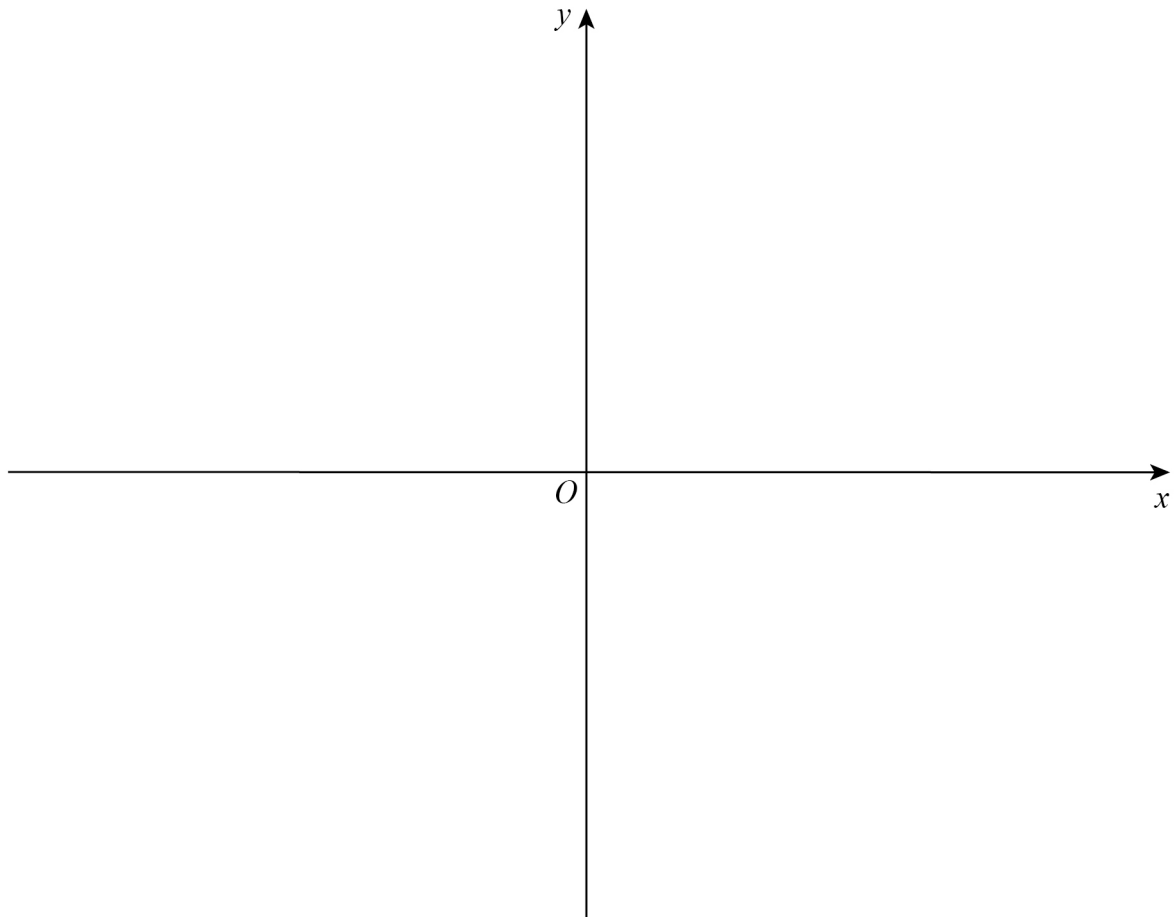
**8 (b) (ii)** Sketch the graph of  $H$  on **Figure 1**

Include the coordinates of any axis intercepts.

Include the asymptotes and their equations.

**[3 marks]**

**Figure 1**



10



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9 The function  $f$  is defined by  $f(x) = \frac{x^2 + 2}{x^2 + 3x}$

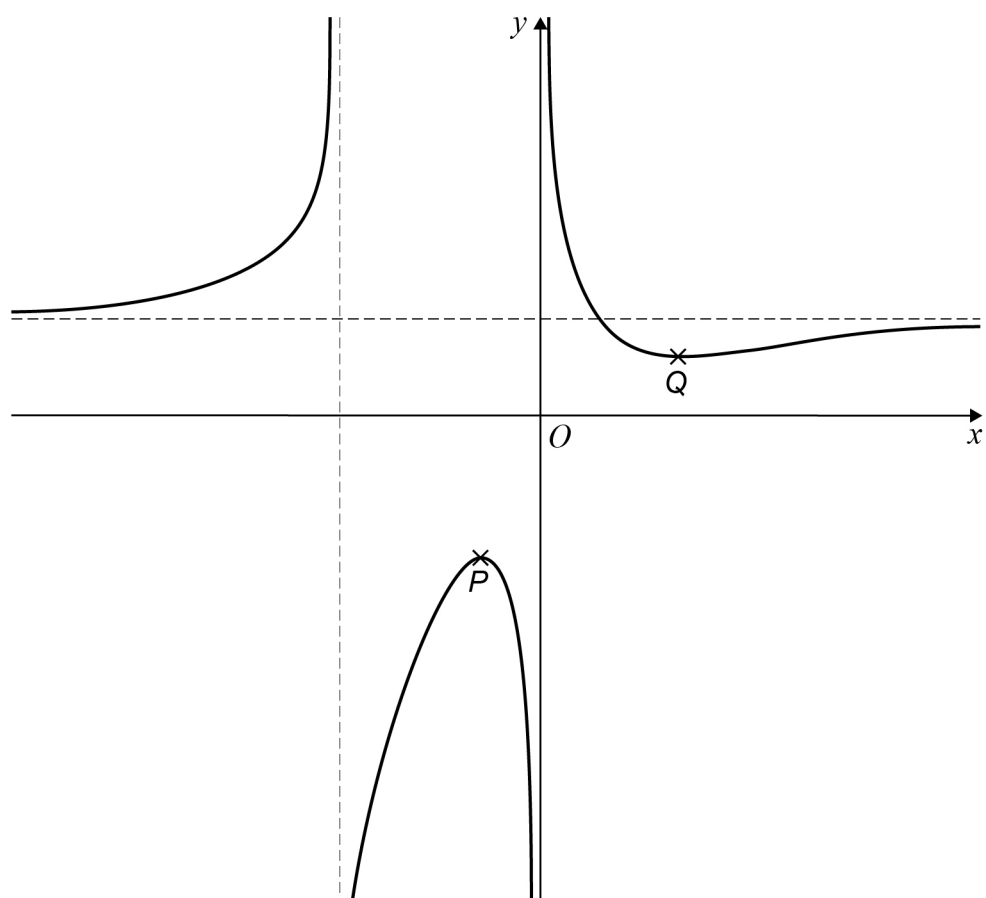
The curve  $C_1$  has equation  $y = f(x)$

The point  $P$  is the local maximum of  $C_1$

The point  $Q$  is the local minimum of  $C_1$

The graph of  $C_1$  is shown in **Figure 2**

**Figure 2**



**[4 marks]**

[illegible]

**Turn over ►**



- 9 (b)** Use your answer to **part (a)** to find the  $y$ -coordinate of  $P$  and the  $y$ -coordinate of  $Q$ .  
Give your answers in an exact form.

**[2 marks]**


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$y$ -coordinate of  $P =$  \_\_\_\_\_

$y$ -coordinate of  $Q =$  \_\_\_\_\_

- 9 (c)** The curve  $C_2$  has equation  $y = \frac{1}{f(x)}$

- 9 (c) (i)** Write down the number of asymptotes of  $C_2$

**[1 mark]**


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Answer \_\_\_\_\_

- 9 (c) (ii)** Write down the  $y$ -coordinate of the point of intersection of  $C_1$  and  $C_2$

**[1 mark]**


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Answer \_\_\_\_\_

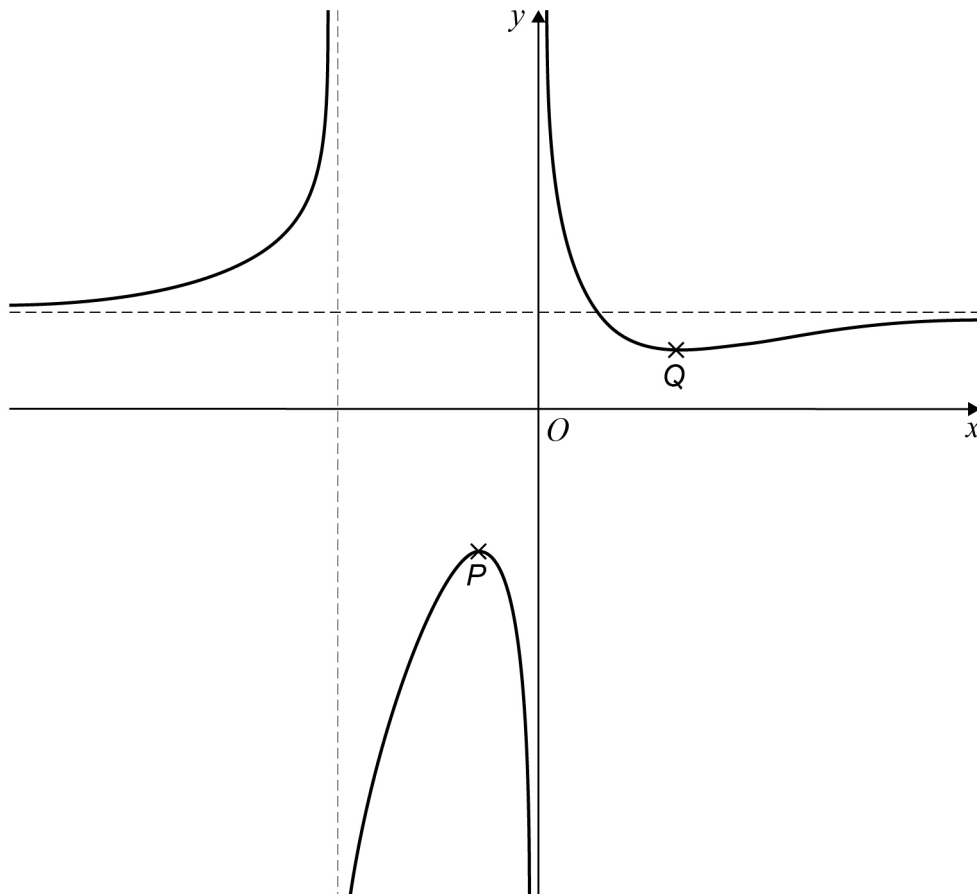


9 (c) (iii) The graph of  $C_1$  is shown again in **Figure 3**

Sketch the graph of  $C_2$  on **Figure 3**

[4 marks]

**Figure 3**



12

Turn over for the next question

Turn over ►



- 10** The complex number  $4 - 5i$  is a root of the quadratic equation

$$z^2 + mz + n = 0$$

where  $m$  and  $n$  are real constants.

- 10 (a)** Find the value of  $m$  and the value of  $n$

**[4 marks]**

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$m =$  \_\_\_\_\_  $n =$  \_\_\_\_\_

- 10 (b)** Solve the equation

$$z^2 + 6z + 153 = 0$$

**[1 mark]**

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Answer \_\_\_\_\_



- 10 (c)** The four roots of the equation

$$(z^2 + mz + n)(z^2 + 6z + 153) = 0$$

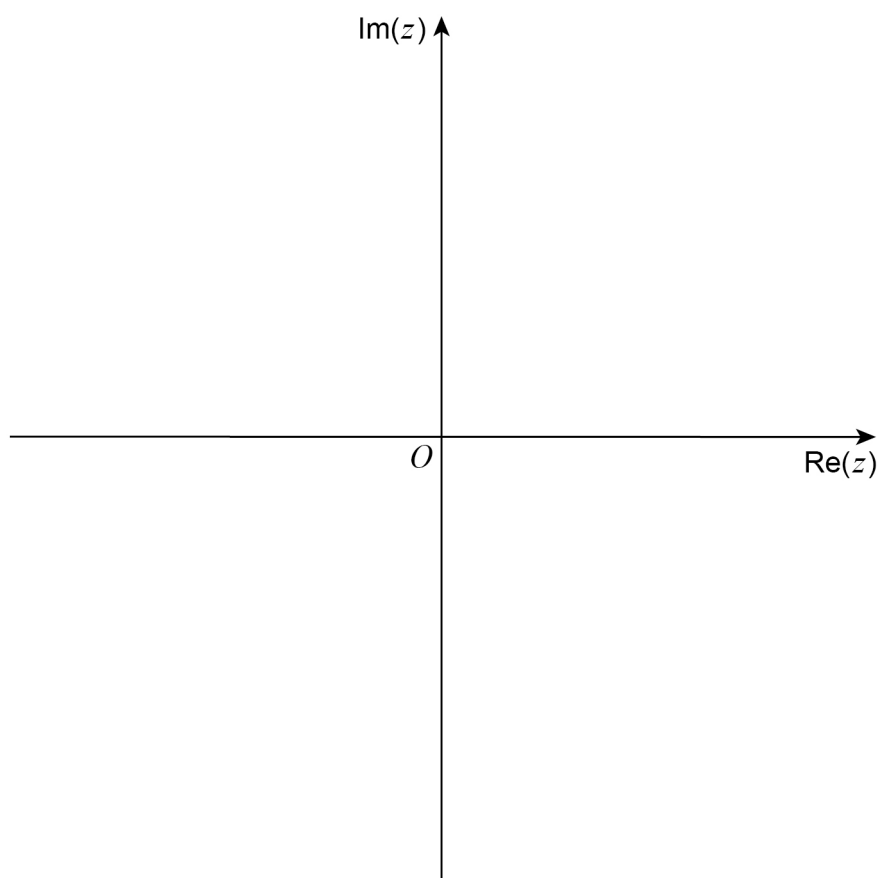
are represented by the points  $P$ ,  $Q$ ,  $R$  and  $S$  on an Argand diagram.

- 10 (c) (i)** Sketch the quadrilateral  $PQRS$  on the Argand diagram in **Figure 4**

You are not required to label the vertices or include the axis intercepts.

**[2 marks]**

**Figure 4**



- 10 (c) (ii)** Calculate the area of quadrilateral  $PQRS$

**[2 marks]**

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Answer \_\_\_\_\_

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**Turn over ►**



**10 (d)** The four roots of the equation

$$(z^2 + mz + n)(z^2 + 6z + 153) = 0$$

also satisfy the equation

$$|z - \alpha| = \beta$$

where  $\alpha$  and  $\beta$  are constants.

**10 (d) (i)** Find the value of  $\alpha$

**[3 marks]**

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$\alpha =$  \_\_\_\_\_



**10 (d) (ii)** Find the value of  $\beta$

**[2 marks]**

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$\beta =$  \_\_\_\_\_

**14**

**END OF QUESTIONS**



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[illegible]