1

(i) Express $x^2 - 5x + 6$ in the form $(x - a)^2 - b$. Hence state the coordinates of the turning point of the curve $y = x^2 - 5x + 6$.

[4]

(ii) Find the coordinates of the intersections of the curve $y = x^2 - 5x + 6$ with the axes and sketch this curve.

[4]

(iii) Solve the simultaneous equations $y = x^2 - 5x + 6$ and $x + y = 2$. Hence show that the line $x + y = 2$ is a tangent to the curve $y = x^2 - 5x + 6$ at one of the points where the curve intersects the axes.

[4]

2 Rearrange the equation $5c + 9t = a(2c + t)$ to make c the subject.

[4]

3

(i) Express $\sqrt{48} + \sqrt{75}$ in the form $a\sqrt{b}$, where a and b are integers.

[2]

(ii) Simplify $\dfrac{7 + 2\sqrt{5}}{7 + \sqrt{5}}$, expressing your answer in the form $\dfrac{a + b\sqrt{5}}{c}$, where a, b and c are integers.

[3]

4 Solve the inequality $5x^2 - 28x - 12 \leqslant 0$.

[4]

5 A circle has diameter d, circumference C, and area A. Starting with the standard formulae for a circle, show that $Cd = kA$, finding the numerical value of k.

[3]

6 Simplify $\dfrac{(4x^5y)^3}{(2xy^2) \times (8x^{10}y^4)}$.

[3]

7 Find the value of each of the following.

(i) $\left(\frac{5}{3}\right)^{-2}$

[2]

(ii) $81^{\frac{3}{4}}$

[2]

8

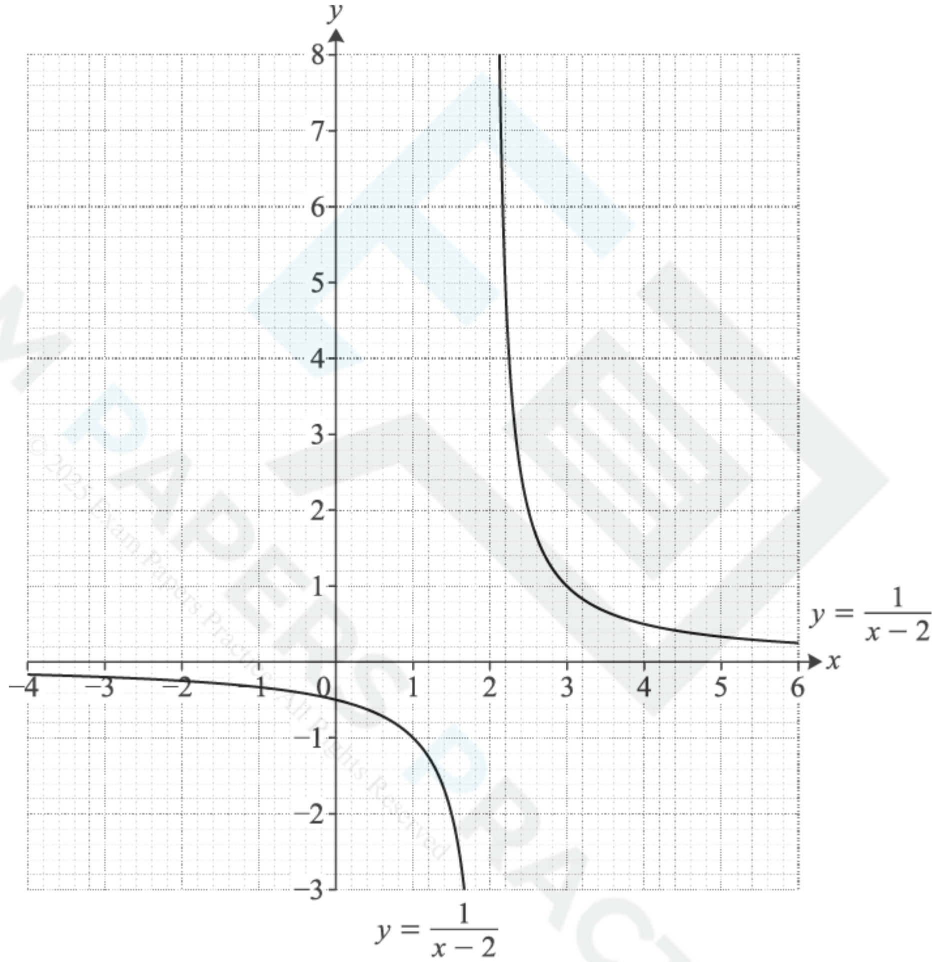


Fig. 12

Fig. 12 shows the graph of $y = \frac{1}{x-2}$.

(i) Draw accurately the graph of $y = 2x + 3$ on the copy of Fig. 12 and use it to estimate the coordinates of the points of intersection of $y = \frac{1}{x-2}$ and $y = 2x + 3$.

[3]

(ii) Show algebraically that the x-coordinates of the points of intersection of $y = \frac{1}{x-2}$ and $y = 2x + 3$ satisfy the equation $2x^2 - x - 7 = 0$. Hence find the exact values of the x-coordinates of the points of intersection.

[5]

(iii) Find the quadratic equation satisfied by the x-coordinates of the points of intersection of $y = \frac{1}{x-2}$ and $y = -x + k$. Hence find the exact values of k for which $y = -x + k$ is a tangent to $y = \frac{1}{x-2}$.

[4]

9 $n - 1$, n and $n + 1$ are any three consecutive integers.

(i) Show that the sum of these integers is always divisible by 3.

[1]

(ii) Find the sum of the squares of these three consecutive integers and explain how this shows that the sum of the squares of any three consecutive integers is never divisible by 3.

[3]

10 Express $3x^2 - 12x + 5$ in the form $a(x - b)^2 - c$. Hence state the minimum value of y on the curve $y = 3x^2 - 12x + 5$.

[5]

11

(i) Express $125\sqrt{5}$ in the form 5^k.

[2]

(ii) Simplify $10 + 7\sqrt{5} + \frac{38}{1 - 2\sqrt{5}}$, giving your answer in the form $a + b\sqrt{5}$.

[3]

12 Rearrange the following formula to make r the subject, where $r > 0$.

$$V = \frac{1}{3}\pi r^2(a+b)$$

[3]

13

(i) Evaluate $(0.2)^{-2}$.

[2]

(ii) Simplify $(16a^{12})^{\frac{3}{4}}$.

[3]

14 Solve the inequality $3x^2 + 10x + 3 > 0$.

[3]

15 Make a the subject of $3(a + 4) = ac + 5f$.

[4]

16

(i) Expand and simplify $(7 - 2\sqrt{3})^2$.

[3]

(ii) Express $\frac{20\sqrt{6}}{\sqrt{50}}$ in the form $a\sqrt{b}$, where a and b are integers and b is as small as possible.

[2]

17

(i) Evaluate $\left(\frac{1}{27}\right)^{\frac{2}{3}}$.

[2]

(ii) Simplify $\frac{(4a^2c)^3}{32a^4c^7}$.

[3]

(i) Find the coordinates of the points of intersection of the curve $y = 2x^2 - 5x - 3$ with the axes. **[3]**

(ii) Find the coordinates of the points of intersection of the curve y = $2x^2 - 5x - 3$ and the line $y = x + 3$. **[4]**

(iii) Find the set of values of k for which the line $y = x + k$ does not intersect the curve $y = 2x^2 - 5x - 3$. **[5]**

19 Fig. 9 shows the curves $y = \frac{1}{x+2}$ and $y = x^2 + 7x + 7$.

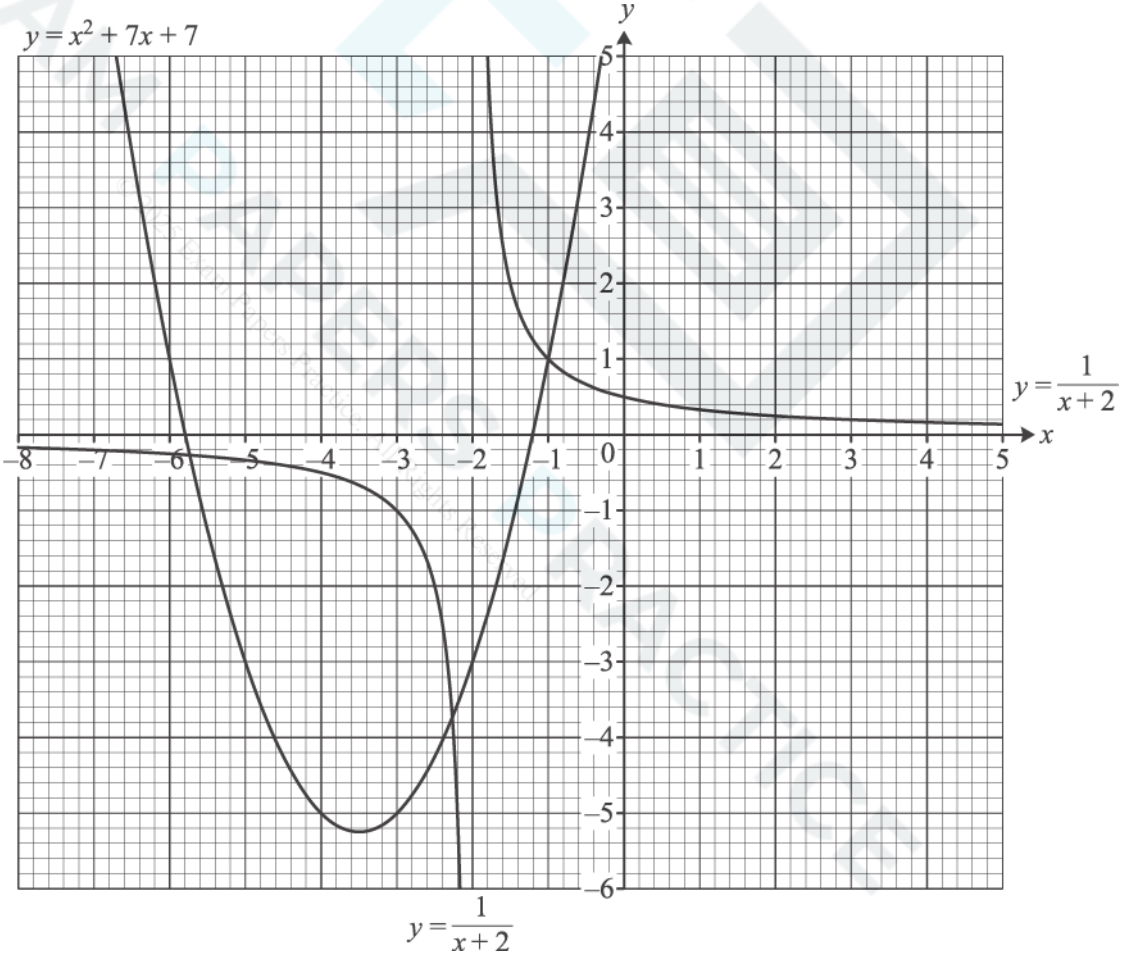


Fig.9

(i) Use Fig. 9 to estimate graphically the roots of the equation $\frac{1}{x+2} = x^2 + 7x + 7$.

[2]

(ii) Show that the equation in part **(i)** may be simplified to $x^3 + 9x^2 + 21x + 13 = 0$. Find algebraically the exact roots of this equation.

[7]

(iii) The curve $y = x^2 + 7x + 7$ is translated by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$.

(*A*) Show graphically that the translated curve intersects the curve $y = \frac{1}{x+2}$ at only one point. Estimate the coordinates of this point.

[2]

(*B*) Find the equation of the translated curve, simplifying your answer.

[2]

20

(i) Solve the equation $(x-2)^2 = 9$.

[2]

(ii) Sketch the curve $y = (x - 2)^2 - 9$, showing the coordinates of its intersections with the axes and its turning point.

[3]

21

(i) Express $\sqrt{50}+3\sqrt{8}$ in the form $a\sqrt{b}$, where a and b are integers and b is as small as possible.

[2]

(ii) Express $\frac{5+2\sqrt{3}}{4-\sqrt{3}}$ in the form $(5c^2d)^3 \times \frac{2c^4}{d^5}$, where c and d are integers.

[3]

22 You are given that $a=\frac{3c+2a}{2c-5}$. Express a in terms of c.

[4]

23

(i) Solve the inequality $\frac{1-2x}{4} > 3$.

[2]

(ii) Simplify $(5c^2d)^3 \times \frac{2c^4}{d^5}$.

[2]

24 Find the coordinates of the point of intersection of the lines $2x + 3y = 12$ and $y = 7 - 3x$.

[4]

25 Find the value of each of the following.

(i) 3° **[1]**

(ii) $9^{\frac{3}{2}}$ **[2]**

(iii) $\left(\frac{4}{5}\right)^{-2}$ **[2]**

26 Differentiate $2x^3 + 9x^2 - 24x$. Hence find the set of values of x for which the function $f(x) = 2x^3 + 9x^2 - 24x$ is increasing.

[4]

27 Use calculus to find the set of values of x for which $x^3 - 6x$ is an increasing function.

[5]

28 Simplify $\frac{(2x^2y)^3 \times 4x^3y^5}{2xy^{10}}$. [2]

29 Two points, A and B, have position vectors **a** = **i** – 3**j** and **b** = 4**i** + 3**j** . The point C lies on the line $y = 1$. The lengths of the line segments AC and BC are equal. Determine the position vector of C. [4]

30 A circle has equation $(x-2)^2 + (y+3)^2 = 25$.

(a) Write down

- The radius of the circle.
- The coordinates of the centre of the circle. **[2]**

(b) Find, in exact form, the coordinates of the points of intersection of the circle with the *y*-axis. **[3]**

(c) Show that the point (1, 2) lies outside the circle. **[2]**

(d) The point P(–1, 1) lies on the circle. Find the equation of the tangent to the circle at P. **[4]**

31

(a) Sketch the graph of $y = \frac{1}{x} + a$, where *a* is a positive constant.

- State the equations of the horizontal and vertical asymptotes.
- Give the coordinates of any points where the graph crosses the axes.

[4]

(b) Find the equation of the normal to the curve $y = \frac{1}{x} + 2$ at the point where $x = 2$. [5]

(c) Find the coordinates of the point where this normal meets the curve again. [3]

32 **In this question you must show detailed reasoning.**

Determine for what values of k the graphs $y = 2x^2 - kx$ and $y = x^2 - k$ intersect. **[6]**

33 **In this question you must show detailed reasoning.**

Find the coordinates of the points of intersection of the curve $y = x^2 + x$ and the line $2x + y = 4$. **[5]**

34 In a chemical reaction, the mass m grams of a chemical at time t minutes is modelled by the differential equation

$$\frac{dm}{dt} = \frac{m}{t(1+2t)}.$$

At time 1 minute, the mass of the chemical is 1 gram.

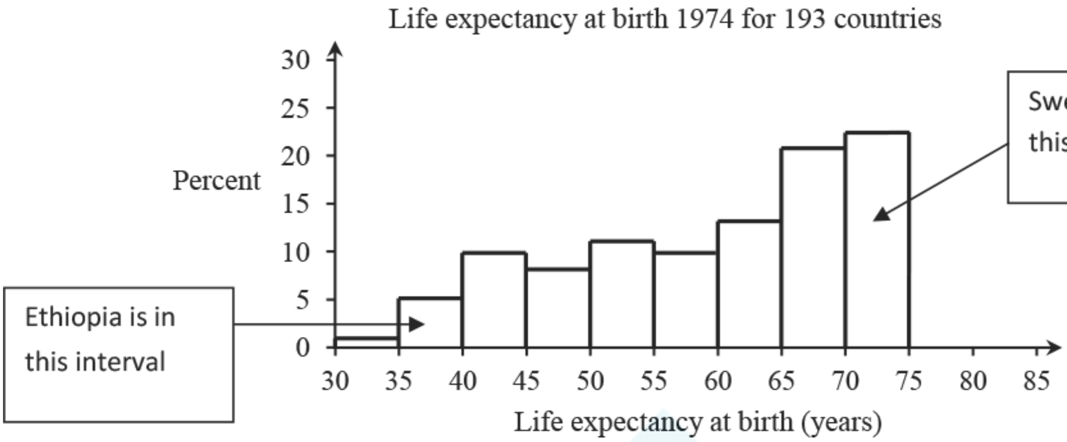
(a) Solve the differential equation to show that $m = \frac{3t}{(1+2t)}$. **[8]**

(b) Hence

(i) find the time when the mass is 1.25 grams, **[2]**

(ii) show what happens to the mass of the chemical as t becomes large. **[2]**

35 Fig. 16.1, Fig. 16.2 and Fig. 16.3 show some data about life expectancy, including some from the pre-release data set (see http://www.ocr.org.uk/Images/308749-units-h630-and-h640-large-data-set-lds-sample-assessment-material.xls).



A book

Fig. 16.1

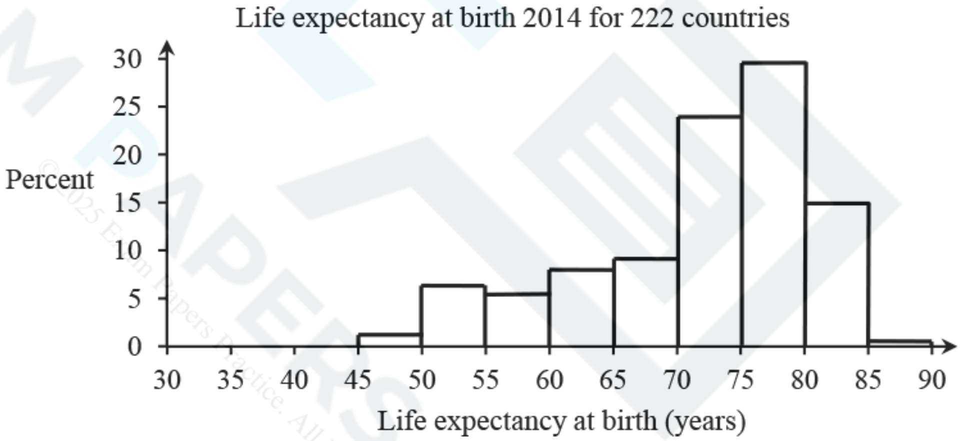


Fig. 16.2

Increase in life expectancy from 1974 to 2014 (years)

Increase in life expectancy for 193 countries from 1974 to 2014	
Number of values	193
Minimum	- 4.618
Lower quartile	6.9576
Median	9.986
Upper quartile	15.873
Maximum	30.742

Source: CIA World Factbook and Gapminder

Fig. 16.3

(a) Comment on the shapes of the distributions of life expectancy at birth in 2014 and 1974. **[2]**

(b) (i) The minimum value shown in the box plot is negative. What does a negative value indicate? **[1]**

(ii) What feature of Fig 16.3 suggests that a Normal distribution would **not** be an appropriate model for increase in life expectancy from one year to another year? **[1]**

(iii) Software has been used to obtain the values in the table in Fig. 16.3. Decide whether the level of accuracy is appropriate. Justify your answer. **[1]**

(iv) John claims that for half the people in the world their life expectancy has improved by 10 years or more. Explain why Fig. 16.3 does not provide conclusive evidence for John's claim. **[1]**

(c) Decide whether the maximum increase in life expectancy from 1974 to 2014 is an outlier. Justify your answer. **[3]**

Here is some further information from the pre-release data set (see http://www.ocr.org.uk/Images/308749-units-h630-and-h640-large-data-set-lds-sample-assessment-material.xls).

Country	Life expectancy at birth in 2014
Ethiopia	60.8
Sweden	81.9

(d) (i) Estimate the change in life expectancy at birth for Ethiopia between 1974 and 2014.

(ii) Estimate the change in life expectancy at birth for Sweden between 1974 and 2014.

(iii) Give **one** possible reason why the answers to parts (i) and (ii) are so different. **[4]**

Fig.16.4 shows the relationship between life expectancy at birth in 2014 and 1974.

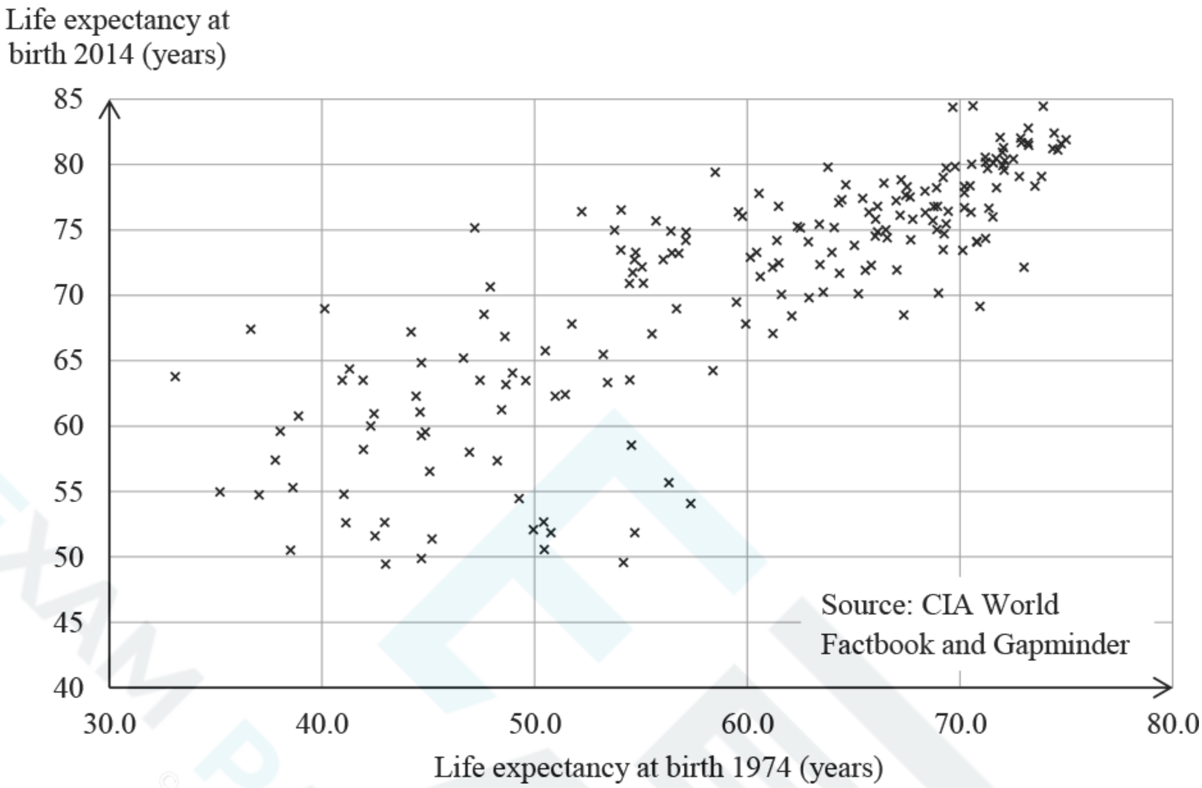


Fig. 16.4

A spreadsheet gives the following linear model for all the data in Fig 16.4.

(Life expectancy at birth 2014) = 30.98 + 0.67 × (Life expectancy at birth 1974)

The life expectancy at birth in 1974 for the region that now constitutes the country of South Sudan was 37.4 years. The value for this country in 2014 is not available.

(e) (i) Use the linear model to estimate the life expectancy at birth in 2014 for South Sudan. **[2]**

(ii) Give two reasons why your answer to part (*i*) is not likely to be an accurate estimate for the life expectancy at birth in 2014 for South Sudan. You should refer to **both** information from Fig 16.4 **and** your knowledge of the large data set. **[2]**

(f) In how many of the countries represented in Fig. 16.4 did life expectancy drop between 1974 and 2014? Justify your answer. **[3]**

36 Show that the area of the region bounded by the curve $y = 3x^{-\frac{3}{2}}$, the lines x = 1 , x = 3 and the x-axis is $6 - 2\sqrt{3}$. **[5]**

37 (a) The graph of $y = 3\sin^2\theta$ for $0° \le \theta \le 360°$ is shown in Fig. 6. On Fig. 6, sketch the graph of $y = 2\cos\theta$ for $0° \le \theta \le 360°$. [2]

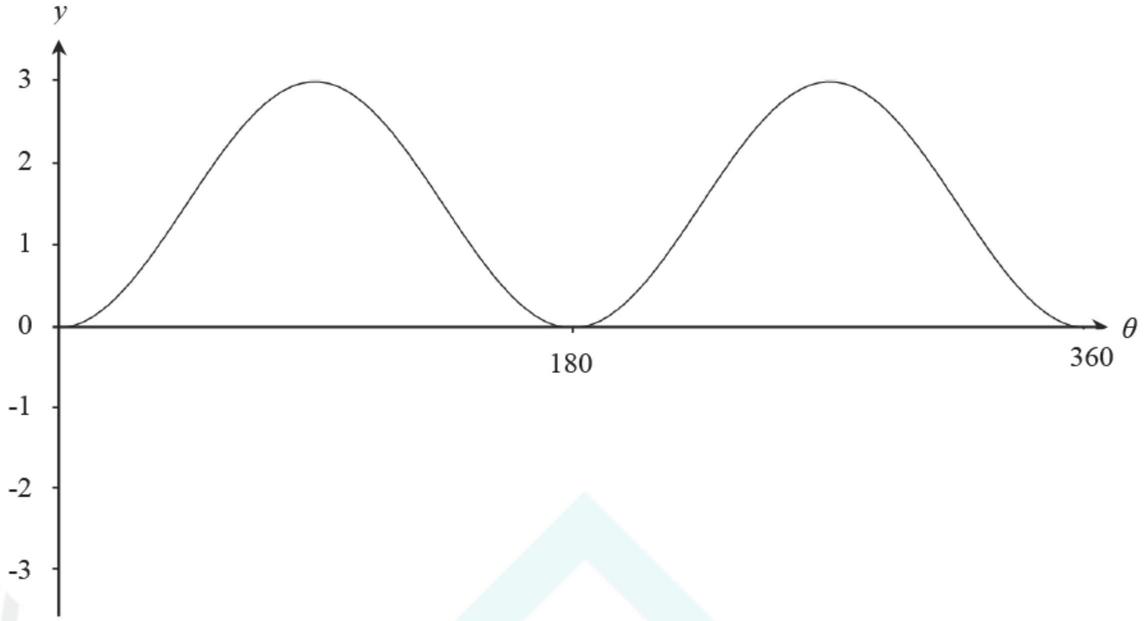


Fig. 6

(b) In this question you must show detailed reasoning.

Determine the values of θ , $0° \leq \theta \leq 360°$, for which the two graphs cross. **[6]**

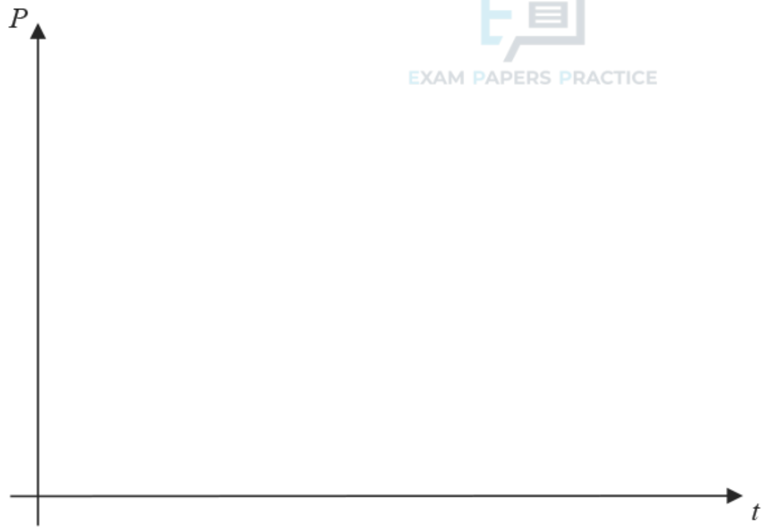
38 In a certain region, the populations of grey squirrels, P_G and red squirrels P_R, at time t years are modelled by the equations:

$$P_G = 10000(1 - e^{-kt})$$

$$P_R = 20000e^{-kt}$$

where $t \geq 0$ and k is a positive constant.

(a) (i) On the axes below, sketch the graphs of P_G and P_R on the same axes.



(ii) Give the equations of any asymptotes. **[4]**

(b) What does the model predict about the long term population of

- grey squirrels
- red squirrels? **[2]**

Grey squirrels and red squirrels compete for food and space. Grey squirrels are larger and more successful than red squirrels.

(c) Comment on the validity of the model given by the equations, giving a reason for your answer. **[1]**

(d) Show that, according to the model, the rate of decrease of the population of red squirrels is always double the rate of increase of the population of grey squirrels. **[4]**

(e) When $t = 3$, the numbers of grey and red squirrels are equal. Find the value of k. **[4]**

39 Fig. 11 shows the curve with parametric equations

$$x = 2\cos\theta,\ y = \sin\theta,\ 0 \le \theta \le 2\pi.$$

The point P has parameter $\frac{1}{4}\pi$. The tangent at P to the curve meets the axes at A and B.

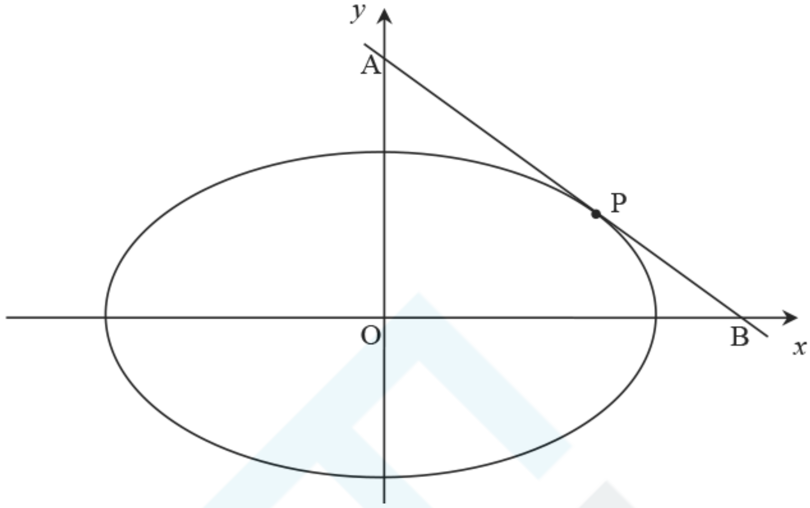


Fig.11

(a) Show that the equation of the line AB is $x + 2y = 2\sqrt{2}$. [6]

(b) Determine the area of the triangle AOB. **[3]**

40 **In this question you must show detailed reasoning.**

Determine the values of k for which part of the graph of $y = x^2 - kx + 2k$ appears below the x-axis. **[4]**

41

Express $\frac{2}{x-1} + \frac{5}{2x+1}$ as a single fraction. **[2]**

42

(a) Express $\cos\theta + 2\sin\theta$ in the form $R\cos(\theta - a)$, where $0 < \alpha < \frac{1}{2}\pi$ and R is positive and given in exact form. [4]

The function f(θ) is defined by $f(\theta) = \frac{1}{(k + \cos\theta + 2\sin\theta)}$, $0 \le \theta \le 2\pi$, k is a constant.

(b) The maximum value of f(θ) is $\frac{(3+\sqrt{5})}{4}$. Find the value of k. [3]

43 The function f(x) is defined by $f(x) = x^4 + x^3 - 2x^2 - 4x - 2$.

(a) Show that $x = -1$ is a root of $f(x) = 0$. **[1]**

(b) Show that another root of $f(x) = 0$ lies between $x = 1$ and $x = 2$. **[2]**

(c) Show that $f(x) = (x + 1)g(x)$, where $g(x) = x^3 + ax + b$ and a and b are integers to be determined. **[3]**

(d) Without further calculation, explain why $g(x) = 0$ has a root between $x = 1$ and $x = 2$. **[1]**

(e) Use the Newton-Raphson formula to show that an iteration formula for finding roots of g(x) = 0 may be written

$$x_{n+1} = \frac{2x_n^{\ 3} + 2}{3x_n^{\ 2} - 2}$$

Determine the root of g(x) = 0 which lies between x = 1 and x = 2 correct to 4 significant figures. **[3]**

44 (See Insert for Specimen 64003.) Show that the larger regular hexagon in Fig. C1 has perimeter $4\sqrt{3}$. **[3]**

45 (See Insert for Specimen 64003.) Show that the two values of *b* given on line 34 are equivalent. **[3]**

46 (See Insert for Specimen 64003.) Fig. 15 shows a unit circle and the escribed regular polygon with 12 edges.

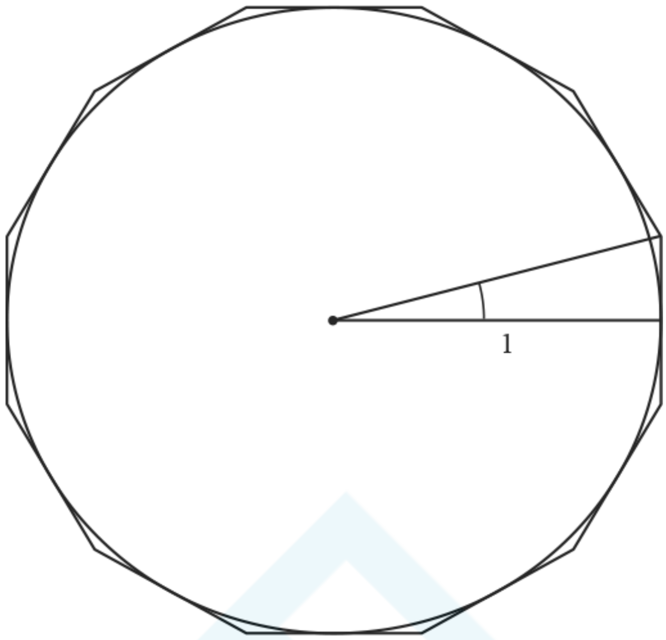


Fig. 15

(a) Show that the perimeter of the polygon is 24 tan15° . **[2]**

(b) Using the formula for tan $(\theta - \phi)$ show that the perimeter of the polygon is $48 - 24\sqrt{3}$. **[3]**

47 (See Insert for Specimen 64003.) On a unit circle, the inscribed regular polygon with 12 edges gives a lower bound for π, and the escribed regular polygon with 12 edges gives an upper bound for π. Calculate the values of these bounds for π, giving your answers:

(*A*) in surd form

(*B*) correct to 2 decimal places. **[3]**

48 **[3]**

Given that $y = 6x + 3 - \frac{5}{x^2}$ find $\frac{dy}{dx}$.

49 You are given that $f(x) = 6x^3 - 25x^2 + 2x + 8$.

(a) Evaluate f(4). **[1]**

(b) **In this question you must show detailed reasoning.**

Express $f(x)$ as the product of three linear factors. **[4]**

50 **In this question you must show detailed reasoning.**

Solve the equation

$$2\tan\theta + \cos\theta = 0$$

in the range $0° < \theta < 360°$. **[7]**

END OF QUESTION PAPER

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$\left(x-\frac{5}{2}\right)^2-\frac{1}{4}$ oe	B3	B1 for a = 5/2 oe and M1 for 6 – *their* a^2 soi;	condone $\left(x-\frac{5}{2}\right)^2-\frac{1}{4}$ oe = 0 condone omission of index – can earn all marks bod M1 for 6 – 4.25 or 6 – 25/2 etc, if bearing some relation to an attempt at 6 – *their* 2.5^2; M0 for just 1.75 etc without further evidence
		i	$\left(\frac{5}{2},-\frac{1}{4}\right)$ oe or ft	B1	accept x = 2.5, y = –0.25 oe **Examiner's Comments** The majority are quite confident in the technique of completing the square, although some struggled with the arithmetic since fractions were involved. Some candidates did not complete the question and omitted to state the coordinates of the turning point; some others made sign errors such as (–2.5, 0.25) after a correct completion of the square.	condone starting again and finding using calculus
		ii	(2, 0) and (3, 0)	B2	B1 each or B1 for both correct plus an extra or M1 for (x – 2)(x – 3) or correct use of formula or for *their* $a \pm \sqrt{their\ b}$ ft from (i)	condone not expressed as coordinates, for both x and y values; accept eg in table or marked on graph
		ii	(0, 6)	B1		

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii	graph of quadratic the correct way up and crossing both axes	B1	ignore label of their tp; condone stopping at y-axis **Examiner's Comments** Apart from the occasional upside down parabola and the odd cubic, most candidates made a good attempt at drawing a sketch of the curve, showing the relevant information about the intersections with the axes. They found the required factorisation straightforward, though a few candidates did resort to using the formula and in several of these cases they failed to recognise that $\sqrt{0.25}$ is equal to 0.5. The quality of the curve was often poor, probably because candidates marked the intersections on the axis first and then tried to draw the curve through them, but it was usually good enough to earn the mark.	condone 'U' shape or slight curving back in/out; condone some doubling / feathering – deleted work sometimes still shows up in scoris; must not be ruled; condone fairly straight with clear attempt at curve at minimum; be reasonably generous on attempt at symmetry
		iii	$x^2 - 5x + 6 = 2 - x$	M1	for attempt to equate or subtract eqns or attempt at rearrangement and elimination of x	accept calculus approach: $y' = 2x - 5$
		iii	$x^2 - 4x + 4$ [= 0]	M1	for rearrangement to zero ft and collection of terms; condone one error; if using completing the square, need to get as far as $(x - k)^2 = c$, with at most one error [$(x - 2)^2 = 0$ if correct]	use of $y' = -1$ M1

Question			Answer/Indicative content	Marks	Part marks and guidance	
		iii	$x = 2$, [$y = 0$]	A1	condone omission of $y = 0$ since already found in (ii) if they have eliminated x, $y = 0$ is not sufft for A1 – need to get $x = 2$ A0 for $x = 2$ and another root	$x = 2$ A1
		iii	'double root at $x = 2$ so tangent' oe; www;	A1	eg 'only one point of contact, so tangent'; or showing $b^2 - 4ac = 0$, and concluding 'so tangent'; www **Examiner's Comments** A good number found $x = 2$ correctly. Some candidates chose to eliminate x rather than y and more often than not went wrong. Many candidates realised that a repeated root meant that the line was a tangent to the curve, but quite a few clearly did not, with some omitting the final step of showing that the line was a tangent to the curve. A small number of candidates justified the tangent by using calculus in order to determine the slope of the line and the curve at their point of intersection.	tgt is y [– 0] = –(x – 2) and obtaining given line A1
			Total	12		

Question			Answer/Indicative content	Marks	Part marks and guidance	
2			$5c + 9t = 2ac + at$	M1	for correct expansion of brackets	
			$5c - 2ac = 9t$ oe	M1	for correct collection of terms, ft eg after M0 for $5c + 9t = 2ac + t$ allow this M1 for $5c - 2ac = -8t$ oe	for each M, ft previous errors if their eqn is of similar difficulty;
			$c(5 - 2a) = at - 9t$ oe	M1	for correctly factorising, ft; must be c × a two-term factor	may be earned before t terms collected
			$[c =] \dfrac{at - 9t}{5 - 2a}$ or $\dfrac{t(a-9)}{5-2a}$ oe as final answer	M1	for correct division, ft their two-term factor **Examiner's Comments** A good number were successful in the rearrangement, but some very poor work was also seen, revealing fundamental misconceptions about algebraic manipulation. Common errors included dividing some terms by a but not others, and confusion of division and subtraction.	treat as MR if t is the subject, with a penalty of 1 mark from those gained, marking similarly
			Total	4		

Question			Answer/Indicative content	Marks	Part marks and guidance	
3		i	$9\sqrt{3}$ www oe as final answer	2	M1 for $\sqrt{48} = 4\sqrt{3}$ or $\sqrt{75} = 5\sqrt{3}$ soi	
		ii	$\frac{39 + 7\sqrt{5}}{44}$ www as final answer	3	M1 for attempt to multiply numerator and denominator by $7 - \sqrt{5}$ B1 for each of numerator and denominator correct (must be simplified) **Examiner's Comments** Simplifying and adding the surds was done correctly by a high proportion of candidates. Most candidates knew how to rationalise a denominator for the second part but mistakes in implementation were common, the denominator being more frequently correct than the numerator.	condone $\frac{39}{44} + \frac{7\sqrt{5}}{44}$ for 3 marks eg M0B1 if denominator correctly rationalised to 44 but numerator not multiplied
			Total	5		

Question			Answer/Indicative content	Marks	Part marks and guidance	
4			$(5x + 2)(x - 6)$	M1	for factors giving at least two out of three terms correct when expanded and collected	or use of formula or completing the square with at most one error (comp square must reach [5]$(x - a)^2 \le b$ oe or $(5x - c)^2 \le d$ oe stage) if correct: $5(x - 2.8)^2 \le 51.2$ or $(x - 2.8)^2 \le 10.24$ or $(5x - 14)^2 \le 256$
			boundary values –0.4 oe and 6 soi	A1	A0 for just $\frac{28 \pm \sqrt{1024}}{10}$	
			$-0.4 \le x \le 6$ oe	A2	may be separate inequalities; mark final answer A1 for one end correct eg $x \le 6$ or for $-0.4 < x < 6$ oe or B1 for $a \le x \le b$ ft their boundary values **Examiner's Comments** A few made basic mistakes in factorising and finding the end-points. Those who sketched the graph of the quadratic usually reached the correct inequality. Some used the quadratic formula, which often led to unsimplified end points. Those who did not sketch often made an error such as '$(5x - 2) \le 0$ or $(x - 6) \le 0$' as their next step after factorising. Unusually, some candidates offered final answers such as –0.4 • 0 • 6.	condone unsimplified but correct $\frac{28 - \sqrt{1024}}{10} \le x \le \frac{28 + \sqrt{1024}}{10}$ etc allow A1 for $-0.4 \le 0 \le 6$ condone errors in the inequality signs during working towards final answer
			Total	4		

Question			Answer/Indicative content	Marks	Part marks and guidance	
5			obtaining a correct relationship in any 3 of C, d, r and A	M2	may substitute into given relationship;	eg M2 for $Cd = 4\pi r^2$ or $\pi d^2 = k\pi r^2$ seen/obtained
			or obtaining a correct relationship in k and no more than 2 other variables		or M1 for at least two of $A = \pi r^2$, $C = \pi d$, $C = 2\pi r$, $d = 2r$ or $r = \frac{d}{2}$ seen or used	condone eg Area = πr^2; allow $A = \pi\left(\frac{d}{2}\right)^2$ to imply $A = \pi r^2$ and $r = \frac{d}{2}$ and so earn M1, if M2 not earned

Question			Answer/Indicative content	Marks	Part marks and guidance	
			convincing argument leading to $k = 4$	A1	must be from general argument, not just substituting values for r or d; may start from given relationship and derive $k = 4$ **Examiner's Comments** Many candidates did not know where to start. Having picked up on the keyword 'circle' many just wrote down the general equation of a circle and nothing else, or offered no response at all. For some candidates, lack of real understanding of algebra meant that when confronted with a different style of question they were unable to find an appropriate strategy. Some students did not remember the required circle formulae, eg $A = 2\pi$ was not uncommon. Those starting with the given form $Cd = kA$ and putting in the correct formulae were often most successful. The squaring of $\frac{d}{2}$ formulae were often most successful. The squaring of $\frac{d^2}{2}$ leading to $k = 2$. Many had several attempts at this question and solutions were often scrappily presented and difficult to follow.	eg M1only for eg $A = \pi r2$ and $C = \pi d$ and so $k = 4$ with no further evidence
			Total	3		

Question			Answer/Indicative content	Marks	Part marks and guidance	
6			$4x^4y^{-3}$ or $\frac{4x^4}{y^3}$ as final answer	3	B1 each 'term'; or M1 for numerator = $64x^{15}y^3$ and M1 for denominator = $16x^{11}y^6$ **Examiner's Comments** Whereas the numerical work with indices is good, the algebraic work is definitely weaker – as was seen in this question. There were still a pleasing number of correct solutions, but quite a few dropped a mark or two here – often for not cubing the 4 in the numerator – and/or for having x^{10} in the denominator.	B0 if obtained fortuitously mark B scheme or M scheme to advantage of candidate, but not a mixture of both schemes
			Total	3		

Question			Answer/Indicative content	Marks	Part marks and guidance	
7		i	$\frac{9}{25}$ or 0.36 isw	2	M1 for numerator or denominator correct or for squaring correctly or for inverting correctly **Examiner's Comments** The first part was very well answered on the whole, with the majority scoring full marks. Most inverted first and attempted to square second.	M1 for eg $\frac{1}{\left(\frac{25}{9}\right)}$ or $\left(\frac{25}{9}\right)^{-1}$ or $\frac{25}{9}$ or for $\left(\frac{3}{5}\right)^2$ or $\frac{3}{5}$ M0 for just $\frac{1}{\left(\frac{5}{3}\right)^2}$
		ii	27	2	M1 for $81^{\frac{1}{4}} = 3$ soi **Examiner's Comments** Again a high proportion of correct answers was seen. Among the common errors were responses from candidates who either thought that $81^{\frac{1}{4}} = \sqrt{3}$ or that they needed to find $(\sqrt[3]{81})^4$. Regrettably, the error $3^3 = 9$ was not rare.	eg M1 for 3^3 M0 for $81^3 = 531441$ (true but not helpful)
			Total	4		

Question			Answer/Indicative content	Marks	Part marks and guidance	
8		i	$y = 2x + 3$ drawn accurately	M1	at least as far as intersecting curve twice	ruled straight line and within 2mm of (2, 7) and (–1, 1)
		i	(–1.6 to –1.7, –0.2 to –0.3)	B1	intersections may be in form $x = \ldots$, $y = \ldots$	
		i	(2.1 to 2.2, 7.2 to 7.4)	B1		if marking by parts and you see work relevant to (ii), put a yellow line here and in (ii) to alert you to look
		i	**Revised tolerances for modified papers for visually impaired candidates (graph in (i) with 6mm squares)** $y = 2x + 3$ drawn accurately	M1	at least as far as intersecting curve twice	ruled straight line and within **3** mm of (2, 7) and (–1, 1)
		i	(–1.6 to –1.**8**, –0.2 to –0.3)	B1	intersections may be in form $x = \ldots$, $y = \ldots$	
		i	(2.1 to 2.**3**, 7.**1** to 7.4)	B1		if marking by parts and you see work relevant to (ii), put a yellow line here and in (ii) to alert you to look
		i			**Examiner's Comments** Almost all candidates were able to draw the line accurately. Omission of one or both of the signs on the negative intersections was quite common; a few reversed the coordinates. A few just wrote the two x-values only.	
		ii	$\frac{1}{x-2} = 2x+3$	M1	or attempt at elimination of x by rearrangement and substitution	may be seen in (i) – allow marks; the part (i) work appears at the foot of the image for (ii) so show marks there rather than in (i)
		ii	$1 = (2x + 3)(x - 2)$	M1	condone lack of brackets	implies first M1 if that step not seen

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii	$1 = 2x^2 - x - 6$ oe	A1	for correct expansion; need not be simplified; NB A0 for $2x^2 - x - 7 = 0$ without expansion seen [given answer]	implies second M1 if that step not seen after $\frac{1}{x-2} = 2x+3$ seen
		ii	$\frac{1 \pm \sqrt{1^2 - 4 \times 2 \times -7}}{2 \times 2}$ oe $\frac{1 \pm \sqrt{57}}{4}$ isw	M1	use of formula or completing square on given equation, with at most one error	completing square attempt must reach at least [2]$(x - a)^2 = b$ or $(2x - c)^2 = d$ stage oe with at most one error
		ii		A1	isw e.g. coordinates; after completing square, accept $\frac{1}{4} \pm \sqrt{\frac{57}{16}}$ or better **Examiner's Comments** Most were able to obtain the correct equation and many went on to solve it successfully, although as expected, there were some errors in using the formula, especially frequently in evaluating the discriminant after correct substitution.	
		iii	$\frac{1}{x-2} = -x + k$ and attempt at rearrangement	M1		
		iii	$x^2 - (k + 2)x + 2k + 1[=0]$	M1	for simplifying and rearranging to zero; condone one error; collection of x terms with bracket not required	e.g. M1 bod for $x^2 - (k + 2)x + 2k$ or M1 for $x^2 - 2kx + 2k + 1[= 0]$
		iii	$b^2 - 4ac = 0$ oe seen or used	M1		= 0 may not be seen, but may be implied by their final values of k

Question			Answer/Indicative content	Marks	Part marks and guidance	
		iii	[k =] 0 or 4 as final answer, both required	A1	SC1 for 0 and 4 found if 3rd M1 not earned (may or may not have earned first two Ms) **Examiner's Comments** After the previous part, most candidates realised that they had to equate the two expressions and manipulate the resulting equation, although many had problems dealing with the 'k' terms ('$kx + 2x = 2kx$' for instance). Most candidates stopped there, but some realised that they needed to use '$b^2 - 4ac = 0$' to establish the final values of k. Some were confused with the k and x terms and were unable to identify the coefficients correctly or made errors in simplifying the equation. A few candidates used their graphs to establish the results for k. A few tried to apply calculus but rarely with any success.	e.g. obtained graphically or using calculus and / or final answer given as a range
			Total	12		

Question			Answer/Indicative content	Marks	Part marks and guidance	
9		i	$3n$ isw	1	accept equivalent general explanation	
		ii	at least one of $(n-1)^2$ and $(n+1)^2$ correctly expanded	M1	must be seen	M0 for just $n^2 + 1 + n^2 + n^2 + 1$
		ii	$3n^2 + 2$	B1		accept even if no expansions / wrong expansions seen
		ii	comment e.g. $3n^2$ is always a multiple of 3 so remainder after dividing by 3 is always 2	B1	dep on previous B1 B0 for just saying that 2 is not divisible by 3 – must comment on $3n^2$ term as well allow B1 for $\frac{3n^2+2}{3} = n^2 + \frac{2}{3}$ **Examiner's Comments** A small number of candidates made no attempt to generalise in either part, and simply gave examples to demonstrate the properties, so, of course, gained no marks. Most earned the mark in the first part for adding the values. In the second part there were some errors in multiplying but many correctly reached $3n^2$ +2. The last mark was often lost due to an incomplete explanation centred on the fact that 2 was not divisible by 3, without making any reference to the fact that $3n^2$ is always divisible by 3.	SC: n, $n + 1$, $n + 2$ used similarly can obtain first M1, and allow final B1 for similar comment on $3n^2 + 6n + 5$
			Total	4		

Question			Answer/Indicative content	Marks	Part marks and guidance	
10			$3(x-2)^2-7$ isw or a = 3, b = 2 c = 7 www	M4.	B1 each for a = 3, b = 2 oe and B2 for c = 7 oe or M1 for $[-]\frac{7}{3}$ or for 5 − *their* a(*their* b)^2 or for $\frac{5}{3}-(\textit{their } b)^2$ soi	condone omission of square symbol; ignore '= 0' may be implied by their answer
			–7 or ft	B1	B0 for (2, –7) **Examiner's Comments** Some who completed the square correctly lost the final mark by giving the minimum point of (2, –7) rather than the minimum y -value. Most common part-correct answers were getting the values of a and b correct but ignoring the multiple of 3 in establishing any value of c. The most common wrong values of b were –6 (dividing the '–12x' by 2) and 4 (taking the 3 out as a common factor and forgetting to divide by 2).	may be obtained by starting again eg with calculus
			Total	5		

Question			Answer/Indicative content	Marks	Part marks and guidance	
11		i	$5^{3.5}$ oe or k = 7/2 oe	2	M1 for $125 = 5^3$ or $\sqrt{5} = 5^{\frac{1}{2}}$ soi Examiner's Comments This question was found to be difficult by many candidates. In the first part, although the correct answer was seen fairly frequently, a significant number of candidates, having correctly shown 125 and √5 to be 5^3 and $5^{\frac{1}{2}}$ respectively, then multiplied the indices to give an answer of $5^{\frac{3}{2}}$. Others found one of the indices correctly, but not the other. Some candidates treated it as though the square root applied to 125 as well.	M0 for just answer of 5^3 with no reference to 125
		ii	attempting to multiply numerator and denominator of fraction by $1+2\sqrt{5}$	M1		some cands are incorporating the $10+7\sqrt{5}$ into the fraction. The M1s are available even if this is done wrongly or if $10+7\sqrt{5}$ is also multiplied by $1+2\sqrt{5}$

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii	denominator = –19 soi	M1	must be obtained correctly, but independent of first M1 **Examiner's Comments** Few correct answers were seen in the second part. Being in a different format from usual, many candidates did not know how to cope with the initial 10 + 7√5. Many multiplied the '10 + 7√5' term by 2 + √5, sometimes losing the denominator altogether. Those who knew they should rationalise the denominator of the fraction often made errors in multiplying the denominator, with 9, –9 or 19 often seen (19 often following the correct 1 – 20). Some who correctly reached this point then only divided the first term in the numerator by –19.	e.g. M1 for denominator of 19 with a minus sign in front of whole expression or with attempt to change signs in numerator
		ii	$8+3\sqrt{5}$	A1		
			Total	5		

Question			Answer/Indicative content	Marks	Part marks and guidance	
12			$r = \sqrt{\dfrac{3V}{\pi(a+b)}}$ oe www as final answer	3	M1 for dealing correctly with 3 and M1 for dealing correctly with π(*a* + *b*), ft and M1 for correctly finding square root, ft *their* 'r^2 ='; square root symbol must extend below the fraction line **Examiner's Comments** There were many good answers in rearranging the formula. Most candidates managed at least one mark; some triple-decker fractions or the use of ÷ signs were seen. The π and the (*a* + *b*) sometimes became separated. The radius was sometimes considered to be ±, and the > sign was used on more than one occasion. It was encouraging to see very few penalties incurred due to a poor square root symbol.	M0 if triple-decker fraction, at the stage where it happens, then ft; condone missing bracket at rh end M0 if ±... or *r* >... for M3, final answer must be correct
			Total	3		

Question			Answer/Indicative content	Marks	Part marks and guidance	
13		i	25	2	M1 for $\left(\frac{10}{2}\right)^2$ or $\left(\frac{1}{0.2}\right)^2$ oe soi or for $\frac{1}{0.04}$ oe **Examiner's Comments** In evaluating $(0.2)^{-2}$, many stopped after evaluating $\frac{1}{0.2^2}$ as $\frac{1}{0.04}$ (or, sadly often, as $\frac{1}{0.4}$). Those who converted to fractions first were more successful in reaching 25.	ie M1 for one of the two powers used correctly M0 for just $\frac{1}{0.4}$ with no other working
		ii	$8a^9$	3	B2 for 8 or M1 for $16^{\frac{1}{4}} = 2$ soi and B1 for a^9 **Examiner's Comments** In the second part, the majority found the power of a correctly, but the $16^{\frac{3}{4}}$ proved more challenging. A surprising number did $\frac{3}{4} \times 16 = 12$ to obtain $12a^9$.	ignore ± eg M1 for 2^3; M0 for just 2
			Total	5		

Question			Answer/Indicative content	Marks	Part marks and guidance	
14			$(3x + 1)(x + 3)$	1	or $3(x + 1/3)(x + 3)$ or for –1/3 and –3 found as endpoints eg by use of formula	
			x –3 [or]	A1		

Question			Answer/Indicative content	Marks	Part marks and guidance	
			$x > -1/3$ oe	1	mark final answers; allow only A1 for $-3 > x > -1/3$ oe as final answer or for $x \leq -3$ and $x \geq -1/3$ if M0, allow SC1 for sketch of parabola the right way up with their solns ft their endpoints **Examiner's Comments** In solving the quadratic inequality, most candidates were able to factorise the quadratic expression correctly, though a few produced incorrect factors. A small number resorted to using the formula to determine the end points, often failing to do so correctly. It was very clear that those candidates who drew a sketch to help them were generally successful in identifying the two different regions. But without a diagram many either just gave the single region between the end points, or having written down two correct inequalities tried to combine them into a 'doubleended' inequality, which, of course, they were unable to do. Another error often seen was to believe that since $(3x + 1)(x + 3) > 0$, then $(3x + 1) > 0$ and/or $(x + 3) > 0$.	A0 for combinations with only one part correct eg $-3 > x -1/3$, though this would earn M1 if not already awarded
			Total	3		

Question			Answer/Indicative content	Marks	Part marks and guidance	
15			$3a + 12$ [= $ac + 5f$]	M1	for expanding brackets correctly	annotate this question if partially correct
			$3a - ac = 5f - 12$ or ft	M1	for collecting *a* terms on one side, remaining terms on other	ft only if two *a* terms
			$a(3 - c) = 5f - 12$ or ft	M1	for factorising *a* terms; may be implied by final answer	ft only if two *a* terms, needing factorising may be earned before 2nd M1
			$[a =]\dfrac{5f - 12}{3 - c}$ oe or ft as final answer	M1	for division by their two-term factor; for all 4 marks to be earned, work must be fully correct **Examiner's Comments** Rearranging the formula was usually done well. Those who found this difficult generally attempted to isolate just one a term and hence scored only the first mark. Other errors seen occasionally included sign errors and a final spoiling of the answer by invalidly 'cancelling' 3 into 12.	
			Total	4		

Question			Answer/Indicative content	Marks	Part marks and guidance	
16		i	$61-28\sqrt{3}$	3	B2 for 61 or B1 for 49 + 12 found in expansion (may be in a grid) and B1 for $-28\sqrt{3}$ if B0, allow M1 for at least three terms correct in $49-14\sqrt{3}-14\sqrt{3}+12$ the correct answer obtained then spoilt earns SC2 only	
		ii	$4\sqrt{3}$	2	M1 for $\sqrt{50}=5\sqrt{2}$ or $\sqrt{300}=10\sqrt{3}$ or $20\sqrt{300}=200\sqrt{3}$ or $\sqrt{48}=2\sqrt{12}$ seen **Examiner's Comments** Most candidates gained at least one mark in the first part for $-28\sqrt{3}$. Those who failed to reach the correct final answer often incorrectly expanded the last terms of the brackets, obtaining $\pm4\sqrt{3}$, 6 or 12 rather than +12. For most candidates the second part was more challenging than the first part. Errors tended to be introduced when rationalising the denominator, with many choosing to multiply by $\sqrt{50}$ or $-5\sqrt{2}$. Those that did rationalise were then unsure how to simplify the numerator, often obtaining large roots which they were unable to simplify accurately. Those that had the most success in this question expressed the $\sqrt{50}$ in the denominator as $5\sqrt{2}$ and were then comfortable dividing surds and cancelling fractions.	

Question			Answer/Indicative content	Marks	Part marks and guidance	
			Total	5		
17		i	$\frac{1}{9}$	2	isw conversion to decimal	
		i			M1 for 9 or for 3^{-2} or for $\frac{1}{3}$	ie M1 for evidence of $(\sqrt[3]{27})^2$ or $1/(\sqrt[3]{27})$ found correctly
		i			Except M0 for 9 from 27/3 or $\sqrt[3]{27}$	
		ii	$2a^2c^{-4}$ or $\frac{2a^2}{c^4}$ as final answer	3	B1 for each element; must be multiplied if B0, allow SC1 for $64a^6c^3$ obtained from numerator or for all elements correct but added **Examiner's Comments** Most candidates knew what to do and handled the indices well. Errors such as $\sqrt[3]{27} = 9$ were seen occasionally in the first part. In the second, the most frequent errors came from failing to cube the 4 or the a^2 correctly.	
			Total	5		

Question			Answer/Indicative content	Marks	Part marks and guidance	
18		i	(0, –3)	B1	condone $y = -3$, isw	if not coordinates, must be clear which is x and which is y
		i	(– 1/2 , 0) and (3, 0) www	B2	condone $y = -1/2$, and 3; **B1** for one correct www or **M1** for $(2x + 1)(x - 3)$ or correct use of formula or reversed coordinates	**Examiner's Comments** Many candidates earned all 3 marks in this part. Some forgot to find the y-intercept. A few used the quadratic formula or completed the square, perhaps not realising that factorising was possible.
		ii	$2x^2 - 6x - 6$[= 0] isw or $x^2 - 3x - 3$ [= 0] or $2y^2 - 18y + 30$ [=0]	M1	for equating curve and line, and rearrangement to zero, condoning one error	allow rearranging to constant if they go on to attempt completing the square
		ii	use of formula or completing the square, with at most one error	M1	no ft from $2x^2 - 6x = 0$ or other factorisable equations	if completing the square must get to the stage of complete square only on lhs as in 9(ii)

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii	$\left(\frac{6\pm\sqrt{84}}{4},\frac{18\pm\sqrt{84}}{4}\right)$ or $\left(\frac{3\pm\sqrt{21}}{2},\frac{9\pm\sqrt{21}}{2}\right)$ oe isw	A2	**A1** for one set of coords or for *x* values correct (or *y*s from quadratic in *y*); need not be written as coordinates	A0 for unsimplified *y* coords eg $\frac{3+\sqrt{21}}{2}+3$ **Examiner's Comments** The majority of candidates chose the straightforward approach and equated the line and curve given. These candidates most often simplified correctly to a required form and applied the formula. This was often very well carried out. A very good number of candidates earned 3 marks using this approach. Some attempted to complete the square. Those who, sensibly, divided through by 2 before doing so were usually successful – those who did not were less successful. Most candidates struggled to find and simplify the *y* -coordinates. Some simply omitted them and the many who attempted them often just wrote the *x* coordinates '+ 3' or failed to convert the 3 being added to a fraction of a common denominator to add to the *x*-coordinate. Some candidates made their solution unnecessarily complicated by rearranging the equation of the line and substituting for *x*. These candidates often omitted to take the solitary *y*-term into account and mostly scored no more than the first two marks.
		iii	$2x^2 - 5x - 3 = x + k$	M1	for equating curve and line	

Question			Answer/Indicative content	Marks	Part marks and guidance	
		iii	$2x^2 - 6x - 3 - k$ [= 0]	M1	for rearrangement to zero, condoning one error, but must include k ;this second M1 implies the first, eg it may be obtained by subtracting the given equations	
		iii	$b^2 - 4ac < 0$ oe for non-intersecting lines	M1	eg allow for just quoting this condition; may be earned near end with correct inequality sign used there allow 'discriminant is negative' if further work implies $b^2 - 4ac$	some may use condition for intersecting lines or for a tangent and then swap condition at the end; only award this M1 and the final A mark if the work is completely clear
		iii	36 – 8 × – (3 + k) [< 0] oe	A1	for correct substitution into $b^2 - 4ac$; no ft from wrong equation; if brackets missing or misplaced, must be followed by a correct simplified version	can be earned with equality or wrong inequality, or in formula – this mark is not dependent on the 3rd M mark;
		iii	$k < -\frac{15}{2}$ oe		isw if 3rd M1 not earned, allow **B1** for $-\frac{15}{2}$ obtained for k with any symbol	
		iii	or, for those using a tangent condition with trials to find the boundary value			mark one mark scheme or another, to the advantage of the candidate, but not a mixture of schemes
		iii	rearrangement with correct boundary value of k eg $2x^2 - 6x + 4.5$ [= 0] or $2x^2 - 6x - (3 - 7.5)$ [= 0]	M2	**M1** for $2x^2 - 5x - 3 = x - 7.5$	M0 for trials with wrong values without further progress, though may still earn an M1 for $b^2 - 4ac < 0$
		iii	showing 36 – 8 × – (3– 7.5) = 0 or 36 – 8 × 4.5 = 0 oe	M1	may be in formula implies previous M2	

Question			Answer/Indicative content	Marks	Part marks and guidance	
		iii	$k < -\frac{15}{2}$ oe	A2	**B1** for $-\frac{15}{2}$ obtained for *k* as final answer with any symbol	
		iii	or, for using tangent with differentiation			
		iii	$y = 4x - 5$	M1		
		iii	[when $y = x + k$ is tgt] $4x - 5 = 1$	M1		
		iii	$x = 1.5$, $y = -6$	A1		
		iii	$-6 = 1.5 + k$ or $k = -7.5$ oe	A1		
		iii	$k < -7.5$ oe	A1		**Examiner's Comments** Some candidates were unable to cope with the constant of the equation they had formed being in terms of *k*. Many equated the line and curve, as before, and found $2x^2 - 6x - 3 - k = 0$ and then, rather than applying $b^2 - 4ac < 0$, they wrote $2x^2 - 6x - 3 = k$ and tried to apply $b^2 - 4ac < 0$ to the left hand side. Those who did work with $2x^2 - 6x - 3 - k = 0$ were almost always successful. Some candidates made sign errors through carelessness. Some introduced wrong brackets into their equation in an attempt to group the *c* term, such as $-(3 - k)$. Some candidates correctly substituted into $b^2 - 4ac < 0$ but were unable to multiply out correctly. The result $36 - 8 - (3 + k)$ was not uncommon amongst these candidates. Other

Question			Answer/Indicative content	Marks	Part marks and guidance	
						candidates used trials on $2x^2 - 6x - 3 - k = 0$ to find the boundary value ie the constant that gave $b^2 - 4ac = 0$. These often scored 3 marks, but sign errors usually resulted in the loss of the final 2 marks. A very few candidates used a calculus approach. In most cases, once $y' = 4x - 5$ had been found, it was equated to 0 and the minimum point established, thinking that this would be helpful, then no further progress was made. Candidates' setting out in this question was often poor and difficult to make sense of – particularly if they had had several attempts or had used trials. Some candidates lost marks as they restarted several times, with each time being worth less than the previous attempt! Candidates should take care in this regard – and indicate which of their attempts they intend to be taken as the answer in these cases.
			Total	12		

Question			Answer/Indicative content	Marks	Part marks and guidance	
19		i	–5.7 to –5.8, –2.2 to –2.3, –1 isw	2	**B1** for 2 correct or for all 3 only stated in coordinate form, ignoring y coordinates	**Examiner's Comments** About the same number of candidates gave the coordinates of intersection of the two graphs as gave the requested roots of the given equation in x. A few misread from the graph and/or struggled with the scale.
		ii	$1=(x+2)(x^2+7x+7)$	M1	condone missing brackets if expanded correctly; or **M1** for correct expansion of $(x+2)(x^2+7x+7)$	
		ii	correct completion with at least one interim stage of working to given answer: $x^3+9x^2+21x+13=0$	A1		
		ii	[$x=-1$ is root so] $(x+1)$ is factor soi	M1	implied by division of cubic by $x+1$	condone some confusion of root/factor for this mark if division of cubic by $x+1$ seen
		ii	correctly finding other factor as $x^2+8x+13$	M2	**M1** for correct division of cubic by $(x+1)$ as far as obtaining x^2+8x (may be in grid) or for two correct terms of $x^2+8x+13$ obtained by inspection	allow seen in grid without + signs
		ii	$\dfrac{-8\pm\sqrt{8^2-4\times13}}{2}$ oe	M1	for use of formula, condoning one error, for $x^2+8x+13=0$	or **M1** for $(x+4)^2=4^2-13$ oe or further stage, condoning one error

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii	$\frac{-8\pm\sqrt{12}}{2}$ isw or $-4\pm\sqrt{3}$ isw and $x=-1$	A1	x = –1 may be stated earlier	**Examiner's Comments** isw wrong simplification or giving as coordinates **Examiner's Comments** In the main this part was completed well, with almost all candidates gaining the first two marks for multiplying by (x+2) and expanding to prove the stated equality. A significant number of candidates were unable to progress further, unsure of how to solve a cubic equation. Stronger candidates produced a well-organised solution, leading directly to the fully factorised expression (sometimes in only a few lines of working, having used the root of x =–1 from the graph and/or part (i) to obtain the first factor). The majority were able to find the correct quadratic following division by (x + 1), with a few using synthetic division and a sizeable minority finding the solution by inspection. At this stage many found the correct final solution, but a significant number failed to include x =–1 in their final solution to this question, or stated incorrectly that (x + 1) was a root.
		iii	(A) drawing the translated quadratic	B1	must be a reasonable translation of given quadratic, only intersecting given curve once; intersections with x axis –3 to –2.5 and 1.5 to 2; ignore above y = 1	

Question			Answer/Indicative content	Marks	Part marks and guidance	
		iii	or showing that the horizontal gap between the relevant parts of the curve is always less than 3			
		iii	estimated coordinates of the point of intersection (1.8 to 2, 0.2 to 0.3)	B1		**Examiner's Comments** Many candidates were able to translate correctly although there were issues with the intersections on the *x*-axis for some. Quite a few candidates pointed out the intersection but did not write down the coordinates as requested.
		iii	$y = x^2 + x - 5$ or $y = \left(x + \frac{1}{2}\right)^2 - \frac{21}{4}$ (B)	2	**M1** for [y =] $(x - 3)^2 + 7(x - 3) + 7$ oe or for simplified equation with 'y =' omitted or for $y = (x - a)(x - b)$ where a and b are the values $3 + \frac{-7 \pm \sqrt{21}}{2}$ oe (may have been wrongly simplified)	M0 for use of estimated roots in (*A*) **Examiner's Comments** Many candidates failed to recognise that they should substitute $(x - 3)$ for x in the original equation, with a variety of different methods attempted. Substituting $(x + 3)$ was quite a common error, as was adding 3 to the original equation, or changing the constant term to –5. Some used estimated roots. Many failed to gain full marks because they omitted 'y =' from their final answer.
			Total	13		

Question			Answer/Indicative content	Marks	Part marks and guidance	
20		i	[x =] 5, [x =] –1 www	2	**M1** for $x - 2 = \pm 3$ or for $(x - 5)(x + 1)$ [=0]	0 for just $x = 5$ or for $x - 2 = 3$
		ii	parabola shape curve the correct way up	1	must extend beyond x-axis;	condone 'U' shape or very slight curving back in/out; condone some doubling / feathering–deleted work sometimes still shows up in rm assessor; must not be ruled; condone fairly straight with clear attempt at curve at minimum; be reasonably generous on attempt at symmetry e.g. condone minimum on y-axis for this mark
		ii	intersecting x-axis at 5 and –1 or ft from (i) and y-axis at –5	1		

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii	turning point (2, –9)	1	seen on graph or identified as tp elsewhere in this part	may be implied by 2 and –9 marked on axes 'opposite' turning point **Examiner's Comments** Most candidates managed to solve the equation. Quite a number of candidates multiplied out the brackets and rearranged to form a quadratic equation in the traditional form. This was then usually solved by factorisation, but occasionally using the formula. Those who used the given form and took the square root of both sides were more inclined to find just one root, by ignoring the possibility that the square root of 9 could be –3. The quality of the parabolas varied enormously, but most candidates determined the coordinates of the turning point and made a good attempt. Some candidates did not consider the turning point and often had skewed parabolas with a minimum on the *y*-axis. A few candidates sketched cubics and received no marks.
			Total	5		

Question			Answer/Indicative content	Marks	Part marks and guidance	
21		i	$11\sqrt{2}$	2	**M1** for $\left[\sqrt{50}=\right]5\sqrt{2}$ or $\left[3\sqrt{8}=\right]6\sqrt{2}$	
		ii	attempting to multiply numerator and denominator of fraction by $4+\sqrt{3}$	M1		
		ii	$2+\sqrt{3}$ or $2+1\sqrt{3}$ or *c* = 2 and *d* = 1	A2	or **B1** for denominator = 13 soi or numerator $26+13\sqrt{3}$	
		ii	or			
		ii	cross-multiplying by $4-\sqrt{3}$ and forming a pair of simultaneous equations in *c* and *d*, with at most one error	M1		

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii	$c = 2$ and $d = 1$	A2	**A1** for one correct	**Examiner's Comments** The first part was nearly always correct with the vast majority scoring at least one mark for correctly stating that $\sqrt{50} = 5\sqrt{2}$. Some candidates had difficulty with $3\sqrt{8}$ and a number incorrectly gave this as $5\sqrt{2}$ which typically came from the incorrect working of $3\sqrt{8} = 3(2\sqrt{2}) = (3+2)\sqrt{2} = 5\sqrt{2}$. In the second part, most candidates clearly knew how to rationalise the denominator with nearly all correctly indicating the need to multiply both numerator and denominator by $(4+\sqrt{3})$; only a small minority incorrectly multiplied by either $(4-\sqrt{3})$ or $\sqrt{3}$. Nearly all correctly achieved a value of 13 for the denominator but some had issues with either expanding or simplifying the numerator. A significant minority who achieved $\frac{26+13\sqrt{3}}{13}$ did not simplify this correctly with $2+13\sqrt{3}$ being a common incorrect answer.
			Total	5		

Question			Answer/Indicative content	Marks	Part marks and guidance	
22			$a(2c-5) = 3c + 2a$ or $2ac - 5a = 3c + 2a$	M1	for multiplying up correctly (may also expand brackets)	annotate this question if partially correct
			$a(2c-5) - 2a = 3c$ or $2ac - 7a = 3c$ or ft	M1	for collecting *a* terms on one side, remaining term[s] on other [need not be simplified]	ft only if two or more *a* terms,
			$a(2c-7) = 3c$ or ft	M1	for factorising *a* terms, need not be simplified; may be implied by final answer	ft only if two or more a terms, needing factorising may be earned before 2nd M1

Question			Answer/Indicative content	Marks	Part marks and guidance	
			$[a =]\frac{3c}{2c-7}$ or simplified equivalent or ft as final answer	M1	for division by their two-term factor (accept a 3 term factor that would simplify to 2 terms); for all 4 marks to be earned, work must be fully correct and simplified and not have a triple-or quadruple-decker answer	candidates whose final answer expresses *c* in terms of *a:* treat as MR after the first common M and mark equivalently, applying MR–1 if they gain further Ms. So that a final answer, correctly obtained, of $[c =]\frac{7a}{2a-3}$ or simplified equivalent earns 3 marks in total **Examiner's Comments** The majority of the candidates were very familiar with the topic of rearranging to make a different variable the subject of a formula, and coped well with this example. Nearly all candidates correctly multiplied by $(2c - 5)$ to give $a(2c - 5) = 3c + 2a$. However it was surprising that a large number of candidates went on to make *c* rather than *a* the subject of the formula (albeit the majority did this correctly and scored 3 of the 4 marks available). Where errors occurred it was usually sign errors from moving terms from one side to the other and a small minority did not simplify their answers fully, giving say an answer of $a = 3c / (2c - 5 - 2)$. It was pleasing to see that the majority of candidates correctly factorised their *a* (or *c*) terms as this has in the past caused issues.

Question			Answer/Indicative content	Marks	Part marks and guidance	
			Total	4		
23		i	$x < -11/2$ oe www as final answer	2	**M1** for $-2x > 11$ oe or $x < 11/-2$	if working with equals throughout, give 2 for correct final answer, 0 otherwise
		ii	$250c^{10}d^{-2}$ or $\frac{250c^{10}}{d^2}$ as final answer	2	**B1** for two correct elements; must be multiplied if B0, allow **SC1** for $125c^6d^3$ obtained from numerator or for all elements correct but added	**Examiner's Comments** Nearly all candidates knew how to solve a linear inequality for the first part, and earned at least one of the two marks. When the rearrangement was done so that that the $2x$ term appeared on the right, already positive (so $-11 > 2x$) the vast majority of candidates went on to get the correct answer. However, when candidates arranged to $-2x > 11$, a considerable number neglected to reverse the inequality sign when dividing by the negative value of 2. While the majority of candidates scored both marks in the second part, a number failed to expand $(5c^2d)^3$ correctly, with many of these failing to cube the 5. It was common for candidates to achieve at least two correct elements – with nearly all getting c^{10} and an equal split between those getting one of 250 or d^{-2}. Some candidates failed to deal with the two d terms correctly in both the numerator and denominator with many of these giving an answer of d^2.
			Total	4		

Question			Answer/Indicative content	Marks	Part marks and guidance	
24			substitution to eliminate one variable	M1	or multiplication or division to make one pair of coefficients the same; condone one error in either method	
			simplification to $ax = b$ or $ax - b = 0$ form, or equivalent for y		or appropriate subtraction / addition; condone one further error in either method	independent of first M1

Question			Answer/Indicative content	Marks	Part marks and guidance	
			(9/7, 22/7) oe or x = 9/7 y = 22/7 oe isw	A2	**A1** each	A0 for just rounded decimals or for –9 / –7 oe **Examiner's Comments** This was a good source of marks for the majority of candidates, who found the demand of solving a pair of simultaneous equations relatively straightforward, although errors in coping with the fractional answer to x to find the y-value were quite common, as was occasionally forgetting to find the y-value. Very few candidates found the y -value first. Those who used substitution and wrote down $2x + 3(7 – 3x) = 12$ nearly always went on to get the correct answer for x - although it was particularly disheartening the number of times that $7x = 9$ became $x = 7/9$. Those who substituted for y and had $y = 7 – 3((12 – 3y)/2)$ were usually less successful, due to the fraction and the number of negative terms in the equation. Elimination methods were less frequently seen and not as successful - candidates often did not multiply all values by the required constant or they added or subtracted their pair of equations incorrectly.
			Total	4		

Question			Answer/Indicative content	Marks	Part marks and guidance	
25		i	1	1		
		ii ii ii	27	2	condone ±27; $\sqrt{729}^{3}$ or or for $\left[9^{\frac{1}{2}} =\right] 3$ or ± 3 soi	
		iii	$\frac{25}{16}$ or $1\frac{9}{16}$ isw	2	**B1** for $\frac{5}{4}$ or $\frac{1}{\frac{16}{25}}$ or $\frac{16}{25}$ oe	B0 for 1.5625 without fractions seen; if this is found, check for possible use of calculator throughout the paper **Examiner's Comments** Nearly all candidates interpreted the zero power correctly in the first part. Most interpreted the fractional power correctly in the second part, although a number of candidates began by attempting to cube 9, which usually ran into difficulties as candidates did not have the assistance of their calculators; they had similar issues when attempting to find the square root 729. The most common error was candidates believing that 3 cubed was 9. Coping with the fraction and negative power in the last part was usually done correctly; notable errors were inverting the fraction whilst losing the power altogether or losing the power from either the numerator or denominator.
			Total	5		

Question			Answer/Indicative content	Marks	Part marks and guidance	
26			$6x^2 + 18x - 24$	B1		
			their $6x^2 + 18x - 24 = 0$ or >0 or $\leqslant 0$	M1		or sketch of $y = 6x^2 + 18x - 24$ with attempt to find x -intercepts
			-4 and $+1$ identified oe	A1		
			$x < -4$ and $x > 1$ cao	A1	or $x \leqslant -4$ and $x \leq 1$ **Examiner's Comments** Most candidates differentiated correctly and identified the correct values of x. The final mark was often lost, either due to a misunderstanding of what had been found — answer given as –4 < x < 1 or poor notation — answer given as –4 > x > 1. Those who used a graphical approach with the derivative generally scored full marks. A few candidates missed the last term out, converted the first plus sign to a minus sign or failed to multiply 2 by 3 correctly, and lost the first mark.	if **B0M0** then **SC2** for fully correct answer
			Total	4		

Question			Answer/Indicative content	Marks	Part marks and guidance	
27			$3x^2 - 6$ seen	B1		
			their $y' = 0$ or $y' > 0$ or $y' \geq 0$	M1	must be quadratic with at least one of only two terms correct	
			$\sqrt{2}$ and $-\sqrt{2}$ identified?	A1	may be implied by use with inequalities or by ± 1.41[4213562] to 3 sf or more	$\lvert x \rvert = \sqrt{2}$implies **A1**
			$x < -\sqrt{2}$ or $x \leq -\sqrt{2}$ isw	A1	if **A1A0A0**, allow **SC1** forfully correct answer in decimal form to 3 sf or more	NB **just** $-\sqrt{2} > x > \sqrt{2}$ or $\sqrt{2} < x < -\sqrt{2}$ or $x > \pm\sqrt{2}$ implies the first **A1** then **A0A0**

Question			Answer/Indicative content	Marks	Part marks and guidance	
			$x > \sqrt{2}$ or $x \geq \sqrt{2}$	A1	or **A2** for $\|x\| > \sqrt{2}$ or $\|x\| \geq \sqrt{2}$ Examiner's Comments The majority of candidates differentiated successfully and went on to identify $\pm\sqrt{2}$correctly. A few neglected the negative root, losing an easy mark. Thereafter candidates went astray in a variety of ways. Many candidates used incorrect forms when writing their inequalities. $x > \pm\sqrt{2}$ was seen frequently and many candidates combined their separate inequalities in illegal ways such as $\sqrt{2} < x < -\sqrt{2}$. These candidates were penalised if the correct inequalities were not seen first. Candidates should realise that it is good practise to write the two inequalities separately first, before any attempt is made to combine them. Some candidates decimalised $\pm\sqrt{2}$ were penalised for having a slight inaccuracy in their answer. A few candidates didn't differentiate at all, thereby ignoring the instruction to use calculus and so made no progress.	
			Total	5		

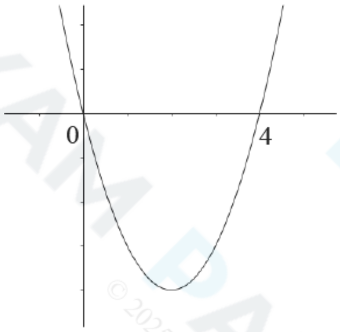
Question			Answer/Indicative content	Marks	Part marks and guidance		
28			$16x^8y^{-2}$ OR $\frac{16x^8}{y^2}$	B2(AO1.1) (AO1.1) [2]	**B1** for two elements correct out of coefficient, power of *x*, power of *y* as part of a product		
			Total	2			
29			Midpoint of $\mathrm{AB} = \left(\frac{5}{2}, 0\right)$ Gradient of perpendicular to $\mathrm{AB} = -\frac{1}{2}$ Perpendicular bisector equation is $y = -\frac{1}{2}\left(x - \frac{5}{2}\right)$ oe Position vector is $\frac{1}{2}\mathbf{i} + \mathbf{j}$ **Alternative method** Suppose C has position vector **c** = *p***i** + **j** $\overrightarrow{\mathbf{AC}}$ = (*p* –1)**i** + 4**j** oe or $\lvert AC\rvert^2 = (p-1)^2 + 4^2$ oe $\overrightarrow{\mathbf{BC}}$ = (*p* – 4)**i** – 2**j** oe or $\lvert BC\rvert^2 = (p-4)^2 + 2^2$ oe $(p-1)^2 + 4^2 = (p-4)^2 + 2^2$ Position vector is $\frac{1}{2}\mathbf{i} + \mathbf{j}$	M1(AO3.1a) M1(AO1.1) A1(AO1.1) A1(AO1.1) [4] M1(AO3.1a) M1(AO1.1) A1(AO1.1) A1(AO1.1)	soi (alongside A1 for $(p-1)^2 + 4^2 = (p-4)^2 + 2^2$)		
			Total	4			

Question			Answer/Indicative content	Marks	Part marks and guidance	
30		a	Radius = 5 (2, –3)	B1(AO1.1) B1(AO1.1) [2]		
		b	$(y + 3)^2 = 21$ or $y^2 + 6y - 12 = 0$ Roots $-3+\sqrt{21}$ and $-3-\sqrt{21}$ $\left(0,-3+\sqrt{21}\right)$ and $\left(0,-3-\sqrt{21}\right)$	M1(AO1.1a) A1(AO1.1) A1(AO1.1) [3]	Substituting $x = 0$ and rearranging For one y-value All correct	
		c	$(1-2)^2 + (2+3)^2 = 1^2 + 5^2$ E.g. This is more than 25 so outside the circle	M1(AO1.1) A1(AO1.1) [2]	Or distance of (1, 2) from their centre Or distance of (1, 2) from centre is $\sqrt{26} > 5$, so outside the circle	

Question			Answer/Indicative content	Marks	Part marks and guidance	
		d	Gradient $\text{CP} = \frac{1-(-3)}{-1-2} = -\frac{4}{3}$ FT their C(entre) Gradient of tangent $= \frac{3}{4}$ FT their grad CP Equation of tangent $y-1=\frac{3}{4}(x-(-1))$ FT their grad 4y = 3x + 7 oe	M1(AO1.1a) M1(AO1.1) M1(AO1.1) A1(AO1.1) [4]		
			Total	11		

Question			Answer/Indicative content	Marks	Part marks and guidance	
31		a	Correct shape for $y=\frac{1}{x}$, translated vertically upward Crosses *x*-axis at $x=-\frac{1}{a}$ Asymptotes *x* = 0 and *y* = *a*	B1(AO1.1) B1(AO2.2a) B2(AO1.1) (AO2.2a) [4]	B1 for one asymptote correct	
		b	$\frac{dy}{dx}=-\frac{1}{x^2}$ When *x* = 2 , gradient = $-\frac{1}{4}$ Gradient of normal = 4 FT their gradient $y-\frac{5}{2}=4(x-2)$FT their gradient 2*y* = 8*x* –11 oe	M1(AO1.1a) M1(AO1.1) M1(AO2.1) M1(AO1.1) A1(AO2.1) [5]	Or gradient of given line is 4 Or $y=\frac{5}{2}$ at the point on the curve where *x* = 2 At least one correct interim step or clear check that $\left(2, \frac{5}{2}\right)$ is on given line Any simplified form	

Question			Answer/Indicative content	Marks	Part marks and guidance	
		c	$2\left(\frac{1}{x}+2\right)=8x-11$	M1(AO3.1a)	Substitution	
			$8x^2 - 15x - 2 = 0$	M1(AO1.1)	Forming quadratic, condone one error	
			Other point is $x = -\frac{1}{8}$, $y = -6$	A1(AO1.1)	BC $x = 2$ not needed in this case	
				[3]		
			Total	12		

Question			Answer/Indicative content	Marks	Part marks and guidance
32			DR		
			$2x^2 - kx = x^2 - k$	B1(AO3.1a)	Equating the two expressions **must** be seen
			$x^2 - kx + k = 0$	M1(AO2.1)	Condone one error in rearranging
			discriminant = $k^2 - 4k$	B1(AO1.2)	
			$k^2 - 4k \geq 0$	M1(AO1.1)	
				M1(AO2.4)	Or give table of values, oe
			$k \leq 0$ or $k \geq 4$	A1(AO2.5) [6]	or $\{k : k \leq 0\} \cup \{k : k \geq 4\}$
			Total	6	

Question			Answer/Indicative content	Marks	Part marks and guidance		
33			DR $y = 4 - 2x$ $4 - 2x = x^2 + x$ $\Rightarrow x^2 + 3x - 4 = 0$ $\Rightarrow x = 1$ or $x = -4$ $y = 2$ or $y = 12$ (1,2) and (–4,12)	M1(AO2.1) M1(AO1.1) A1(AO1.1) A1(AO1.1) A1(AO2.5) [5]	Eliminating x or y must be seen Form a quadratic equation For final A mark, corresponding values of x and y must be expressed as coordinates from well set out correct solution	Or $y^2 - 14y + 24 = 0$ SC1 for one pair of coordinates only	
			Total	5			

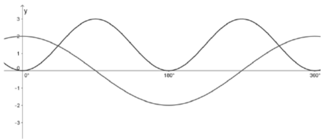
Question			Answer/Indicative content	Marks	Part marks and guidance	
34		a	$\int \frac{dm}{m} = \int \frac{dt}{t(1+2t)}$ $\frac{1}{t(1+2t)} \equiv \frac{A}{t} + \frac{B}{1+2t}$ ⇒ 1 ≡ $A(1 + 2t) + Bt$ $t = 0 \Rightarrow A = 1$ $t = -½ \Rightarrow 1 = -½ B \Rightarrow B = -2$ ⇒ $\int \frac{dm}{m} = \int \left(\frac{1}{t} - \frac{2}{1+2t}\right) dt$ ⇒ $\ln m = \ln t - \ln(1 + 2t) + c$ $t = 1$, $m = 1 \Rightarrow c = \ln 3$ $\Rightarrow \ln m = \ln\left(\frac{3t}{1+2t}\right)$ $\Rightarrow m = \frac{3t}{(1 + 2t)}$	M1(AO1.1a) M1(AO3.1b) M1(AO1.1) A1A1(AO1.1 1.1) B1FT(AO2.1) M1(AO1.1) E1(AO2.1) [8]	separating variables using partial fractions substituting values, equating coeffs or cover up $A = 1$, $B = -2$ FT their *i*, *ii*, condone no *c* evaluating constant of integration AG	
		b	i $1.25 = \frac{3t}{(1+2t)}$ ⇒ $1.25 + 2.5t = 3t$ ⇒ $t = 1.25 \div 0.5 = 2.5$ minutes	M1(AO1.1a) A1(AO1.1) [2]	Enter text here.	
		b	ii $m = \frac{3}{\left(\frac{1}{t} + 2\right)}$ →1.5 [grams]	M1(AO3.1b) A1(AO2.2a) [2]	Enter text here.	
			Total	12		

Question			Answer/Indicative content	Marks	Part marks and guidance	
35		a	Comment about shape of distribution for first graph Comment about shape of distribution for second graph	B1(AO2.2b) B1(AO2.2b) [2]	Comments can be combined e.g Both distributions negatively skewed gets both marks e.g. 1974 distribution has greater spread than 2014 gets both marks	If zero scored, **SC1** for "The 2014 distribution is shifted to the right of the 1974 distribution" oe
		b	i Life expectancy went down [between 1974 and 2014] in [at least] one country	E1(AO2.2a) [1]	**NOT** increase in life expectancy is negative	
		b	ii The box plot is not symmetrical.	B1(AO3.5b) [1]		
		b	iii Not appropriate with reason	E1(AO2.4) [1]	e.g. [some] values of life expectancy are estimates The values of life expectancy are not available to this level of accuracy	
		b	iv Comment about life expectancy at birth data for countries and not individual people	B1(AO2.4) [1]		

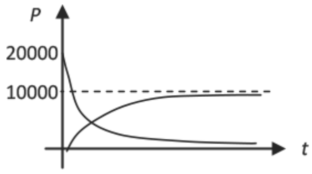
Question			Answer/Indicative content	Marks	Part marks and guidance	
		c	Use of Q3 + 1.5 × (Q3 – Q1) 15.873 + 1.5(8.9154) = 29.2461 (years) The maximum value is an outlier as 30.742>29.2461.	M1(AO1.2) M1(AO1.1) A1(AO1.1) [3]		
		d	*i* approx 60.8 – 37.5= 23.3 (years)	M1(AO3.1b)	Attempt to estimate change in life expectancy at birth soi.	
		d	ii Change in life expectancy for Sweden approx 81.9 –72.5 = 9.4 (years)	A1(AO1.1 1.1)	FT 'their 37.5 between 35 - 40'	
		d	iii E.g. Countries with a lower life expectancy in 1974 have greater opportunity to increase life expectancy in 2014.	A1 E1 (AO3.2a) [4]	FT 'their 72.5 between 70 – 75' OR Countries with a higher life expectancy in 1974 have less opportunity to increase life expectancy in 2014.	
		e	i 30.98 + 0.67 × 37.4 = 56.0 (years)	M1(AO3.4) A1(AO1.1) [2]		

Question			Answer/Indicative content	Marks	Part marks and guidance	
		e	ii E.g. Large amount of scatter at the lower values [and South Sudan is 37.4]. E.g. Not having the data value could indicate that there are problems in the country which could mean it does not follow the pattern for other countries	E1(AO3.5b) E1(AO3.5b) [2]	E1 Reason inferred from Fig 16.4 E1 For knowing why data may be missing	
		f	Correct method Clearly explained 6	M1(AO3.1b) E1(AO2.4) A1(AO1.1) [3]	e.g. draw "$y = x$" on graph e.g. The value on the vertical axis must be lower than the one on the horizontal axis FT their correct method	
			Total	20		

Question			Answer/Indicative content	Marks	Part marks and guidance		
36			$\int_1^3 3x^{-\frac{3}{2}}\,dx$	M1(AO1.1a)	Attempt to integrate (ignore missing limits)	Do not award any A- marks if M0 is given	
			$\left[-6x^{-\frac{1}{2}}\right]_1^3$	A1(AO1.1)	Correct integration		
				A1(AO1.1)	Correct limits seen at some point		
			$\frac{-6}{\sqrt{3}} - \frac{-6}{\sqrt{1}}$	M1(AO1.1)	Substitution of limits (condone one error)	Given answer must be seen to score E1	
			$\frac{-6}{\sqrt{3}} + 6$	E1(AO2.1)	Correct intermediate step using surds which follows from the substitution of limits and is not identical to given answer and completion		
			$6 - 2\sqrt{3}$ AG	[5]			
			Total	5			

Question			Answer/Indicative content	Marks	Part marks and guidance	
37		a		B1(AO1.1a) B1(AO1.1) [2]	Correct shape and symmetry for cosine graph. Correct maximum and minimum values	
		b	DR 2cosθ = 3sin$^2\theta$ 2cosθ = 3(1– cos$^2\theta$) 3cos$^2\theta$ + 2cosθ – 3 = 0 $\cos\theta = \frac{-1}{3} + \frac{\sqrt{10}}{3}$ θ = 43.9°, 316.1° cosθ = $\frac{-1}{3} - \frac{\sqrt{10}}{3} < -1$ gives no solution	B1(AO1.2) M1(AO3.1a) M1(AO1.1) A1(AO1.1) A1(AO1.1) E1(AO2.4) [6]	Correct use of identity **must be seen** Rearranging to zero **must be** seen, condone one error Solve quadratic Or state that graph in part (a) only shows two solutions	

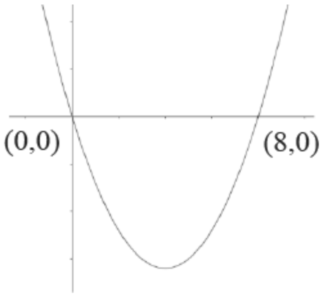
Question			Answer/Indicative content	Marks	Part marks and guidance
			Total	8	

Question			Answer/Indicative content	Marks	Part marks and guidance
38		a	*A* 	M1(AO1.1) M1(AO1.1)	P_G shape through O P_R shape through (0, 20000), [condone graphs for –ve *t*]
		a	*B* asymptote for P_G = 10 000 Asymptote for P_R = 0	A1(AO2.2a) A1(AO2.2a) [4]	Or p = 10 000 Or p = 0
		b	Red squirrels zero Grey 10 000	B1(AO3.4) B1(AO3.4) [2]	
		c	One relevant comment evaluating the validity of the model	B1(AO3.5a)	E.g. One of • Grey population increases as would be expected [since grey squirrels are larger and more successful] • Red population decreases as would be expected [since red

Question			Answer/Indicative content	Marks	Part marks and guidance
				[1]	squirrels have to compete with the larger grey squirrels for food] • Number of squirrels tends to a limit as would be expected [since there is limited food and space] • Would expect grey population to grow slower at first • Would expect red population to fall slower at first

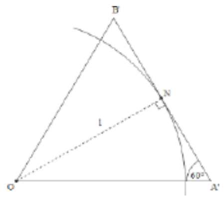
Question			Answer/Indicative content	Marks	Part marks and guidance	
		d		M1(AO3.1b)	Attempts to differentiate either or both	
			$\frac{dP_G}{dt} = 10\ 000ke^{-kt}$	A1(AO1.1)		
			$\frac{dP_R}{dt} = -20\ 000ke^{-kt}$	A1(AO1.1)		
			so $\frac{dP_R}{dt} = -2\frac{dP_G}{dt}$	E1(AO2.1)	Or in words	
				[4]		
		e	10 000 $(1 - e^{-3k}) = 20\ 000\ e^{-3k}$	M1(AO1.1a)		
			$\Rightarrow 1 - e^{-3k} = 2\ e^{-3k}$			
			$\Rightarrow e^{-3k} = \frac{1}{3}$	A1(AO1.1)		
			$\Rightarrow -3k = \ln\left(\frac{1}{3}\right)$	M1(AO1.1)	Taking natural logs of both sides	
			$\Rightarrow k = -\frac{1}{3}\ln\left(\frac{1}{3}\right) = 0.366$ or $\frac{1}{3}\ln 3$	A1cao(AO2.1)		
				[4]		
			Total	15		

Question			Answer/Indicative content	Marks	Part marks and guidance	
39		a	P is $(\sqrt{2}, \frac{\sqrt{2}}{2})$ $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$ $= \frac{\cos\theta}{-2\sin\theta}$ When $\theta = \frac{\pi}{4}, \frac{dy}{dx} = -\frac{1}{2}$ Equation of tangent is $(y - \frac{\sqrt{2}}{2}) = -\frac{1}{2}(x - \sqrt{2})$ $\Rightarrow \quad y = -\frac{1}{2}x + \frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{2}$ $\Rightarrow \quad x + 2y = 2\sqrt{2}$	B1(AO1.1) M1(AO3.1a) A1(AO1.1) A1(AO1.1) B1(AO2.1) E1(AO1.1) **[6]**	oe AG	
		b	When x = 0, y = $\sqrt{2}$ so A is (0, $\sqrt{2}$) When y = 0, x = $2\sqrt{2}$ so B is ($2\sqrt{2}$, 0) Area of triangle $= \frac{1}{2}\sqrt{2} \times 2\sqrt{2} = 2$units^2	B1(AO1.1) B1(AO1.1) B1(AO1.1) **[3]**		
			Total	9		

Question			Answer/Indicative content	Marks	Part marks and guidance	
40			DR discriminant = $k^2 - 8k$ $\Rightarrow k^2 - 8k < 0$  	B1(AO1.2) M1(AO1.1) E1(AO2.4)	Or give table of values, oe	
			$\Rightarrow k > 0$ or $k < 8$	A1(AO2.5) [4]		or $(\infty, 0) \cup (8, \infty)$ or $\{k : k < 0\} \cup \{k : k > 8\}$
			Total	4		
41			$\frac{2(2x+1)+5(x-1)}{(x-1)(2x+1)}$ $= \frac{9x-3}{(x-1)(2x+1)}$	M1(AO1.1) A1(AO1.1) [2]	Numerator should be simplified but need not be factorised, and denominator may be expanded, but mark final answer	
			Total	2		

Question			Answer/Indicative content	Marks	Part marks and guidance	
42		a	cos θ + 2sin θ ° R cos(θ – α) ⇒ Rcos α = 1, Rsin α = 2 ⇒ R2 = 5, R = $\sqrt{5}$ tan α = 2, α = 1.107	M1(AO1.1a) B1(AO1.1) M1A1(AO1.1 1.1) [4]		
		b	max value is $\frac{1}{\left(k - \sqrt{5}\right)}$ $\frac{1}{\left(k - \sqrt{5}\right)} = \frac{\left(3+\sqrt{5}\right)}{4}$ $4 = 3k - 5 + k\sqrt{5} - 3\sqrt{5}$ [This is indep of $\sqrt{5}$ so] k = 3	M1(AO3.1a) M1(AO1.1) A1(AO1.1) [3]		
			Total	7		

Question			Answer/Indicative content	Marks	Part marks and guidance	
43		a	$f(-1) = (-1)^4 + (-1)^3 - 2(-1)^2 - 4(-1) - 2$ $= 1 - 1 - 2 + 4 - 2 = 0$	E1(AO1.1) [1]		
		b	$f(1) = 1 + 1 - 2 - 4 - 2 = -6$ or 'negative' $f(2) = 16 + 8 - 8 - 8 - 2 = 6$ or 'positive' change of sign $\Rightarrow$ root between 1 and 2	B1(AO1.1) E1(AO2.4) [2]	both correct allow no mention of continuity of f AG	
		c	long division or equating coeffts $\Rightarrow g(x) = x^3 - 2x - 2$ so $a = -2$, $b = -2$	M1(AO1.1) A1A1(AO2.2a 1.1) [3]		
		d	Clear explanation E.g. $f(x) = (x + 1)g(x)$ For the root of $f(x) = 0$ between 1 and 2, RHS is also zero hence $g(x) = 0$	E1(AO2.4) [1]		
		e	$x_{n+1} = x_n - \frac{g(x_n)}{g'(x_n)}$ $= x_n - \frac{x_n^3 - 2x_n - 2}{3x_n^2 - 2}$ $= \frac{3x_n^3 - 2x_n - x_n^3 + 2x_n + 2}{3x_n^2 - 2}$ $= \frac{2x_n^3 + 2}{3x_n^2 - 2}$ Root 1.769 (4sf)	M1(AO1.1) E1(AO2.4) A1(AO2.2a) [3]	AG BC	
			Total	10		

Question			Answer/Indicative content	Marks	Part marks and guidance
44			 Show diagram which was previously fig 13 Angle $A'N$ = tan 30° OR $\tan 30^\circ = \frac{1}{A'N}$ $A'N = \tan 30^\circ = \frac{1}{\sqrt{3}}$ **Alternative method** using the equilateral triangle $OA'B'$ of side length $2a$: $(2a)^2 = a^2 + 1 \Rightarrow a^2 = \frac{1}{3}$ $a = A'N = \frac{1}{\sqrt{3}}$ Evidence of 6 × A'B' or 12 × A'N = $4\sqrt{3}$	M1(AO3.1a) A1(AO1.1) M1(AO3.1a) M1(AO1.1) E1(AO2.4) [3]	Soi AG
			Total	3	

Question			Answer/Indicative content	Marks	Part marks and guidance	
45			$\left(\frac{\sqrt{6}-\sqrt{2}}{2}\right)^2 = \frac{8-2\sqrt{12}}{4}$ $= \frac{8-4\sqrt{3}}{4} = 2-\sqrt{3}$ $\frac{\sqrt{6}-\sqrt{2}}{2}$ is positive so it is equal to $\sqrt{2-\sqrt{3}}$	M1(AO3.1a) A1(AO1.1) E1(AO2.1) [3]	Attempt to square Answer in exact form Completion of argument to show the two values are equal	
			Total	3		

Question			Answer/Indicative content	Marks	Part marks and guidance	
46		a	Angle = 360 ÷ 24 = 15 Edge length = 2 tan15° Perimeter = 12 × 2 tan15° = 24 tan15°	M1(AO1.1) E1(AO2.1) [2]	AG	
		b	tan15° = tan (45° – 30°) $= \frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}$ $\left[= \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{\left(\sqrt{3}-1\right)^2}{2}\right]$ **Alternative method** tan15° = tan (60° – 45°) $= \frac{\sqrt{3}-1}{1+\sqrt{3}}$ $\left[= \frac{2\sqrt{3}-4}{-2}\right]$ Perimeter = 12 × 2 tan15° $= 48 - 24\sqrt{3}$	B1(AO3.1a) M1(AO1.1) B1(AO3.1a) M1(AO1.1) E1(AO2.1) [3]	Exact values of tan 45° and tan30° used Exact values of tan 60° and tan15° used Correct completion AG	
			Total	5		

Question			Answer/Indicative content	Marks	Part marks and guidance	
47			*(A)* Lower bound: $3\left(\sqrt{6}-\sqrt{2}\right)$	B1(AO1.1)	Half perimeter (from text)	
			Upper bound: $24-12\sqrt{3}$	B1(AO1.1)		
			(B) = 3.11 and 3.22	B1(AO1.1)	Both as decimals	
				[3]		
			Total	3		
48			$-2 \times 5x^{-2-1}$	M1(AO1.1)	soi	
			6	B1(AO1.1)	or correct differentiation of $6x + 3$	
			$\left[\frac{dy}{dx}=\right]6+10x^{-3}$	A1(AO1.1)		Allow equivalent form, but constant must be simplified to 10.
				[3]		
			Total	3		

Question			Answer/Indicative content	Marks	Part marks and guidance	
49		a	[384 – 400 + 8 + 8 =] 0	B1(AO1.1) [1]		
		b	Long division or equating coefficients $6x^2 - x - 2$ seen $(x - 4)(3x \pm 2)(2x \pm 1)$ $(x - 4)(3x - 2)(2x + 1)$	M1(AO2.1) A1(AO1.1) M1(AO1.1) A1(AO1.1) [4]	DR	
			Total	5		

Question			Answer/Indicative content	Marks	Part marks and guidance	
50			$\frac{2\sin\theta}{\cos\theta}+\cos\theta=0$ 2 sinθ + 1 – sin$^2\theta$ = 0 $\sin\theta=1\pm\sqrt{2}$ $\sin\theta=1+\sqrt{2}$ has no roots since $-1\le\sin\theta\le1$ If $\sin\theta=1-\sqrt{2},\ \theta=-24.47$ or $-155\text{-}53$ 204 335	M1(AO1.1) M1(AO3.1a) A1(AO1.1) E1(AO2.3) A1(AO1.1) A1(AO3.2a) A1(AO1.1) [7]	DR Use of identity Multiplication by cos θ and use of Pythagoras Both answers from correct factorizing or correct use of quadratic formula allow 204.5 or 204.47 allow 335.5 or 335.53	Ignore extra values outside range. Deduct one mark if extra values in range. If A0A0 allow SC1 for both correct answers given to greater precision.
			Total	7		