1 James tries to solve the inequality $x^2 - 5x - 14 \geq 0$. He writes his answer as

$$\{x: x \leq 2\} \cup \{x: x < 7\}.$$

Correct all the errors in his answer. **[3]**

2 (a) Sketch the curve $y = e^{2x}$. [2]

(b) Describe fully the transformation that maps the curve $y = e^{x}$ onto the curve $y = e^{2x}$. [2]

(c) Find the equation of the tangent to $y = e^{2x}$ at the point where $x = 3$, giving your answer in the form $y = e^{a}(bx + c)$ where a, b and c are integers. [6]

3

(a) Use the binomial expansion to show that $(1-2x)^{-\frac{1}{2}} \approx 1+x+\frac{3}{2}x^2$ for sufficiently small values of x. [2]

(b) For what values of x is the expansion valid? [1]

(c) Find the expansion of $\sqrt{\frac{1+2x}{1-2x}}$ in ascending powers of x as far as the term in x^2. [3]

(d) Use $x = \frac{1}{20}$ in your answer to part (c) to find an approximate value for $\sqrt{11}$. [2]

4 (a) Prove the identity $\sec\theta - \cos\theta \equiv \tan\theta \sin\theta$. [4]

(b) Hence or otherwise solve the equation $\sec\theta - \cos\theta = \frac{1}{2}\tan\theta$ for $0 \leq \theta < 2\pi$. [4]

5 **(a)** Solve the differential equation

$$\frac{dy}{dx} = y(1+y)(1-x),$$

given that $y = 1$ when $x = 1$. Give your answer in the form $y = f(x)$, where f is a function to be determined. **[9]**

(b) By considering the sign of $\frac{dy}{dx}$ near (1, 1), or otherwise, show that this point is a maximum point on the curve $y = f(x)$. **[3]**

6 You are given that $gf(x) = |3x - 1|$, for $x \in \mathbb{R}$.

(a) Given that $f(x) = 3x - 1$, express $g(x)$ in terms of x. **[1]**

(b) State the range of $gf(x)$. **[1]**

(c) Solve the inequality $|3x - 1| > 1$. **[4]**

7 (a) In this question you must show detailed reasoning.

Determine the exact values of k for which the curves $y = x^2 - kx$ and $y = 3(k + 1) + kx - x^2$ touch. **[6]**

(b) Determine whether or not there is a value of k for which the curves cross on the y-axis. **[4]**

8 **In this question you must show detailed reasoning.**

A geometric series has first term $(b^2 - 13)$, common ratio $\frac{1}{b}$ and sum to infinity -6.

Find the possible values of the common ratio. **[9]**

9 (a) Write down the exact values of tan 45° and tan 60°. [1]

(b) In this question you must show detailed reasoning.

Show that $\tan 15° = 2 - \sqrt{3}$. [4]

10

(i) Find the value of $\left(1\frac{7}{9}\right)^{-\frac{1}{2}}$. [3]

(ii) Simplify $\frac{(6x^5y^2)^3}{18y^{10}}$. [2]

11

(i) Express $\frac{5-x}{(2-x)(1+x)}$ in partial fractions. [3]

(ii) Hence or otherwise find the first 3 terms of the binomial expansion of $\frac{5-x}{(2-x)(1+x)}$ in ascending powers of x. [5]

12 The equation of a circle is

$$x^2 - 4x + y^2 + 6y - 12 = 0.$$

Find

(a) the coordinates of the centre of the circle, **[2]**

(b) the radius of the circle. **[2]**

13 **In this question you must show detailed reasoning.**

Given that

$$(1 + ax)^n = 1 + 6x - 6x^2 + \ldots,$$

where a and n are constants, find the values of a and n. **[6]**

14 The function f (x) is defined by f(x) = $x^4 - 3x^3 + 3x^2$ for $x \in \mathbb{R}$.

(a) Show that the only stationary point on the curve y = f(x) is a minimum point at the origin. **[7]**

(b) Explain why f(x) does not have an inverse function. **[1]**

The function g(x) is defined by g(x) = $x^4 - 3x^3 + 3x^2$ for $0 \leq x \leq 2$.

(c) State the domain and range of the inverse function $g^{-1}(x)$. **[2]**

15 **(a)** Find the sum of all the even numbers from 2 to 1000. **[3]**

(b) The sum of n consecutive even numbers, starting at 2, is less than 110.

(i) Show that $n^2 + n < 110$. **[1]**

(ii) Find the set of possible values of n given that $n \neq 0$. **[3]**

16 In a chemical reaction, compound B is formed from compound A and other compounds. The mass of B at time t minutes is x kg. The total mass of A and B is always 1 kg. Sadiq formulates a simple model for the reaction in which the rate at which the mass of B increases is proportional to the product of the masses of A and B.

(a) Show that the model can be written as $\frac{dx}{dt} = kx(1-x)$ where k is a constant. **[1]**

Initially, the mass of B is 0.2 kg.

(b) Solve the differential equation, expressing x in terms of k and t. **[7]**

After 15 minutes, the mass of B is measured to be 0.9 kg.

(c) Find the value of k, correct to 3 significant figures. **[2]**

(d) Find the mass of B after 30 minutes. **[1]**

(e) Explain what the model predicts for the mass of A remaining for large values of t. **[1]**

17 **In this question you must show detailed reasoning.**

Fig. 8 shows the curve with parametric equations

$$x = 7\cos\theta + 2\cos 2\theta,\ y = 2 + \sin\theta,\ (0 \leq \theta \leq 2\pi).$$

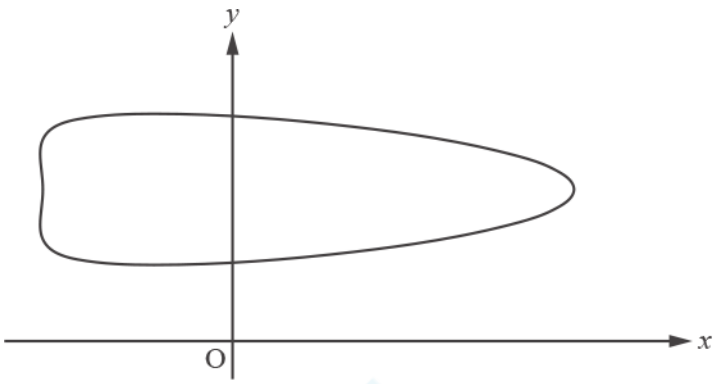


Fig. 8

(a) Find the coordinates of the point on the curve with the greatest y-coordinate. **[4]**

(b) Determine the exact y-coordinates of the points where the curve crosses the y-axis. **[6]**

18 **In this question you must show detailed reasoning.**

A curve has equation $y = x - 5 + \frac{1}{x-2}$. The curve is shown in Fig. 4.

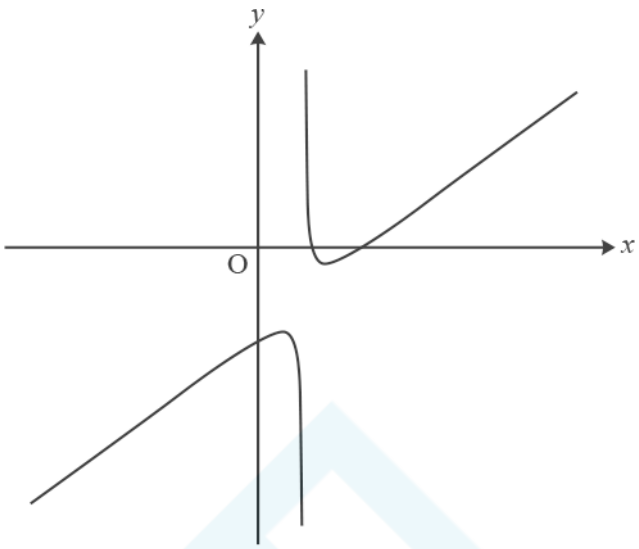


Fig. 4

(a) Determine the coordinates of the stationary points on the curve. **[5]**

(b) Determine the nature of each stationary point. [3]

(c) Write down the equation of the vertical asymptote. [1]

(d) Deduce the set of values of x for which the curve is concave upwards. [1]

(a) Express $\frac{(x^2-8x+9)}{(x+1)(x-2)^2}$ in partial fractions. **[5]**

(b) Express y in terms of x given that

$$\frac{dy}{dx} = \frac{y(x^2 - 8x + 9)}{(x+1)(x-2)^2}$$ and $y = 16$ when $x = 3$.

[7]

20(a) **In this question you must show detailed reasoning.**

A 5-sided spinner can give scores of 1, 2, 3, 4 or 5. After observing a large number of spins, Elaine models the probability distribution of X, the score on the spinner, as shown in Fig. 12.

x	1	2	3	4	5
$P(X = x)$	0.2	0.3	p	p	q

Fig. 12

When the spinner is spun twice, the probability of obtaining a total score of 9 is 0.06.

Given that $q < 2p$, determine the values of p and q. **[6]**

(b) The spinner is spun 10 times. Calculate the probability that exactly one 5 is obtained. **[2]**

(c) Elaine's teacher believes that the probability that the spinner shows a 1 is greater than 0.2. The spinner is spun 100 times and gives a score of 1 on 28 occasions.

Conduct a hypothesis test at the 5% level to determine whether there is any evidence to suggest that the probability of obtaining a score of 1 is greater than 0.2. **[7]**

21 **In this question you must show detailed reasoning.**

Solve the equation $3\cos\theta + 8\tan\theta = 0$ for $0° < \theta < 360°$, giving your answers correct to the nearest degree. **[6]**

22(a) Fig. 10.1 shows a sample collected from the large data set.

$$\text{BMI is defined as } \frac{\text{mass of person in kilograms}}{\text{square of person's height in metres}}.$$

Sex	Age in years	Mass in kg	Height in cm	BMI
Male	38	77.6	164.8	28.57
Male	17	63.5	170.3	21.89
Male	18	68.0	172.3	22.91
Male	18	57.2	172.2	19.29
Male	19	77.6	191.2	21.23
Male	24	72.7	177.0	23.21
Male	25	92.5	177.9	29.23
Male	26	70.4	159.4	27.71
Male	31	77.5	174.0	25.60
Male	34	132.4	182.2	39.88
Male	38	115.0	186.4	33.10
Male	40	112.1	171.7	38.02

Fig. 10.1

Calculate the mass in kg of a person with a BMI of 23.56 and a height of 181.6 cm, giving your answer correct to 1 decimal place. [2]

(b) Fig. 10.2 shows a scatter diagram of BMI against age for the data in the table. A line of best fit has also been drawn.

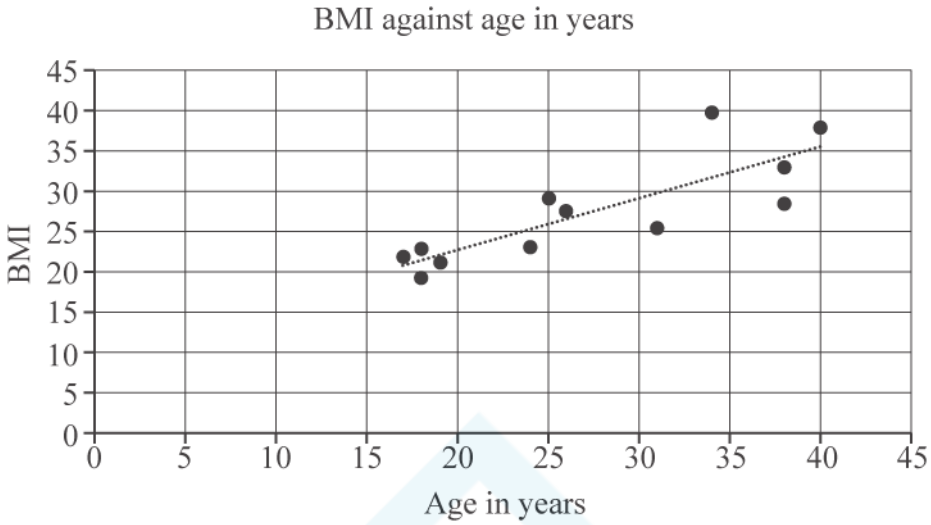


Fig. 10.2

Describe the correlation between age and BMI. [1]

(c) Use the line of best fit to estimate the BMI of a 30-year-old man. [1]

(d) Explain why it would not be sensible to use the line of best fit to estimate the BMI of a 60-year-old man. [1]

(e) Use your knowledge of the large data set to suggest two reasons why the sample data in the table may not be representative of the population. [2]

(f) Once the data in the large data set had been cleaned there were 196 values available for selection. Describe how a sample of size 12 could be generated using systematic sampling so that each of the 196 values could be selected in the sample. [2]

23 **In this question you must show detailed reasoning.**

A circle has centre (2, – 1) and radius 5.

A straight line passes through the points (1, 1) and (9, 5).

Find the coordinates of the points of intersection of the line and the circle. **[8]**

24(a) **Please refer to Insert H640-03 Maths B November 2020 to answer this question.**

Show that the only stationary point on the curve $y = \frac{\ln x}{x}$ occurs where x = e, as given in line 45. **[3]**

(b) Show that the stationary point is a maximum. **[3]**

(c)

It follows from part **(b)** that, for any positive number a with $a \neq \mathrm{e}$, $\dfrac{\ln \mathrm{e}}{\mathrm{e}} > \dfrac{\ln a}{a}$.

Use this fact to show that $\mathrm{e}^a > a^{\mathrm{e}}$. **[2]**

25 The graph of $y = |1 - x| - 2$ is shown in Fig. 2.

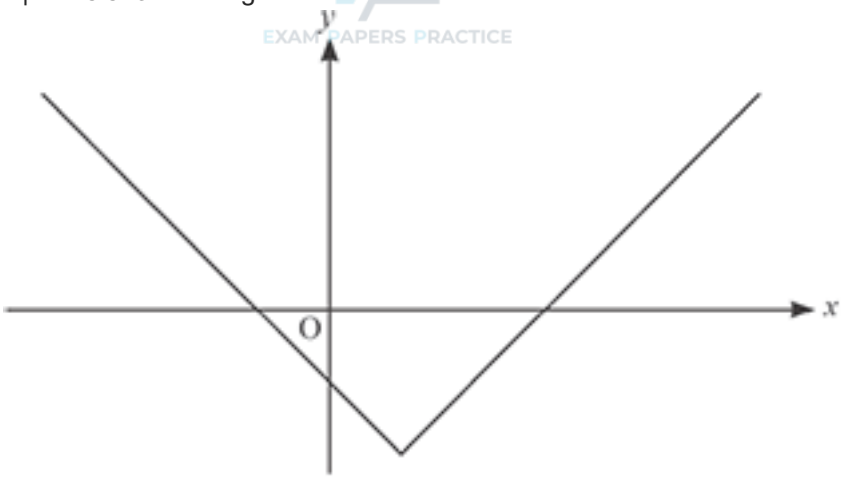


Fig. 2

Determine the set of values of x for which $|1 - x| > 2$. **[4]**

26 **In this question you must show detailed reasoning.**

Show that the equation $2 \log_2 (x + 8) - \log_2 (x + 6) = 3$ has only one root. **[5]**

27 The point A has coordinates (–1, –2) and the point B has coordinates (7, 4). The perpendicular bisector of AB intersects the line $y + 2x = k$ at P.

Determine the coordinates of P in terms of k. **[7]**

28(a) **In this question you must show detailed reasoning.**

A curve has equation $y = 4x^3 - 6x^2 - 9x + 4$.

Sketch the gradient function for this curve, clearly indicating the points where the gradient is zero. **[4]**

(b) Find the set of values of x for which the gradient function is decreasing. Give your answer using set notation. **[2]**

29(a) Douglas wants to construct a model for the height of the tide in Liverpool during the day, using a cosine graph to represent the way the height changes.

He knows that the first high tide of the day measures 8.55 m and the first low tide of the day measures 1.75 m.

Douglas uses t for time and h for the height of the tide in metres. With his graph-drawing software set to degrees, he begins by drawing the graph of $h = 5.15 + 3.4 \cos t$.

Verify that this equation gives the correct values of h for the high and low tide. **[1]**

(b) Douglas also knows that the first high tide of the day occurs at 1 am and the first low tide occurs at 7.20 am. He wants t to represent the time in hours after midnight, so he modifies his equation to $h = 5.15 + 3.4 \cos(at + b)$.

(i) Show that Douglas's modified equation gives the first high tide of the day occurring at the correct time if $a + b = 0$. **[1]**

(ii) Use the time of the first low tide of the day to form a second equation relating a and b. **[1]**

(iii) Hence show that $a = 28.42$ correct to 2 decimal places. **[2]**

(c) Douglas can only sail his boat when the height of the tide is at least 3 m.

Use the model to predict the range of times that morning when he cannot sail. **[3]**

(d) The next high tide occurs at 12.59 pm when the height of the tide is 8.91 m.

Comment on the suitability of Douglas's model. **[2]**

30 To answer this question, please refer to the **Insert for H640-03 June 2019**.

A typical tube of toothpaste measures 5.4 cm across the straight edge at the top and is 12 cm high. It contains 75 ml of toothpaste so it needs to have an internal volume of 75cm^3.

Comment on the accuracy of the formula $V = \frac{2}{3}\pi r^2 h$, as given in line 41, for the volume in this case. **[3]**

31(a) **In this question you must show detailed reasoning.**

Express $\ln 3 \times \ln 9 \times \ln 27$ in terms of $\ln 3$. **[2]**

(b) Hence show that $\ln 3 \times \ln 9 \times \ln 27 > 6$. **[2]**

32 A circle has centre C (10, 4). The x-axis is a tangent to the circle, as shown in Fig. 6.

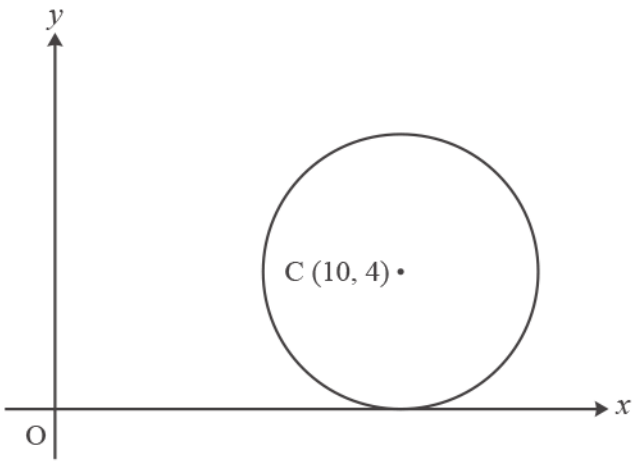


Fig. 6

Show that the line $y = x$ is not a tangent to the circle. **[4]**

33 Express $\frac{1}{(x+2)(x+3)}$ in partial fractions. [3]

34 Solve the equation $4x^{-\frac{1}{2}} = 7$, giving your answer as a fraction in its lowest terms. [3]

35 David puts a block of ice into a cool-box. He wishes to model the mass m kg of the remaining block of ice at time t hours later. He finds that when $t = 5$, $m = 2.1$, and when $t = 50$, $m = 0.21$.

David at first guesses that the mass may be inversely proportional to time. Show that this model fits his measurements. **[3]**

36 **In this question you must show detailed reasoning.**

Show that the equation $x = 7 + 2x^2$ has no real roots. **[3]**

37 (See Insert for H640/03, Practice 4.)

Assume now that the area of the equilateral triangle shown in Fig. C1.1 is one unit of area.

(a) Find the area of the first iteration, shown in Fig. C1.2. **[2]**

(b) Find the increase in area between the iterations shown in Figs C1.2 and C1.3. **[1]**

(c) Find the area of the Koch snowflake. [4]

38 A curve has parametric equations

$$x = 2\cot\theta, \qquad y = 2\sin^2\theta,$$

for values of θ for which both x and y are defined.

(a) For what values of θ, in the interval $0 \leq \theta \leq 2\pi$, is x undefined? [2]

(b) Show that the cartesian equation of the curve is $y = \frac{A}{x^2 + B}$ where A and B are positive integers to be determined. [3]

(c) Ali says that the curve lies completely above the x-axis. Determine whether Ali is correct. **[2]**

39 **In this question you must show detailed reasoning.**

Find the set of values of x for which the line $y = 5x - 6$ lies below the curve $y = x^2$. **[4]**

40 **In this question you must show detailed reasoning.**

The curve $y = \ln x$ passes through the point (a, b), where $a > 1$.

The area A is bounded by the x-axis, the line $x = a$ and the curve $y = \ln x$.

The area B is bounded by the x-axis, the y-axis, the line $y = b$ and the curve $y = \ln x$.

The area A is equal in magnitude to the area B.

(a) Show that a satisfies the equation $pa \ln a + qa + r = 0$, where p, q and r are constants to be determined. **[7]**

The value of a is found using the Newton-Raphson method on a spreadsheet. The output is shown in Fig. 15.

r	x_r
0	4
1	5.177399
2	4.931531
3	4.921571
4	4.921554

Fig. 15

Heidi states that the value of a is 4.921554 correct to 6 decimal places.

(b) Determine whether she is correct. **[2]**

41 Chione uses the equation $V = 22000 - a\sqrt{t}$ to model the value of her caravan, where V is the value in pounds t years after purchase.

(a) The model must give the value of the caravan after 4 years as £12 000. Find the value of a. **[1]**

(b) Find the rate at which the value is changing when the caravan is 4 years old. **[3]**

(c) Explain the limitations of this model for large values of t. **[1]**

Chione creates a second model $V = be^{-0.15t} + c$ in which the initial value is £22 000 and the value after a long time tends to £1000.

(d) Find the values of the constants b and c. **[2]**

Chione wishes to find the times other than $t = 0$ at which the two models give the same value.

(e) Show that these times satisfy the equation $t = -\frac{20}{3}\ln\left(1 - \frac{5}{21}\sqrt{t}\right)$. **[3]**

Chione uses fixed point iteration with this equation and $t_0 = 5$. The table shows some of her values.

t_0	5
t_1	
t_2	
t_3	5.157 893 470
t_4	5.187 562 897
t_5	5.210 144 461

(f) Find the missing values t_1 and t_2. **[2]**

Fig. 14 shows the graphs of $y = t$ and $y = -\frac{20}{3}\ln\left(1 - \frac{5}{21}\sqrt{t}\right)$.

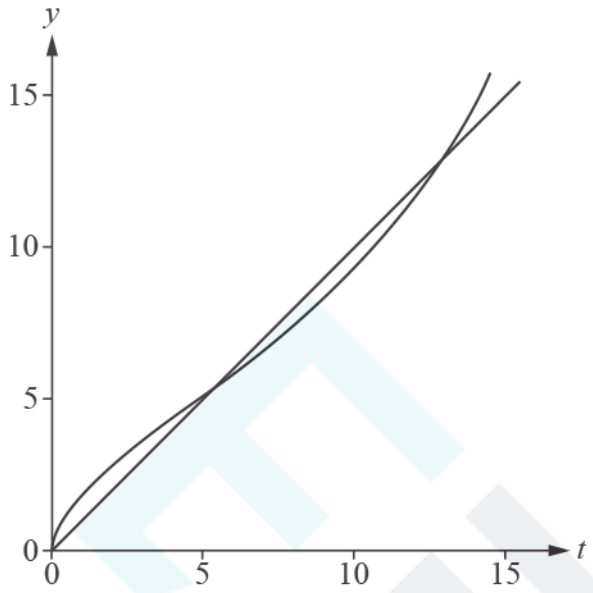
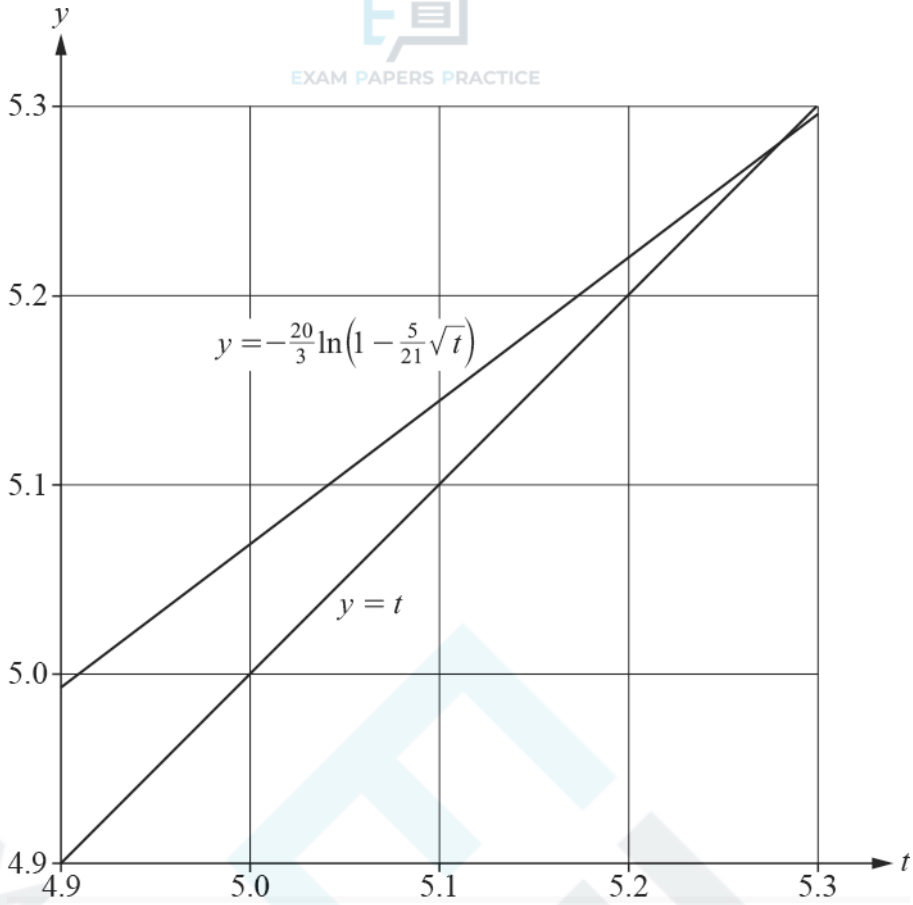


Fig. 14

(g) Sketch the staircase diagram for Chione's iteration on the diagram below. [1]



(h) Chione notices that t_4 and t_5 both round to 5.2 so argues that the root of the equation is 5.2 correct to 1 decimal place. Comment on the validity of her argument. **[1]**

(i) Determine whether this fixed point iteration can be used to find the root near to 12. **[2]**

42 Find the set of values of k for which the equation $2x^2 + kx + 8 = 0$ has distinct real roots. **[3]**

43 The circle C has equation $x^2 + y^2 - 8x + 10y + 16 = 0$.

(a) Find the coordinates of the centre of the circle and the radius of the circle. **[3]**

(b) Show that the circle touches the x-axis. **[1]**

(c) The point P(7, −1) lies on the circle. Find the equation of the tangent to the circle at P, giving your answer in the form $ax + by = c$. **[4]**

44 **In this question you must show detailed reasoning**

A bag contains blue discs and red discs. There are 15 blue discs and an unknown number of red discs. There are more red discs than there are blue discs. A disc is taken at random from the bag and not replaced. A second disc is then taken at random from the bag.

Calculate the probability that 2 blue discs are taken, given that two discs of the same colour are taken. **[8]**

(a) (see insert for practice3 H640/03) The differential equation $\frac{dP}{dQ} = \frac{1}{k}\frac{P}{Q}$ is given on line 42. Find the general solution, giving Q as a function of P. [3]

(b) (see insert for practice3 H640/03) Hence show that, when the PED is constant, a 5% increase in price results in the demand changing by a percentage which is independent of the original price, as stated in lines 43–44. [3]

Use the substitution $u = x + 1$ to find $\int (5x+2)\sqrt{x+1}\,\mathrm{d}x$. Give your answer in the form $kx(x + 1)^p + c$ where k, p and c are constants. **[7]**

47 Prove that $x^2 + x + 2 > 1$ for all real values of x. [3]

48

The curve $y = (x - 1)^2$ maps onto the curve C_1 following a stretch scale factor $\frac{1}{2}$ in the x-direction.

(a) Show that the equation of C_1 can be written as $y = 4x^2 - 4x + 1$. **[2]**

The curve C_2 is a translation of $y = 4.25x - x^2$ by $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$.

(b) Show that the normal to the curve C_1 at the point (0, 1) is a tangent to the curve C_2. **[7]**

(See Insert for Jun18 64003.) Line 8 states that $\frac{a+b}{2} \geqslant \sqrt{ab}$ for a, b $\geq$ 0.

Explain why the result cannot be extended to apply in each of the following cases.

(a) One of the numbers *a* and *b* is positive and the other is negative. **[1]**

(b) Both numbers *a* and *b* are negative. **[1]**

50 **In this question you must show detailed reasoning.**

Fig. 7 shows the curve $y = 5x - x^2$.

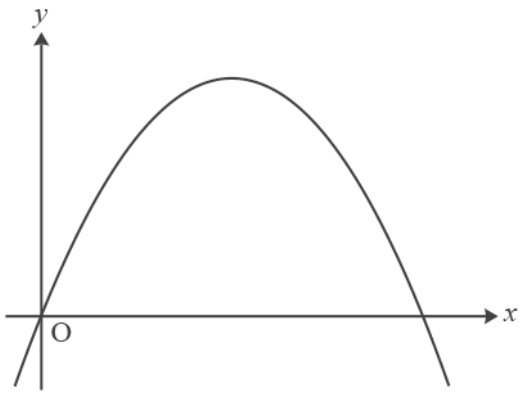
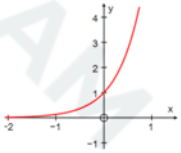


Fig. 7

The line $y = 4 - kx$ crosses the curve $y = 5x - x^2$ on the x–axis and at one other point.

Determine the coordinates of this other point. **[8]**

END OF QUESTION PAPER

Question			Answer/Indicative content	Marks	Guidance
1			Boundary values –2, 7 $\{x : x \leq -2\} \cup \{x : x \geq 7\}$	B1(AO 3.1a) B1(AO 2.3) B1(AO 2.5) [3]	BC For both of $x < -2$, $x > 7$ or better For completely correct answer expressed in correct set notation Ignore use of either 'or' or 'and' here Must be ≤ and ≥ for this mark
			Total	3	
2		a		G1(AO 1.2) G1(AO 1.2) [2]	Correct shape with x-axis as asymptote (0, 1) clearly shown
		b	Stretch in the x-direction Stretch scale factor $\frac{1}{2}$	B1(AO 1.1a) B1(AO 1.1a) [2]	'Stretch' must be seen at least once for any marks to be awarded, but the word needn't be repeated

Question			Answer/Indicative content	Marks	Guidance
		c	$\frac{dy}{dx} = 2e^{2x}$	M1(AO 1.1a)	Allow for any ke^{2x}
				A1(AO 1.1b)	Correct $2e^{2x}$
			When $x = 3$, $\frac{dy}{dx} = 2e^6$	A1(AO 1.1a)	
			and $y = e^6$	B1(AO 1.1a)	
			Tangent is $y - e^6 = 2e^6 (x - 3)$	M1(AO 1.1a)	Allow method if 403 or better used for e^6
			$y = 2e^6x - 5e^6$		
			$y = e^6 (2x - 5)$	A1(AO 2.1)	Must be in this form
				[6]	
			Total	10	

Question			Answer/Indicative content	Marks	Guidance
3		a	$(1-2x)^{-\frac{1}{2}} \approx 1+\left(-\frac{1}{2}\right)(-2x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(-2x)^2+\ldots$ $=1+x+\frac{3}{2}x^2$ **AG**	M1(AO 1.1b) A1(AO 2.1) [2]	Correct form required; allow sign errors Must be correctly obtained
		b	Valid when $\lvert x \rvert < \frac{1}{2}$	E1(AO 2.3) [1]	
		c	$(1+2x)^{\frac{1}{2}} = 1+x-\frac{1}{2}x^2+\ldots$ $\left(1+x-\frac{1}{2}x^2\right)\left(1+x+\frac{3}{2}x^2\right)$ = 1 + 2x + 2x^2 Alternative method $\sqrt{\frac{1+2x}{1-2x}} = \sqrt{\frac{(1+2x)(1+2x)}{(1-2x)(1+2x)}} = \frac{1+2x}{\sqrt{1-4x^2}}$ $=(1+2x)\left(1+\left(-\frac{1}{2}\right)(-4x^2)+\ldots\right)$ = 1 + 2x + 2x^2	B1(AO 1.1a) M1(AO 1.1a) A1(AO 1.1b) M1 M1 A1 [3]	Product of their expansions attempted Converting to rational numerator form Expand denominator and multiply out

Question			Answer/Indicative content	Marks	Guidance
		d	$\sqrt{\frac{1.1}{0.9}} = \frac{\sqrt{11}}{3} \approx 1 + 2\times\frac{1}{20} + 2\times\left(\frac{1}{20}\right)^2$ $\sqrt{11} \approx 3.315$	M1(AO 2.1) A1(AO 2.2a) [2]	Obtaining an expression involving $\sqrt{11}$ Or $\frac{663}{200}$
			Total	8	

Question			Answer/Indicative content	Marks	Guidance
4		a	$\sec\theta - \cos\theta \equiv \frac{1}{\cos\theta} - \cos\theta$	B1(AO 1.2)	Use of definition of sec
			$\equiv \frac{1-\cos^2\theta}{\cos\theta}$	M1(AO 2.1)	Manipulation of fractions
			$\equiv \frac{\sin^2\theta}{\cos\theta} \equiv \frac{\sin\theta}{\cos\theta}\sin\theta$	M1(AO 1.2)	Use of at least one trig identity
			$\equiv \tan\theta \sin\theta$ AG	E1(AO 2.1)	Clear and complete argument
				[4]	
		b	$\tan\theta\sin\theta = \frac{1}{2}\tan\theta \Rightarrow \tan\theta\left(\sin\theta - \frac{1}{2}\right) = 0$	M1(AO 1.1a)	Factorising
			$\tan\theta = 0$ gives $\theta = 0, \pi$	A1(AO 1.1b)	Both values
			$\sin\theta = \frac{1}{2}$ gives $\theta = \frac{1}{6}\pi, \frac{5}{6}\pi$	M1(AO 1.1a)	At least one value from arcsin
				A1(AO 1.1b)	Both values correct
			Alternative method		
			$1 - \cos^2\theta = \frac{1}{2}\sin\theta \Rightarrow \sin^2\theta = \frac{1}{2}\sin\theta$	M1	Obtaining quadracit in $\sin\theta$
			$\sin\theta = 0$, gives $\theta = 0, \pi$	A1	Both values
			$\sin\theta = \frac{1}{2}$ gives $\theta = \frac{1}{6}\pi, \frac{5}{6}\pi$	M1	At least one value from arcsin
				A1	Both values correct
				[4]	

SCI for sole answer $\frac{1}{6}\pi$ www

Question			Answer/Indicative content	Marks	Guidance
			Total	8	

Question			Answer/Indicative content	Marks	Guidance
5		a	$\int \frac{1}{y(1+y)}\,\mathrm{d}y = \int (1-x)\,\mathrm{d}x$	M1(AO 1.1a)	Separating the variables
			$\frac{1}{y(1+y)} = \frac{1}{y} - \frac{1}{1+y}$	M1(AO 3.1a)	Correct form $\frac{A}{y} + \frac{B}{1+y}$ used
				A1(AO 1.1b)	Correct partial fractions
			$\ln y - \ln(1+y) = x - \frac{1}{2}x^2 + c$	A1(AO 1.1b)	oe, e.g. RHS $-\frac{1}{2}(1-x)^2 + c$
			$\ln 1 - \ln(1+1) = 1 - \frac{1}{2} + c$	M1(AO 1.1a)	Use of (1, 1) to find c
			$c = -\frac{1}{2} - \ln 2$	A1(AO 1.1b)	
			$\ln y - \ln(1+y) = x - \frac{1}{2}x^2 - \frac{1}{2} - \ln 2$		
			$\ln\left(\frac{2y}{1+y}\right) = x - \frac{1}{2}x^2 - \frac{1}{2}$		
			$\frac{2y}{1+y} = \mathrm{e}^{x - \frac{1}{2}x^2 - \frac{1}{2}}$	M1(AO 1.1a)	Writing in non-logarithmic form
			$2y = (1+y)\mathrm{e}^{x - \frac{1}{2}x - \frac{1}{2}x^2}$		
			$y = \frac{\mathrm{e}^{x - \frac{1}{2}x^2 - \frac{1}{2}}}{2 - \mathrm{e}^{x - \frac{1}{2}x^2 - \frac{1}{2}}}$	M1(AO 1.1a)	Making y the subject
				A1(AO 1.1b)	Correct $y = \mathrm{f}(x)$
				[9]	

Question			Answer/Indicative content	Marks	Guidance
		b	At, (1, 1), $\frac{dy}{dx} = 1 \times 2(1-1) = 0$	B1(AO 3.1a)	
			Near (1, 1), $y(1 + y)$ is positive and $(1 – x)$ is positive for $x < 1$ but negative for $x > 1$	M1(AO 1.1b)	Determining sign of gradient on each side
			$\frac{dy}{dx} > 0$ for $x < 1$ and $\frac{dy}{dx} < 0$ for $x > 1$ so maximum point	E1(AO 2.2a)	Clear deduction seen
			Alternative method		
			At, (1, 1), $\frac{dy}{dx} = 1 \times 2(1-1) = 0$	B1	
			$\frac{d^2y}{dx^2} = (1+2y)\frac{dy}{dx}(1-x) + (y+y^2)(-1)$	M1	Differentiation using product rule and evaluation at (1, 1)
			$\frac{d^2y}{dx^2} = 3 \times 0 + 2(-1) = -2 < 0$		Condone –2 not seen if clearly negative
			(1, 1) is a maximum point	E1	Must follow both 1st and 2nd derivatives
				[3]	
			Total	12	

Question			Answer/Indicative content	Marks	Guidance
6		a	[g(x) =] $\lvert x \rvert$	B1(AO 1.1) [1]	
		b	All non-negative real numbers	B1(AO 1.1) [1]	Allow any reasonable notation, e.g. $y \geq 0$
		c	**Either** $\lvert 3x - 1 \rvert > 1$ $\Rightarrow 3x - 1 > 1$ or $3x - 1 < -1$ **Or** $(3x - 1)^2 > 1$ $9x^2 - 6x > 0 \Rightarrow 3x(3x - 2) > 0$ Hence $x > \frac{2}{3}, x < 0$ $\{x : x < 0\} \cup \left\{x : x > \frac{2}{3}\right\}$	 M1(AO 1.1) M1(AO 1.1) M1 M1 A1(AO 1.1) A1(AO 2.5) [4]	 Allow for sketch of $y = (3x - 1)^2$ and $y = 1$ Allow for critical values $x = 0, \frac{2}{3}$ For either of these inequalities Final answer must be in this set notation form or else stated as '$x < 0$ or $x > \frac{2}{3}$'
			Total	6	

Question			Answer/Indicative content	Marks	Guidance	
7		a	DR At intersections, $x^2 - kx = 3(k + 1) + kx - x^2$	M1(AO 3.1a)	Forming quadratic	
			$2x^2 - 2kx - 3(k + 1) = 0$	M1(AO 1.1)	0 on one side of quadratic	
			For touching, '$b^2 - 4ac = 0$'	M1(AO 2.1)	Forming quadratic from discriminant	
			$4k^2 + 24(k + 1) = 0 \Rightarrow k^2 + 6k + 6 = 0$	A1(AO 2.2a)	Correct quadratic in 3-term form	
			$k = \frac{-6 \pm \sqrt{12}}{2}$	B1(AO 1.1)	Use of formula or completing the square	$(k + 3)^2 = 3$
			$k = -3 \pm \sqrt{3}$	B1(AO 1.1)		
				[6]		
		b	On y-axis $x = 0$, so $y = 0^2 - 0k = 0$	M1(AO 3.1a)	Use of $x = 0$ to find y	
			If they cross on the y-axis, they cross at the origin	B1(AO 2.2a)		
			$0 = 3(k + 1) + k0 - 0^2$	M1(AO 1.1)	Use of $x = 0$ and $y = 0$	
			$k = -1$ [so a value exists]	A1(AO 2.1)		
				[4]		
			Total	10		

Question			Answer/Indicative content	Marks	Guidance
8			DR $\frac{b^2-13}{1-\frac{1}{b}}=-6$	M1(AO 3.1a)	Use of sum to infinity to form an equation
			$\frac{b(b^2-13)}{b-1}=-6$	M1(AO 3.1a)	Starting to clear fractions
			$b(b^2-13) = -6(b-1)$	A1(AO 1.1)	Correct equation without fractions
			$b^3 - 7b - 6 = 0$	M1(AO 1.1)	Cubic with zero on one side
			$(-1)^3 - 7 \times (-1) - 6 = 0$	M1(AO 2.1)	Use of factor theorem to search for factor
			$(b + 1)(b^2 - b - 6) = 0$	A1(AO 1.1)	Correct fact
			$(b + 1)(b + 2)(b - 3) = 0$	M1(AO 1.1)	Method for solving quadratic
			Roots –1, –2, 3 –1 cannot be the common ratio of a geometric sequence with a sum to infinity	B1(AO 2.3)	Rejection of root that does not make sense in the context
			Possible common ratios are $-\frac{1}{2}$ and $\frac{1}{3}$	A1FT(AO 3.2a)	Follow through their values of b
				[9]	
			Total	9	

Question			Answer/Indicative content	Marks	Guidance	
9		a	$\tan 45^0 = 1$ and $\tan 60^0 = \sqrt{3}$	B1(AO 1.2) [1]		
		b	$\tan(60-45) = \dfrac{\tan 60 - \tan 45}{1 + \tan 60 \times \tan 45}$	M1(AO 3.1a)	DR	Other correct methods eg use of double angle formula are acceptable
			$\dfrac{\sqrt{3}-1}{1+\sqrt{3}}$	M1(AO 1.1)	Substitution of their surds in correct compound angle formula	
			Multiply numerator and denominator by $\sqrt{3}-1$ eg $\dfrac{3-2\sqrt{3}+1}{3-1}$	M1(AO 1.1)		
			$= 2 - \sqrt{3}$	A1(AO 2.1) [4]	AG Convincing arithmetic to given result	
			Total	5		

Question			Answer/Indicative content	Marks	Guidance
10		i	$\frac{3}{4}$oe	3 [3]	**B2** for $\frac{3}{a}$ or $\frac{c}{4}$ or $\pm\frac{3}{4}$ or **M2** for $\left(\frac{4}{3}\right)^{-1}$ or $\left(\frac{9}{16}\right)^{\frac{1}{2}}$ or $\sqrt{\frac{9}{16}}$ or **M1** for $\frac{1}{\left(1\frac{7}{9}\right)^{\frac{1}{2}}}$ or $\left(\frac{16}{9}\right)^{-\frac{1}{2}}$ or $\frac{4}{3}$ isw wrong conversion to decimals **Examiner's Comments** Not many candidates dropped marks in the first part. Those who did usually lost out due to their inability to convert a mixed number into an improper fraction, preventing them from scoring any of the marks. Candidates scoring 0 often seemed to have little idea with indices, but these were a minority. Some candidates reached $\frac{1}{\left(1\frac{7}{9}\right)^{\frac{1}{2}}}$, gaining a mark for this, but then did not know how to proceed with their triple-decker fraction.
		ii	$12x^{15}y^{-4}$ or $\frac{12x^{15}}{y^4}$	2 [2]	**B1** for two elements correct if B0, allow **M1** for expanded numerator $= 6^3x^{15}y^6$ or $216x^{15}y^6$ **Examiner's Comments** In the second part, the vast majority of candidates coped well, the main mistakes were usually due to the misapplication of the rules of indices, adding when the powers should be multiplied. What was concerning was the minority of candidates who could not multiply or divide the numerical values forming the coefficient.
			Total	5	

Question			Answer/Indicative content	Marks	Guidance
11		i	$\frac{5-x}{(2-x)(1+x)} = \frac{A}{2-x} + \frac{B}{1+x}$ $\Rightarrow 5 - x = A(1 + x) + B(2 - x)$ $x = 2 \Rightarrow 3 = 3A$, $A = 1$ $x = -1 \Rightarrow 6 = 3B \Rightarrow B = 2$	M1 A1 A1 [3]	Cover up, substitution or equating coefficients Examiner's Comments Part (i) was answered extremely well with the vast majority of candidates correctly expressing $\frac{5-x}{(2-x)(1+x)}$ in partial fractions.

Question			Answer/Indicative content	Marks	Guidance
		ii	$\frac{A}{2-x}=\frac{A}{2}(1-\frac{1}{2}x)^{-1}$ $=\frac{A}{2}(1+(-1)(-\frac{1}{2}x)+\frac{(-1)(-2)}{2!}(-\frac{1}{2}x)^2+...)$	B1 M1	Or equivalent All three correct **unsimplified** binomial coefficients (not nCr) soi **for either expansion** i.e. 1, –1 and $\frac{(-1)(-2)}{2}$. Or correct simplified coefficients seen
			$=A\left(\frac{1}{2}+\frac{1}{4}x+\frac{1}{8}x^2+...\right)$	A1ft	Ignore any subsequent incorrect terms – ft their *A* from (i) only
			$\frac{B}{1+x}=B(1+x)^{-1}=B(1-x+x^2+...)$	A1ft	Ignore any subsequent incorrect terms – ft their *B* from (i) only
			$\frac{5-x}{(2-x)(1+x)}=\frac{5}{2}-\frac{7}{4}x+\frac{17}{8}x^2+...$	A1	www cao – ignore any higher order terms stated – isw after correct expansion seen
				[5]	**Examiner's Comments** In part (ii) most candidates used their answer to part (i) in their attempt to find the binomial expansion of $\frac{5-x}{(2-x)(1+x)}$ although some candidates did (with varying degrees of success) attempt to expand $(5-x)(2-x)(1+x)^{-1}$ directly. Whilst the majority of candidates correctly dealt with the expansion of $\frac{2}{1+x}$ (and so scored at least two marks in this part) it was surprising how many candidates (at this level) struggled in re-writing $\frac{1}{2-x}$ as $\frac{1}{2}\left(1-\frac{1}{2}x\right)^{-1}$. In some cases it was clear that candidates either did not realise or even recognise that the 2 inside the bracket had to be removed before this term could be binomially expanded. Those candidates who expanded both terms correctly usually went on to score full marks.
			Total	8	

Question			Answer/Indicative content	Marks	Guidance
12		a	$(x-2)^2 + (y+3)^2$ seen (2, –3)	M1(AO1.1) A1(AO1.1b) [2]	ignore other working
		b	$(x-2)^2 + (y+3)^2 - 2^2 - 3^2 - 12 = 0$ or better seen $r = 5$	M1(AO1.1b) A1 (AO 1.1b) [2]	
			Total	2	
13			$na = 6$ $\frac{n(n-1)}{2!}a^2 = -6$ Substitution of $a = \frac{6}{n}$ in second equation oe $18(n-1) = -6n$ soi $n = ¾$ $a = 8$	M1(AO3.1a) A1(AO2.1) M1(AO1.1b) A1(AO1.1b) A1(AO1.1b) A1(AO1.1b) [6]	
			Total	6	

Question			Answer/Indicative content	Marks	Guidance
14		a	At stationary point: $4x^3 - 9x^2 + 6x = 0$	M1(AO1.1a)	Attempt to differentiate & equate to 0
			$x(4x^2 - 9x + 6) = 0 \Rightarrow x = 0$ or $4x^2 - 9x + 6 = 0$	M1(AO1.1a)	Factorising the cubic
			Discriminant of quadratic is $(-9)^2 - 4 \times 24 = -15 < 0$	M1(AO1.1a)	oe (quadratic formula, completing the square, complex roots from calculator)
			So $x = 0$ gives the only stationary point	A1(AO2.2a)	
			f (0) = $0^4 - 3 \times 0^3 + 3 \times 0^2 = 0$, so the point is the origin	B1(AO2.1)	f(0) or y must be shown to be zero
			$f''(x) = 12x^2 - 18x + 6 \Rightarrow f''(0) = 6 > 0$	M1(AO1.1a)	Or other valid method, e.g. sign of $f'(x)$ for $x < 0$ and $x > 0$
			So the origin is a minimum point	A1(AO2.2a) [7]	AG Must establish minimum clearly
		b	f is not a one-to-one function	B1(AO2.4) [1]	

Question			Answer/Indicative content	Marks	Guidance
		c	g(0) = 0 and g(2) = 4 Domain of $g^{-1}(x)$ is range of g(x), so $0 \leq x \leq 4$ Range of $g^{-1}(x)$ is domain of g(x), so $0 \leq g^{-1}(x) \leq 2$	B1(AO2.2a) B1(AO1.2) [2]	Must not be in terms of y Must not be an interval for x
			Total	10	

Question			Answer/Indicative content	Marks	Guidance	
15		a	$n = 500$	B1(AO3.1a)		
			$\frac{500}{2} \times 1002$	M1(AO1.1)	Use of formula for sum of AP	
			250 500	A1(AO1.1)		
				[3]		
		b	(i) $\frac{n}{2}(2+2n) < 110$			At least one line of working leading to given answer is required
			So $n + n^2 < 110$	E1(AO2.4)	AG	
				[1]		
			(ii) $n^2 + n - 110 < 0$ $\Rightarrow (n + 11)(n - 10) < 0$	M1(AO1.1a)	Or roots of quadratic are 10 and –11	
			[–11<] $x < 10$	M1(AO1.1)	Or relevant sketch of quadratic	
			{1, 2, 3, 4, 5, 6, 7, 8, 9} oe	A1(AO1.1)	Correct set of numbers clearly stated	
				[3]		
			Total	4		

Question			Answer/Indicative content	Marks	Guidance
16		a	$\frac{dx}{dt}$ is the rate at which x is increasing Mass of B is x, so mass of A is $(1 - x)$ $\frac{dx}{dt} \propto x(1-x)$, so $\frac{dx}{dt} = kx(1-x)$	B1(AO2.1) [1]	Must indicate where terms come from AG
		b	$\int \frac{1}{x(1-x)}\,dx = \int k\,dt$ $\frac{1}{x(1-x)} = \frac{A}{x} + \frac{B}{1-x}$ $1 \equiv A(1-x) + Bx \Rightarrow A = 1, B = 1$ $\int \left(\frac{1}{x} + \frac{1}{1-x}\right) dx = \int k\,dt \Rightarrow \ln x - \ln(1-x) = kt + c$ $t = 0,\ x = 0.2 \Rightarrow c = \ln\frac{1}{4}$ (oe) $\frac{4x}{1-x} = e^{kt}$ (oe) $x = \frac{e^{kt}}{4 + e^{kt}}$	M1(AO3.1a) M1(AO3.1a) A1(AO2.2a) M1*(AO1.1a) M1dep*(AOs3.1a) M1dep*(AO1.1b) A1(AOs2.5) [7]	Separation of variables Find partial fractions (may be implied) Condone sign error, but must have two ln terms and +c Use of initial conditions; may be done after equation is rearranged Rearrange equation to remove logs; may be done before finding c oe, but must be of the form $x = f(t)$
		c	$t = 15$, $x = 0.9 \Rightarrow 36 = e^{15k}$ (oe) $k = 0.239$ to 3sf	M1(AO3.1) A1(AO1.1b) [2]	Substitute values in their solution

Question			Answer/Indicative content	Marks	Guidance
		d	$t = 30 \Rightarrow$ mass of B is $\frac{e^{0.239\times30}}{4+e^{0.239\times30}} = 0.997\,\text{kg}$	B1(AO3.4) [1]	
		e	As $t \to$ ro, $x \to 1$ and so $1 - x \to 0$, so the model predicts there is a very small amount of A remaining when t is large	B1(AO3.5a) [1]	May evaluate x for large t (eg $t = 100$)
			Total	12	

Question			Answer/Indicative content	Marks	Guidance
17		a	DR $\frac{dy}{dx} = 0$ when $\frac{dy}{d\theta} = \cos\theta = 0$	M1(AO3.1a)	
			$\theta = \frac{1}{2}\pi,\ \frac{3}{2}\pi$	M1(AO1.1)	
			$x = -2$	A1(AO1.1)	
			$y = 3$	A1(AO3.2a)	
			Alternative solution		
			Maximum y occurs when sin $\theta = 1$	M1	
			$y = 3$	A1	
			$\theta = \frac{1}{2}\pi$	M1	
			$x = -2$	A1	
				[4]	

Question			Answer/Indicative content	Marks	Guidance
		b	DR $7\cos\theta + 2\cos 2\theta = 0 \Rightarrow 4\cos^2\theta + 7\cos\theta - 2 = 0$	M1(AO1.1a)	Use of double angle formula
			$(4\cos\theta - 1)(\cos\theta + 2) = 0$	M1(AO1.1)	Method for solving quadratic
			$\cos\theta = \frac{1}{4}$ or -2	A1(AO1.1)	
			Reject $\cos\theta = -2$	B1(AO3.2a)	May be seen later
			$\sin^2\theta = 1 - \cos^2\theta = 1 - \left(\frac{1}{4}\right)^2 = \frac{15}{16}$	M1(AO3.1a)	
			$y = 2 \pm \frac{\sqrt{15}}{4}$	A1(AO2.2a)	oe exact form
				[6]	
			Total	10	

Question			Answer/Indicative content	Marks	Guidance
18		a	$\frac{dy}{dx} = 1 - \frac{1}{(x-2)^2}$	M1(AO 1.1a)	Attempt to differentiate with one term correct
				A1(AO 1.1)	Correct derivative
			$1 - \frac{1}{(x-2)^2} = 0$ at stationary points	M1(AO 1.1a)	
			$x - 2 = \pm 1$ so $x = 1, 3$	A1(AO 2.2a)	Both values of x
			(1, –5) (3, –1)	A1(AO 1.1) [5]	Both values of y – ft *their* x

Examiner's Comments

Most candidates were able to score full marks here following correct differentiation and solution of what ended up as a quadratic equation. Many different ways were used to solve the equation usually without any wrong working. Examiners were pleased to see correct notation used in this question.

Question			Answer/Indicative content	Marks	Guidance
		b	$\frac{d^2y}{dx^2} = \frac{2}{(x-2)^3}$	M1(AO 1.1a)	OR Allow consideration of gradient either side of stationary point for **M1** Correct gradients above and below each tp **A1** Correct convincing conclusions (possibly with sketches) **A1**
			$x = 3$ $\frac{d^2y}{dx^2} > 0$ (2) so minimum	A1(AO 2.4)	
			$x = 1$ $\frac{d^2y}{dx^2} < 0$ (–2) so maximum	A1(AO 2.4)	
				[3]	

x	f'(x)	x	f'(x)
0.5	0.56	2.5	-3.00
0.6	0.49	2.6	-1.78
0.7	0.41	2.7	-1.04
0.8	0.31	2.8	-0.56
0.9	0.17	2.9	-0.23
1	0.00	3	0.00
1.1	-0.23	3.1	0.17
1.2	-0.56	3.2	0.31
1.3	-1.04	3.3	0.41
1.4	-1.78	3.4	0.49
1.5	-3.00	3.5	0.56

Examiner's Comments

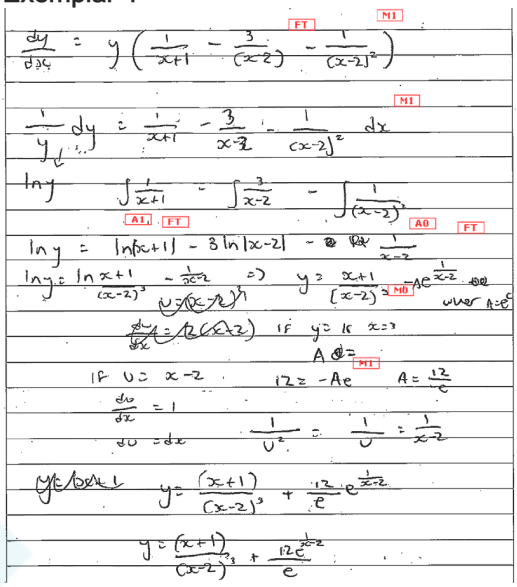
Again most candidates were successful in classifying the stationary points with use of the second derivative being the most common method. A few considered the gradient either side of each turning point and then reasoned their way to a correct conclusion.

Question			Answer/Indicative content	Marks	Guidance
		c	$x = 2$	B1(AO 1.2) [1]	 Examiner's Comments There appeared to be confusion as to the meaning of vertical asymptote which led to a low success rate for this part.
		d	$x > 2$	A1(AO 2.2a) [1]	FT *their* (c) if region is to right of *their x* value Examiner's Comments As in part (c), a large proportion of the candidates struggled with this part. AfL The OCR B (MEI) H640 specification defines the terms "concave upwards" and "concave downwards" as those that will be used in examination questions.
			Total	10	

Question			Answer/Indicative content	Marks	Guidance
19		a	$\frac{A}{(x+1)}+\frac{B}{(x-2)}+\frac{C}{(x-2)^2}$ $x^2 - 8x + 9 = A(x - 2)^2 + B(x + 1)(x - 2) + C(x + 1)$ $A = 2$ $B = -1$ $C = -1$	B1 (AO 3.1a) M1 (AO 2.1) A1 (AO 1.1) A1 (AO 1.1) A1 (AO 1.1) [5]	may be seen later $\frac{2}{(x+1)}-\frac{1}{(x-2)}-\frac{1}{(x-2)^2}$ Examiner's Comments Candidates who did well recognised the correct form of partial fractions and were able to work successfully to find the coefficients. Candidates who did less well made algebraic slips in clearing the fractions or made slips in arithmetic when finding the coefficients.
		b	$\int\frac{dy}{y}=\int\frac{x^2-8x+9}{(x+1)(x-2)^2}dx$ soi use of their partial fractions in integration $\ln\lvert y\rvert = 2\ln\lvert x+1\rvert - \ln\lvert x-2\rvert + \frac{1}{x-2} + c$	M1* (AO 3.1a) M1* (AO 2.1) A1 (AO 1.1) A1 (AO 1.1)	allow omission of integral signs and/or omission of dy and/or dx allow one sign error and/or one coefficient error A1 for any correct natural log integral on RHS FT *their* $\frac{2}{x+1}$ or *their* $\frac{-1}{x-2}$ A1 for $\frac{1}{x-2}$ FT *their* $\frac{k}{(x-2)^2}$

condone use of brackets instead of modulus signs; these two **A** marks are only available following the award of **both M** marks

Question	Answer/Indicative content	Marks	Guidance	
	substitution of $y = 16$ and $x = 3$	M1dep* (AO 1.1)	expression must include + c and must include at least one natural log term; may be awarded after exponentiating	may be awarded following collection of like terms, which may contain errors NB $c = -1$
	correctly exponentiate both sides of their equation	M1 (AO 1.1)		
	$y = \frac{(x+1)^2}{x-2}e^{\frac{3-x}{x-2}}$ oe	A1 (AO 2.1)	eg $\frac{(x+1)^2}{x-2}e^{\frac{1}{x-2}}e^{-1}$	
		[7]	Examiner's Comments Candidates who did well recognised the need to use their result from part (a). They separated the variables successfully and were then able to integrate and substitute the values of x and y to find the constant of integration. Candidates who did very well were able to go on and find a correct expression for y. Candidates who did less well rearranged incorrectly when they attempted to separate the variables, or were unable to integrate the quadratic term correctly. They made slips in exponentiating both sides of their equation, usually assuming that the operation is distributive.	

Question			Answer/Indicative content	Marks	Guidance
					Exemplar 4  In this response FT marks have been credited for the use of their partial fractions and separation of variables. One A mark has been credited FT, but the integration of the quadratic term went astray. The exponentiation of both sides was incorrect, but in spite of this, the method mark for substitution was subsequently earned.
			Total	12	

Question			Answer/Indicative content	Marks	Guidance	
20	a		$2p + q + 0.2 + 0.3 = 1$ **soi oe**	B1 (AO2.1)	allow **M1** if 2 omitted	
			$2 \times p \times q = 0.06$ **soi**	M1 (AO3.1a)		**NB** if 2 omitted, **A0** for $2p^2 - 0.5p + 0.6 = 0$ or $2q^2 + q + 0.24 = 0$ which have no real root
			eliminate p or q with a correct substitution from one of *their* equations	M1 (AO1.1)	eg $2 \times \frac{0.03}{q} + q = 0.5$ or $2p + \frac{0.03}{p} = 0.5$	
			$q^2 - 0.5q + 0.06 = 0$ or $2p^2 - 0.5p + 0.03 = 0$ **oe**	A1 (AO1.1)		
			$q = 0.2$ or 0.3 **and** $p = 0.15$ or 0.1	A1 (AO1.1)	may be implied by eg $q = 0.2$ or 0.3 **and** $2p = 0.3$ or 0.2	
			($q < 2p$ so) $q = 0.2$ and $p = 0.15$	A1 (AO3.2a) [6]		
	b		$10 \times q \times (1 - q)^9$ **soi**	M1 (AO1.1)	**FT** *their* q where $0 < q < 1$	
			0.27 or 0.268 or awrt 0.2684 **isw**	A1 (AO1.1) [2]		

Question			Answer/Indicative content	Marks	Guidance	
	c		H_0: $p = 0.2$	B1 (AO1.1)	both hypotheses; allow equivalent in words or eg P(1) = 0.2	allow any parameter as long as clearly defined as probability
			H_1: $p > 0.2$			
			p is the probability that the spinner shows a 1 (on any given spin) **oe**	B1 (AO2.5)	k = 27, 28 or 29	**M0** for P($X = k$) **NB** P($X = 28$) = 0.014… P($X = 27$) = 0.020168…
			use of $X \sim$ B(100,0.2) where x is the number of 1s obtained in 100 spins to obtain P($X \geq k$) or P($X \leq k$)	M1 (AO3.3)	or critical region is $X \geq 28$	
			P($X \leq 27$) = awrt 0.97	A1 (AO1.1)	or 28 is in critical region	**FT** *their* probability, dependent on award of first **M1**
			or P($X \geq 28$) = awrt 0.034	M1 (AO3.4)	must have the correct probability or correct critical region for the last two **A** marks	
			0.034 < 0.05 or 0.97 > 0.95			do not allow eg conclude / prove / indicate or other assertive statement instead of suggest; **A0** if answer spoiled
			significant or reject H_0 or accept H_1; may be embedded in conclusion in context	A1 (AO1.1)		
			there is sufficient evidence to **suggest** (at 5% level) that the probability of a score of 1 is greater than 0.2	A1 (AO2.2b) [7]		
			Total	**15**		

Question			Answer/Indicative content	Marks	Guidance	
21			$3\cos\theta + 8\frac{\sin\theta}{\cos\theta}[=0]$	M1* (AO1.2)		
			$3\cos^2\theta + 8\sin\theta = 0$	M1dep* (AO1.1)	multiplication of all terms of their equation by cos θ	Condone sign errors and number errors for the three M marks
			$3(1-\sin^2\theta) + 8\sin\theta = 0$	M1 (AO2.4)	dependent on award of previous **M1**	
			$3\sin^2\theta - 8\sin\theta - 3 = 0$	A1 (AO1.1)		
			$\sin\theta = -\frac{1}{3}$ WWW	A1 (AO1.1a)	from factorising or quadratic formula	May see $\sin\theta = 3$
			$\theta = 341, 199$	A1 (AO2.4) [6]	No wrong values; accept 199.47, 340.53	
			Total	6		

Examiner's Comments

Almost all candidates were able to begin this question by replacing *tanθ* with $\frac{\sin\theta}{\cos\theta}$, and then going on to multiply through the equation by cos*θ*. Some candidates lost the '8' or arrived at '24', and only a minority ended up with the correct quadratic equation in sin*θ*. Most arriving at this point were able to discard the root of 3, though not all could correctly derive the 2 correct angles from $\sin^{-1}\left(-\frac{1}{3}\right)$.

Question			Answer/Indicative content	Marks	Guidance
22	a		Mass = 23.56×1.816^2 =77.7 [kg]	M1 (AO1.1) A1 (AO1.1) [2]	Allow M1 for cm used here CAO Examiner's Comments Most candidates were able to find the mass correctly, though some did not realise the height is squared in the formula, and others used the height in cm rather than m. The correlation in part (b) was generally described correctly as 'positive' or by a description such as 'as age increases so does BMI'. Part (c) was usually done correctly, but few gave the answer that (d) would require extrapolation, which is not sensible. Some candidates did correctly explain that the data only went up to an age of 45, and so the line of best fit could not be used at 60, while others gave an incomplete argument, that the lack of data at (or about) age 60 meant the line of best fit should not be used. For part (e), a good number of candidates correctly pointed out that the sample data included no females, and the range of ages used was restricted. There were no completely correct answers to part (f); some candidates gained a mark for saying that every *n*th value should be taken (where *n* was usually 16), but none explained the need for a random starting point.
	b		Positive [correlation]	B1 (AO1.1) [1]	Ignore, eg, weak or strong Accept 'as age increases so does BMI' oe
	c		$28 \leq BMI < 30$	B1 (AO3.4) [1]	
	d		extrapolation	B1 (AO3.5b) [1]	Or equivalent

Question			Answer/Indicative content	Marks	Guidance	
	e		eg females in population wider age range in population scatter diagram for whole population shows very weak negative correlation	B1 (AO2.2b) B1 (AO2.2b) [2]	any two distinct comments	
	f		Eg Generate a random number, *n*, between 1 and (eg 20) inclusive and select the *n*th item in the data set OR select every *m*th (eg 16th value). For valid solution.	M1 (AO1.2) A1 (AO1.1) [2]	Random no between 1 and 9 and every 17th item would also work.	Random no between 1 and 31, and then every 15th item also works, etc.
			Total	9		

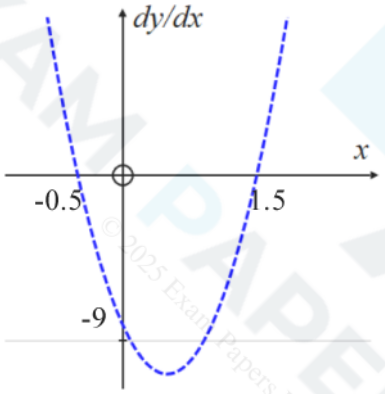
Question			Answer/Indicative content	Marks	Guidance	
23			Circle: equation is $(x-2)^2+(y+1)^2=5^2$ oe Line: $m=\frac{1}{2}$ $y-5=(\text{their }\frac{1}{2})(x-9)$ oe Substitution of their $y = ½(x + 1)$ in their attempt at equation of circle $5x^2–10x–75 = 0$ oe $x = 5$ or $x = – 3$ (5, 3) and (– 3, – 1)	M1 (AO1.1) A1 (AO1.1) B1 (AO1.1) M1 (AO1.1a) M1 (AO3.1a) A1 (AO1.1) A1 (AO1.1) A1 (AO2.5) [8]	Allow sign error, reversed 1 and –2, 5 not squared. Or $y – 1 =$ (their $\frac{1}{2}$)$(x – 1)$ Or their $x = 2y – 1$ Or $5y^2 – 10y – 15 = 0$ oe Or $y = 3$ and $y = – 1$ If A0A0, sc1 for (5, 3) or (– 3, – 1) only	Sight of $y = ½(x + 1)$ $x^2 – 2x – 15 = 0$
			Total	8		

<u>Examiner's Comments</u>
Most candidates were able to write down the equation of the circle, though sign errors were seen both inside and between the brackets. The straight line equation was usually found correctly, and most candidates made an attempt to eliminate one variable. Although some candidates made errors in the manipulation, a good number arrived at the correct points of intersection.

Question			Answer/Indicative content	Marks	Guidance	
24	a		$\frac{dy}{dx} = \frac{\frac{x}{x} - \ln x}{x^2}$ $\frac{1-\ln x}{x^2} = 0 \Rightarrow \ln x = 1 \Rightarrow x = e$	M1 (AO1.1a) A1 (AO1.1) E1 (AO2.2a) [3]	M1 for attempt to use quotient rule (allow one error) Convincing completion (AG)	Subbing x = e gets M0
	b		$\frac{dy}{dx} = \frac{1}{x^2} - \frac{\ln x}{x^2}$ $\frac{d^2y}{dx^2} = -\frac{2}{x^3} - \frac{\frac{x^2}{x} - 2x\ln x}{x^4}$ $= -\frac{2}{x^3} - \frac{x - 2x\ln x}{x^4}$ or $\frac{x^2\left(-\frac{1}{x}\right) - (1-\ln x)2x}{x^4}$ $= \frac{-3 + 2\ln x}{x^3}$ When $x = e$, $\frac{d^2y}{dx^2} = -\frac{1}{e^3} < 0$ hence maximum	M1 (AO1.1a) A1 (AO1.1) E1 (AO2.1) [3]	Attempt to differentiate again (allow one error) Correct second derivative Correct conclusion from correct working	OR M1 subst (must be seen) values either side of e into derivative A1 correct conclusion about sign of gradient E1 correct conclusion from correct working regarding maximum (-0.05)
	c		$\frac{\ln e}{e} > \frac{\ln a}{a} \Rightarrow \frac{1}{e} > \frac{\ln a}{a}$ or $a\ln e > e\ln a$ $e^{\frac{a}{e}} > a$ hence $e^a > a^e$ or $\ln e^a > \ln a^e$ $e^a > a^e$	M1 (AO3.1a) A1 (AO2.4) [2]	Convincing completion (AG)	
			Total	8		

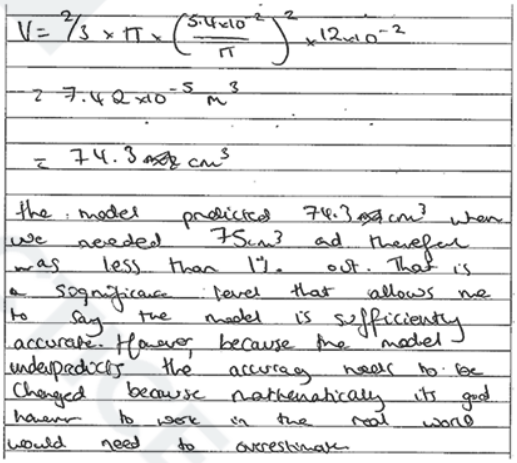
Question			Answer/Indicative content	Marks	Guidance
25			Attempt to find where graph crosses x-axis	M1 (AO3.1a)	E.g. $x = -1$ or $x = 3$ seen or $(1 - x)^2 > 2^2$
			Both $x = -1$ and $x = 3$ seen	A1 (AO1.1)	May be in final answer
			$x < -1$ or $x > 3$ OE	B2 (AO2.2a)	**B1** if equals included in inequalities but otherwise correct (both inequalities needed for B1 or B2)
				(AO2.2a) [4]	Allow , (comma) for 'or' but do not allow 'and'
					<u>Examiner's Comments</u> There was difficulty in dealing with the inequalities in Question 2, with some candidates losing marks by writing 'and' between their two inequalities.
			Total	4	
26			Using laws of logs $\log_2(x+8)^2 - \log_2(x+6) = 3$ $\log_2 \frac{(x+8)^2}{(x+6)} = 3$ $\frac{(x+8)^2}{(x+6)} = 2^3$ $(x+8)^2 = 8(x+6)$ $x^2 + 8x + 16 = 0$ Discriminant is $8^2 - 4 \times 1 \times 16 = 0$ So there is only one solution	M1(AO3.1a) M1(AO3.1a) A1(AO1.1) M1(AO2.1) A1(AO2.2a) [5]	**DR** At least one correct use of laws of logs Clearing logs to obtain 2^3 or 8 seen in an equation Correct quadratic Attempt to find the discriminant of their quadratic (allow one slip) Correct argument from zero discriminant or repeated root found $x = -4$ Allow M1 for an attempt to solve their quadratic
			Total	3	

Question			Answer/Indicative content	Marks	Guidance	
27			EITHER Midpoint AB is (3, 1)	B1(AO3.1a)	Seen or implied	
			Gradient of AB is $\frac{4-(-2)}{7-(-1)}\left(=\frac{6}{8}=\frac{3}{4}\right)$	M1(AO3.1a)	Must be correct way up – allow one slip	
			Gradient of perpendicular $-\frac{4}{3}$	M1(AO1.1a)	negative reciprocal FT their gradient of AB	Allow this M1 for showing the perpendicular to the given line is
			Equation of perpendicular bisector $y-1=-\frac{4}{3}(x-3)$	M1(AO1.1a)	FT their gradient. Condone A or B used instead of M. Do not allow if their gradient of AB used.	
			Solve the simultaneous equations $4x + 3y = 15$ $2x + y = k$	M1(AO1.1a)	Attempt to eliminate one variable	
			$y = 15 - 2k$, $x=\frac{1}{2}(3k-15)$	A1(AO1.1)	cao	
			giving $\mathrm{P}\left(\frac{1}{2}(3k-15), 15-2k\right)$	A1(AO1.1) [7]	cao	
			OR P is of the form $(x, k - 2x)$ and is equidistant from A and B $(x + 1)^2 + (y + 2)^2 = (x - 7)^2 + (y - 4)^2$	B1 M1 M1	Seen or implied Finding at least one distance	
			$(x + 1)^2 + (k - 2x + 2)^2 = (x - 7)^2 + (k - 2x - 4)^2$ $-8x + 1 = 60 - 12k$	M1 M1	Equating distances Substituting for y Attempt to simplify	
			So $x=\frac{1}{2}(3k-15)$			
			and $y = 15 - 2k$	A1	cao	
			P is $\left(\frac{1}{2}(3k-15), 15-2k\right)$	A1 [7]	cao	

Question			Answer/Indicative content	Marks	Guidance	
			Total	7		
28	a		$\frac{dy}{dx} = 12x^2 - 12x - 9$ When $\frac{dy}{dx} = 12x^2 - 12x - 9 = 0$ $3(2x + 1)(2x - 3) = 0$ so $x = -0.5, 1.5$  	M1(AO1.1a) M1(dep)(AO1.1a) A1(AO1.1a) B1(AO1.1) [4]	DR Attempt to differentiate seen Attempt to solve their $\frac{dy}{dx} = 0$ Both values seen – may be indicated on the graph Correct shape through (0, –9)	SC For cubic graph of the function drawn with M0M0A0 allow SC1 for correct shape with minimum when x = 1.5, and maximum when x = -0.5
	b		Min point of gradient function when $\frac{d^2y}{dx^2} = 24x - 12 = 0$ so $x = \frac{1}{2}$ Gradient is decreasing for $\left\{x : x < \frac{1}{2}\right\}$	M1(AO3.1a) A1(AO2.5) [2]	DR Attempt to find the vertex (including completing the square or symmetry argument) Inequality correctly formed and expressed as a set. Allow either < or ≤	
			Total	9		

Question			Answer/Indicative content	Marks	Guidance	
29	a		$h_{max} = 5.15 + 3.4 \times 1 = 8.55$ $h_{min} = 5.15 - 3.4 \times 1 = 1.75$ These are the correct h values for high and low tide	B1(AO3.4) [1]	Choosing cos t = ±1 to give both values must be seen Allow without further comment	Allow for using given h values to find cos$t$ = ±1 only if there is a comment that these are max and min values for cos$t$
	b	i	When $t = 1$ $8.55 = 5.15 + 3.4\cos(a + b)$ So $\cos(a + b) = 1$ giving $a + b = 0$	B1(AO3.3) [1]	Correctly relating high tide, $t = 1$ and cos 0	Accept 8.55 or cos $t = 1$ as evidence of high tide
		ii	Minimum when $(at + b) = 180°$ and $t = 7\frac{1}{3}$ So $\frac{22}{3}a + b = 180$	B1(AO3.3) [1]	Condone the use 7.2 hours here	Allow for $1.75 = 5.15 + 3.4\cos\left(\frac{22}{3}a + b\right)$
		iii	Solve simultaneously to give a = 28.42 to 2 dp	M1(AO3.3) A1(AO3.3) [2]	Attempt to solve simultaneous equations: may be **BC** **AG** (value of b not needed here)	[b = –28.42]
	c		Substitute $h = 3$ $3 = 5.15 + 3.4\cos(28.4t - 28.4)$ $\cos(28.4t - 28.4) = -\frac{43}{68}$ 28.4t – 28.4 = 129.2, 230.8 t = 5.55, 9.13 He does not sail between 5.33 am and 9.08 am	M1(AO3.4) A1(AO3.4) A1(AO3.2a) [3]	Attempting to solve trig equation or inequality At least one correct [decimal] value for t Both times correct. Need not convert to hours and minutes. Must indicate between these times	

Question			Answer/Indicative content	Marks	Guidance	
	d		EITHER The model predicts every high tide 8.55 m. The next high tide 8.91 is higher than that so not perfect model	B1(AO3.4) E1(AO3.5b) [2]	Allow for a comment about the maximum height being wrong. FT their values	
			OR Time difference between high tide and low tide is 6 hr 20 minutes, and between low tide and the next high tide is 5 hours and 40 minutes. The model gives these times as equal, so not perfect model	B1 E1 [2]	Allow for a comment that the time of the next high tide is wrong. FT their values	
			OR tide reaches 8.91 m when cos (at + b) = 1.105 which is impossible	B1 E1 [2]	Allow for a comment that the height predicted cannot reach 8.91 m. FT their values	
			OR When t = 12.983 h = 8.35 which is less than the given value of 8.91 m so the model in not suitable		Allow for a comment that the height predicted is not 8.91 m. FT their values	Allow for t = 13 but not t = 12.59
			Total	10		

Question			Answer/Indicative content	Marks	Guidance
30			$r = \frac{5.4}{\pi}$ V =74.2 to 74.3 or r = 1.727 or 1.73 Suitable comment consistent with working E.g. • Not exactly right but the measurements may not be exact • The volume / radius is too small but it's close • Allowing for approximation the formula seems accurate	B1 (AO3.1a) B1 (AO1.1) E1 (AO3.5b) [3]	soi by 1.71[88], 1.72 If r from V = 75 and h = 12 Dependent on both B marks Examiner's Comments Very often done well, with a simple minimal comment about being accurate sufficing for full marks. The most common approach was to find the volume when $r = \frac{5.4}{\pi}$, although some did find the value of r from volume=75 and proceeded to correctly compare with the decimal version of $\frac{5.4}{\pi}$. The most common error seen was the use of $r = \frac{5.4}{2}$. Exemplar 9  $V = \frac{2}{3} \times \pi \times \left(\frac{5.4\times10^{-2}}{\pi}\right)^2 \times 12\times10^{-2}$ $= 7.42\times10^{-5}\ m^3$ $= 74.3\ cm^3$ the model predicted 74.3 cm³ when we needed 75cm³ and therefore was less than 1%. out. That is a significance level that allows me to say the model is sufficiently accurate. However because the model underpredicts the accuracy needs to be changed because mathematically it's good however to work in the real world would need to overestimate. I included this example as the candidate had only scored 1 mark so far in the comprehension and given 'No response' to half of the 8 previous questions. But notice

Question			Answer/Indicative content	Marks	Guidance
					that, with presumably a very sketchy understanding of the whole situation, the candidate could still score full marks on this part – the final question on the paper. The comprehension is definitely an area where resilience is to be encouraged however difficult previous questions have been.
			Total	3	
31	a		DR $\ln 3^2$, $\ln 3^3$ seen $\ln 3 \times 2\ln 3 \times 3\ln 3 = 6(\ln 3)^3$	B1 (AO1.1a) B1 (AO2.2a) [2]	Examiner's Comments With this being a detailed reasoning question, it was expected that candidates show enough steps in their answers to provide detail of why each step follows. In this part, many went straight to $\ln 3 \times 2\ln 3 \times 3\ln 3$ without showing the intermediate step. This meant that those who got to $6(\ln 3)^3$ only scored 1 mark. Other fairly common mistakes were in misplacing the brackets in the final answer.
	b		DR $3 > e$ so $\ln 3 > 1$ $(\ln 3)^3 > 1$ so $6(\ln 3)^3 > 6$	M1 (AO2.2a) E1 (AO2.4) [2]	Must mention e Convincing completion (answer given) Examiner's Comments Not very many candidates were successful in this part, with only a small proportion even using the fact that $e < 3$. Many tried finding numerical answers via their calculators rather than working with e.
			Total	4	

Question			Answer/Indicative content	Marks	Guidance	
32			Where $y = x$ meets the circle $(x-10)^2 + (x-4)^2 = 16$	M1 (AO3.1a)	For sub'n of $y = x$ into their circle equation	
			$2x^2 - 28x + 116 = 16$ or	M1 (AO1.1)	For expanding and collecting like terms	
			$x^2 - 14x + 50 = 0$ or $2x^2 - 28x + 100 = 0$	M1 (AO1.1)	Rearranging to 3 term quadratic =0	'No real roots' not enough for E1 Allow E1 for correct reasoning following omission of '=0'
			$b^2 - 4ac = -4$ or -16 so no meeting points i.e. not a tangent	E1 (AO2.4)	Or $(x-7)^2 + 1 = 0$ no meeting points hence not a tangent	
			Alternative method		**Alternative method 2**	
			Angle between $y = x$ and x-axis = 45°	M1	gradient of normal is –1	
			Let θ be the angle between the x-axis and the line joining (0, 0) and (10, 4). $\tan\theta = 0.4$	M1	Line joining point of contact (k, k) to centre (10, 4) must have gradient –1	
			$\theta = 21.80$ (2 d.p.)			
			If $y = x$ was a tangent θ would be 22.5° hence not a tangent	M1	$k = 7$	
				E1	(7, 7) does not lie on circle	
					<u>Examiner's Comments</u>	
					The common method for this part was to solve their equation from the part above simultaneously with $y = x$ and the method was generally competently shown the difficulty coming with finishing the question off with many leaving their answer as discriminant < 0, or 'no real roots' or giving the complex roots and not completing by stating 'hence it is not a tangent'.	
			Total	4		

Question			Answer/Indicative content	Marks	Guidance
33			$\frac{1}{(x+2)(x+3)} = \frac{A}{(x+2)} + \frac{B}{(x+3)}$ $1 = A(x + 3) + B(x + 2)$ $x = –3 \Rightarrow 1 = –B \Rightarrow B = –1$ $x = –2 \Rightarrow 1 = A$ so $\frac{1}{(x+2)(x+3)} = \frac{1}{(x+2)} - \frac{1}{(x+3)}$	M1 (AO1.1a) M1 (AO1.1) A1 (AO1.1) [3]	Method marks are implied by correct answer. For clearing the fractions For one appropriate substitution For correct completion Examiner's Comments Practically all candidates found the correct partial fractions in this part with only a handful missing the – sign.
			Total	3	
34			$4 = 7x^{\frac{1}{2}}$ or $\frac{x^{\frac{1}{2}}}{4} = \frac{1}{7}$ Square both sides $x = \frac{16}{49}$	M1 (AO1.1a) M1 (AO1.1a) A1 (AO1.1) [3]	Order of M marks may vary. For getting their x term in numerator eg $\frac{x}{16} = \frac{1}{49}$ $\frac{4}{\frac{1}{x^2}} = 7$ not suff for this mark Examiner's Comments This was done correctly by many candidates, but a significant number went wrong. Some candidates did not notice that the index was negative while others thought it also applied to the '4'. Some candidates failed to square the expression to remove the square root. A small number of candidates made no progress at all on this question.
			Total	3	

Question			Answer/Indicative content	Marks	Guidance	
35			$m = \frac{k}{t}$, so $t = 5$, $m = 2.1 \Rightarrow k = 10.5$ t	M1 (AO2.1)	Using algebraic expression to represent proportionality and one pair of values in attempt to find k	oe with $t = 50$, $m = 0.21$
			When $t = 50$, $m = \frac{10.5}{50} = 0.21$, oe	M1 (AO2.1)	Uses the model to predict the value of m for the other value of t, or uses the other pair of values to check the value of k	e.g. $\therefore m = \frac{10.5}{t}$
			EITHER The model fits the measurements because the prediction agrees with given value OR The model fits the measurements because the same value of k is obtained in each case	A1 (AO2.2a)	Makes suitable statement about consistency of results	
				[3]		
			Alternative argument When the value of t is multiplied by 10, the value of m is divided by 10	M2	Argument in words need not reference the constant of proportionality	
			So [consistent with the model that] mass is inversely proportional to time.	A1	Must make a clear conclusion about inverse proportionality	
				[3]		
					<u>Examiner's Comments</u> The clearest solutions established the value of the constant of proportionality (10.5). Many candidates argued that going from one data point to the other, the value of t is multiplied by 10 and the value for m is divided by ten. Where the argument was clear, this also obtained full marks.	
			Total	3		

Question			Answer/Indicative content	Marks	Guidance	
36			DR Rearrange as $2x^2 - x + 7 = 0$ Discriminant is $(-1)^2 - 4 \times 2 \times 7$ $= -55 < 0$ so no real roots **Alternative method** Rearrange as $2x^2 - x + 7 = 0$ Attempt to complete the square $2(x - 0.25)^2 + 6.875 = 0$ $x - 0.25 = \pm\sqrt{-3.4375}$ so no real roots **Second alternative method** Rearrange as [$y =$] $2x^2 - x + 7 = 0$ Differentiate $\frac{dy}{dx} = 4x - 1 = 0$ Stationary point at (0.25, 6.875) Stationary point is minimum so $y \geq 6.875$ so is never zero	M1 (AO1.1a) M1 (dep) (AO1.1a) A1 (AO2.2a) [3] M1 M1 (dep) A1 [3] M1 M1 (dep) A1 [3]	Must be clearly argued from a correct discriminant which need not be evaluated if clearly negative Allow for $2(x - 0.25)^2 + ...$ soi Must be clearly argued from correct working Must equate to zero Must be clearly argued from correct working <u>Examiner's Comments</u> The majority of candidates were able to answer this question well. A few did not attempt to rearrange the equation into the required form and this cost them all the marks here. A small number simply quoted the complex roots of the equation, but without comment this received credit only for the first method mark.	Expression for discriminant must be precise if not evaluated
			Total	3		

Question			Answer/Indicative content	Marks	Guidance
37		a	Area of each little triangle $= \frac{1}{9}$ unit Area $= 1\frac{1}{3}$ [units]	M1 (AO 2.2a) A1 (AO 1.1) [1]	**or** total additional area $= \frac{1}{3}$
		b	$\frac{12}{81}$	B1 (AO 2.2a) [1]	oe $\frac{4}{27}$
		c	$1+\frac{3}{9}+\frac{4\times3}{9\times9}+\frac{4\times4\times3}{9\times9\times9}+K$ $1+\frac{\frac{3}{9}}{1-\frac{4}{9}}$ $\frac{8}{5}$ [units]	M1 (AO 3.1a) M1 (AO 2.2a) M1 (AO 3.1a) A1 (AO 1.1) [5]	4 times for new number of triangles Triangle area divided by 9 Use of GP formula
			Total	7	

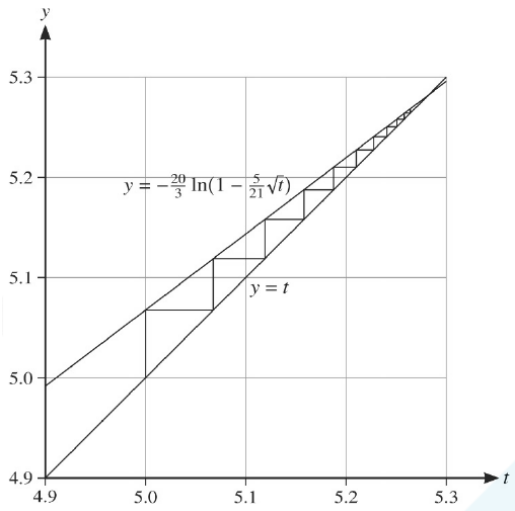
Question			Answer/Indicative content	Marks	Guidance
38		a	0, 2π π	B1(AO 1.2) B1(AO 1.2) [2]	
		b	$\text{cosec}^2\theta = 1 + \cot^2\theta$, so $\frac{2}{y} = 1 + \left(\frac{x}{2}\right)^2$ $\frac{2}{y} = \frac{4+x^2}{4}$ $y = \frac{8}{x^2+4}$ Alternative solution 1 $x^2 = 4\cot^2\theta = \frac{4\cos^2\theta}{\sin^2\theta}$ $x^2 = \frac{2(2-y)}{\frac{1}{2}y}$ $y = \frac{8}{x^2+4}$ Alternative solution 2 $\frac{A}{x^2+B} = \frac{A}{4\cot^2\theta+B} = 2\sin^2\theta$ $8\cos^2\theta + 2B\sin^2 = A$ so $B = 4$ and $A = 8$, giving $y = \frac{8}{x^2+4}$	M1(AO 3.1a) M1(AO 1.1) A1(AO 2.2a) M1 M1 A1 M1 M1 A1 [3]	

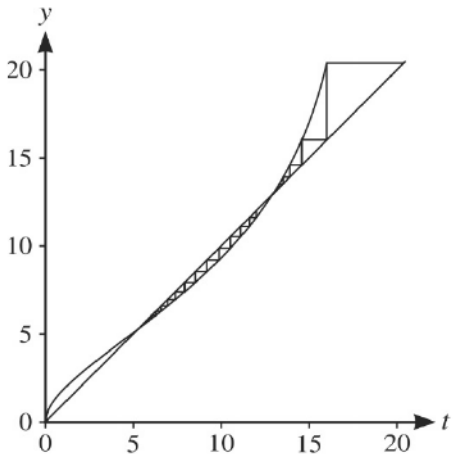
Question			Answer/Indicative content	Marks	Guidance	
		c	$\frac{8}{x^2+4} > 0$ for all x So Ali is correct [because $y > 0$ for all x]	M1(AO 2.1) A1(AO 2.3) [2]	or $2\sin^2\theta \geq 0$ and when $2\sin^2\theta = 0$, x is undefined	If zero scored, SC1 for $x^2 + 4$ is never negative oe
			Total	7		
39			DR $x^2 > 5x - 6$ $x^2 - 5x + 6 > 0$ $(x - 2)(x - 3) > 0$ $x < 2$ or $x > 3$	M1(AO 3.1a) M1(AO 1.1) A1(AO 1.1) A1(AO 2.2a) [4]	Correct factors or values 2, 3 oe, e.g. $\{x : x < 2 \cup x > 3\}$	
			Total	4		

Question			Answer/Indicative content	Marks	Guidance
40		a	ln $a = b$	B1 (AO 1.1)	
			Area B: $\int_0^b e^y dy = e^b - e^0$	M1(AO 3.1a)	or $a \times b$ – their $a\ln a - a + 1$
			$= a - 1$	A1(AO 1.1)	
			Area A: $\int_1^a \ln x \, dx = [x \ln x - x]_1^a$	M1(AO 2.1)	
			$= a\ln a - a + 1$	A1(AO 1.1)	
			Their $a - 1$ = their $a\ln a - a + 1$	M1(AO 3.1a)	
			$a\ln a - 2a + 2 = 0$	A1(AO 1.1)	
				[7]	
		b	Evaluation of their f(4.921554 – δ) and their f(4.921554 + δ)	M1 (AO 2.1)	$\delta \leq 0.000\ 000\ 5$
			eg – 0.000000079882 and 0.000000513742 seen to 2 or more sf plus correct conclusion: sign change, so Heidi is correct	A1(AO 2.2a)	
				[2]	
			Total	9	

Question			Answer/Indicative content	Marks	Guidance
41		a	12000 = 22000 – $a\sqrt{4}$ a = 5000	B1 (AO 3.3) [1]	cao
		b	$\frac{dV}{dt} = -5000 \times \frac{1}{2} \times t^{-\frac{1}{2}}$ $t = 4 \Rightarrow \frac{dV}{dt} = -1250$, so the caravan is losing value at a rate of £1250 per year when t = 4	M1 (AO 3.4) A1 (AO 1.1b) A1 (AO 3.4) [3]	Attempt to differentiate Correct derivative, in any form Must interpret the negative value of the derivative and must state units
		c	For large values of t, [more than 19.36 years] the value of V is negative which is not possible	E1 (AO 3.5b) [1]	Any argument based on negative values for [sufficiently] large t
		d	c = 1000 22 000 = $be^0 + c$ so b = 21000	B1 (AO 3.3) B1 (AO 3.3) [2]	cao cao

Question			Answer/Indicative content	Marks	Guidance	
		e	Agree when $22\,000 - 5\,000\sqrt{t} = 21\,000e^{-0.15t} + 1000$			
			i.e. when $e^{-0.15t} = \frac{21\,000 - 5000\sqrt{t}}{21\,000} = 1 - \frac{5}{21}\sqrt{t}$	M1 (AO 2.1)	Equating their models and attempting to isolate the exponential term	$e^{-0.15t}$ = K need not be simplified at this stage
			$-0.15t = \ln\left(1 - \frac{5}{21}\sqrt{t}\right)$	M1 (AO 2.1)	Taking logs of both sides	
			$t = -\frac{1}{0.15}\ln\left(1 - \frac{5}{21}\sqrt{t}\right)$ $= -\frac{20}{3}\ln\left(1 - \frac{5}{21}\sqrt{t}\right)$	A1 (AO 2.1)	AG; complete argument needed	
				[3]		
		f	t_1 = 5.067572851	B1 (AO 1.1b)	BC	
			t_2 = 5.118888519	B1 (AO 1.1b)	Allow answers which agree when rounded to 3sf	
				[2]		

Question			Answer/Indicative content	Marks	Guidance	
		g		B1 (AO 1.1b) [1]	Must show at least two steps beginning at $t = 5$	
		h	The argument is false – the sequence is increasing and could [and in this case does] increase so that the root rounds to 5.3 or higher	B1 (AO 2.3) [1]	Accept an argument based on calculation of additional terms	Do not accept "false" without a reason

Question			Answer/Indicative content	Marks	Guidance	
		i		B1 (AO 2.4)	Sketch showing two staircases, one starting above the root and diverging and one starting below the root and converging on the root near 5	
			So the iteration will not find the root near 12	B1 (AO 2.2a)	Correct conclusion for correct reason	
			Alternative solution 1 Sequence starting above the root, e.g.13, 13.035, 13.089, 13.174, 13.310, 13.532, 13.909 and sequence starting below the root, e.g. 12, 11.611, 11.118, 10.530, 9.874, 9.193, 8.532, K	B1	Increasing sequence starting above and decreasing sequence starting below	
			So the iteration will not find the root near 12	B1	Correct conclusion for correct reason	
			Alternative solution 2 Near the root, the gradient of $y = -\frac{20}{3}\ln\left(1-\frac{5}{21}\sqrt{t}\right)$ is greater than 1	B1	For correct relevant statement about the gradient	Accept 'steeper than the line' for 'greater than 1'
			So the iteration will not find the root near 12	B1	Correct conclusion for correct reason	
				[2]		
			Total	16		

Question			Answer/Indicative content	Marks	Guidance	
42			Discriminant $k^2 - 4 \times 2 \times 8 > 0$	M1 (AO 1.1a)	May be implied by $k^2 > 64$ oe without working; allow for $\geq$ used	Allow as part of the quadratic formula
			$k > 8$	A1 (AO 1.1b)	oe	
			or $k < -8$	A1 (AO 1.1b) [3]	oe	
			Total	3		

Question			Answer/Indicative content	Marks	Guidance	
43		a	Circle is $(x-4)^2 + (y+5)^2 = 25$	M1 (AO 1.1a)	Attempting to complete the square for both x and y	
			Centre is (4, –5).	A1 (AO 2.2a)		
			Radius is 5	A1 (AO 2.2a)		
				[3]		
		b	The y-coordinate of the centre is –5 and the radius is 5 so the perpendicular distance of the centre from the x-axis is equal to the radius so the axis is a tangent	B1 (AO 2.1)	For a complete argument with reference to both the radius and the value of the y -coordinate	A diagram without explanation is not sufficient for the mark
			Alternative solution $y = 0$ in equation gives $x^2 - 8x + 16 = 0 \Rightarrow (x-4)^2 = 0$ so $x = 4$ is a repeated root, i.e. the x-axis is a tangent	B1	Clear argument based on the repeated root needed	
				[1]		
		c	Gradient of radius to P is $\frac{-1-(-5)}{7-4} = \frac{4}{3}$	M1 (AO 3.1a)		
			Gradient of tangent is $-\frac{3}{4}$	M1 (AO 3.1a)	For use of $m_1 m_2 = -1$	
			Equation of tangent is $y-(-1) = -\frac{3}{4}(x-7)$	M1 (AO 1.1a)	Must use their tangent gradient	
			Rearranging: $3x + 4y = 17$	A1 (AO 1.1b)	cao and must be in this form	
				[4]		
			Total	8		

Question			Answer/Indicative content	Marks	Guidance	
44			$\frac{15}{n}\times\frac{14}{n-1}+\frac{n-15}{n}\times\frac{n-1-15}{n-1}=2\times\frac{15}{n}\times\frac{n-15}{n-1}$	M1 (AO3.1b)	P(BB) + P(RR) = P(one of each)	*n* is the total number of discs
				M1 (AO2.1)	Two of three terms correct	
				A1 (AO1.1)	All correct	
			Multiply through to obtain quadratic in *n*	M1 (AO2.1)		
			$n2 - 61n + 900 = 0$	A1 (AO1.1)		
			n = 36 (not 25) since more red discs	B1 (AO3.2b)	so there are 21 red discs	
			$\frac{\frac{15}{36}\times\frac{14}{35}}{\frac{15}{36}\times\frac{14}{35}+\frac{21}{36}\times\frac{20}{35}}$	M1 (AO2.1)	FT their 44	
			$^1/_3$	A1 (AO1.1) [8]	Accept decimal equivalent to 2 or more dp	
			OR			
			$\frac{15}{15+r}\times\frac{14}{14+r}+\frac{r}{15+r}\times\frac{r-1}{14+r}=2\times\frac{15}{15+r}\times\frac{r}{14+r}$	M1	P(BB) + P(RR) =	*r* is the number of red discs
				M1	P(one of each)	
				A1	Two of three terms correct All correct	
			Multiply through to obtain quadratic in *r*	M1		
			$r^2 - 31r + 210 = 0$	A1		
			r = 21 (not 10) since *r* > 15	B1		
			$\frac{\frac{15}{36}\times\frac{14}{35}}{\frac{15}{36}\times\frac{14}{35}+\frac{21}{36}\times\frac{20}{35}}$	M1	FT their 21 (> 15)	
			$^1/_3$	A1 [8]	Accept decimal equivalent to 2 or more dp	
			Total	8		

Question			Answer/Indicative content	Marks	Guidance
45		a	$\int \frac{1}{P}\,\mathrm{d}P = \frac{1}{k}\int \frac{1}{Q}\,\mathrm{d}Q$	M1 (AO 1.1a)	Separation of variables
			$\ln P = \frac{1}{k}\ln Q\ [+c]$	M1 (AO 1.1)	Integration
			$Q = AP^k$ [where A is a constant]	A1 (AO 2.5)	oe; must include constant here
				[3]	
		b	$P_1 = 1.05P_0$	M1 (AO 3.1b)	notation may vary, e.g. just $1.05P$
			$Q_0 = AP_0^k$ and $Q_1 = A \times 1.05^k P_0^{\,k}$	M1 (AO 3.4)	oe
			Percentage change in Q is $100\left(\frac{A\times 1.05^k P_0^{\,k} - AP_0^{\,k}}{AP_0^{\,k}}\right)$		
			$= 100(1.05^k - 1)$ and so is independent of original price	E1 (AO 2.1)	Convincing completion
				[3]	
			Total	6	

Question			Answer/Indicative content	Marks	Guidance
46			$\frac{du}{dx}=1$	B1 (AO 1.1a)	Or du = dx
			$\int(5u-3)u^{\frac{1}{2}}\,du$	M1 (AO 1.1)	Complete substitution for x and dx
			$\int\left(5u^{\frac{3}{2}}-3u^{\frac{1}{2}}\right)du$	M1 (AO 1.1)	
			$2u^{\frac{5}{2}}-2u^{\frac{3}{2}}\ [+c]$	M1 (AO 1.1)	
			$2(x+1)^{\frac{5}{2}}-2(x+1)^{\frac{3}{2}}+c$	M1 (AO 1.1)	
			$\{2(x+1)-2\}(x+1)^{\frac{3}{2}}+c$	M1 (AO 1.1)	Taking out factor $(x+1)^{\frac{3}{2}}$
			$2x(x+1)^{\frac{3}{2}}+c$	A1 (AO 2.1)	Correct answer in correct form
				[7]	
			Total	7	

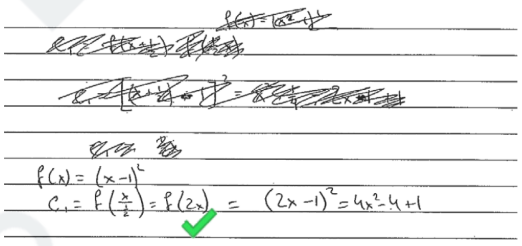
Question			Answer/Indicative content	Marks	Guidance	
47			$x^2+x+2=\left(x+\frac{1}{2}\right)^2+1\frac{3}{4}$	M1 (AO 3.1a)	Attempt to complete square	Or equivalent steps, following initial rearrangement as
				A1 (AO 1.1)	Correct completed square form	
			Minimum value is $1\frac{3}{4}$ hence always greater than 1	E1 (AO 2.4)	Conclusion clearly explained	$x^2 + x + 1 > 0$
			Alternative solution 1			
			$y=x^2+x+2 \Rightarrow \frac{dy}{dx}=2x+1$	M1		Or equivalent steps, following initial rearrangement as
			Minimum occurs when $x=-\frac{1}{2}$	A1		
			Minimum value is $1\frac{3}{4}$ hence always greater than 1	E1	Conclusion clearly explained	$x^2 + x + 1 > 0$
			Alternative solution 2			
			Discriminant of $x^2 + x + 1$ is $1 – 4$	M1	Must be working on $x^2 + x + 1 > 0$	
			$= –3$	A1		
			Discriminant negative so $x^2 + x + 1$ is never zero,			
			when $x = 0$, $x^2 + x + 1 = 1$, hence always positive.	E1	Complete argument	
				[3]		
			Total	3		

Question			Answer/Indicative content	Marks	Guidance
48		a	Using $y = f\left(\frac{x}{a}\right)$ $y = \left(\frac{x}{\frac{1}{2}} - 1\right)^2 = (2x-1)^2$ $= 4x^2 - 4x + 1$	M1 (AO1.1a) A1 (AO2.1) [2]	Allow for 2 instead of $^1/_2$ used for method mark or attempt to write equation of quadratic that touches axis at (0.5, 0) **AG** Must be a convincing argument that references either stretch or f($2x$) or similar $(2x-1)^2$ seen is sufficient for M1
		b	EITHER C_2 is $y = 4.25x - x^2 - 3$ Normal to $y = 4x^2 - 4x + 1$ $\frac{dy}{dx} = 8x - 4$	B1 (AO3.1a) M1 (AO1.1a)	Finding the equation of C_2. Any form Finding the derivative

Examiner's Comments

Many candidates were credited one of two marks for this question as $(2x-1)^2$ was seen. The best answers used function notation to explain the effect of the stretch in the x-direction. The following exemplar shows a candidate who is not sure what algebra is needed to achieve the given transformation. The given answer is used to identify the correct method and the notation makes it very clear that the full argument is given.

Exemplar 3



Question	Answer/Indicative content	Marks	Guidance
	At (0.1) $\frac{dy}{dx} = -4$		
	Gradient of normal is $\frac{1}{4}$	M1 (AO1.1b)	Finding negative reciprocal of their gradient
	(0, 1) on line so equation of normal is $y = \frac{1}{4}x + 1$	A1 (AO1.1a)	FT their value for derivative
	Intersection of normal and C_2 $\frac{1}{4}x + 1 = 4.25x - x^2 - 3$	M1 (AO3.1a)	Attempt to solve simultaneous equations
	$4x^2 - 16x + 16 = 0$ EITHER $(x-2)^2 = 0$ OR discriminant $16^2 - 4 \times 4 \times 16 = 0$	A1 (AO1.1b)	Repeated factor or root, or zero discriminant seen.
	Repeated root so the normal is a tangent to C_2	E1 (AO3.2a) [7]	Must interpret their solution in the context.
	OR		
	C_2 is $y = 4.25x - x^2 - 3$	B1	Finding the equation of C_2. Any form
	Normal to $y = 4x^2 - 4x + 1$ $\frac{dy}{dx} = 8x - 4$	M1	Finding the derivative
	At (0.1) $\frac{dy}{dx} = -4$		

Question	Answer/Indicative content	Marks	Guidance
	Gradient of normal is $\frac{1}{4}$	M1	Finding negative reciprocal of their gradient
	Equation of normal is $y = \frac{1}{4}x + 1$	A1	FT their value for derivative
	Point on C_1 where gradient is $\frac{1}{4}$ $\frac{dy}{dx} = 4.25 - 2x = \frac{1}{4}$	M1	Attempting to find the point on C_1 where tangent parallel to the normal found.
	giving $x = 2$ $y = 1.5$	A1	Both coordinates required
	EITHER So the equation of the tangent is $y - \frac{3}{2} = \frac{1}{4}(x - 2)$ Which is the same equation as the normal to C_1	E1	Correct equation for the tangent in form that makes it clear it is the same line as the normal.
	OR show that point (2, 1.5) lies on normal So the normal to C_1 is a tangent to C_2	(E1) [7]	
	SPECIAL CASE when the candidate tries to show that the normal to C_2 is a tangent to C_1		
	C_2 is $y = 4.25x - x^2 - 3$	B1	Finding the equation of C_2. Any form
	Normal to $y = 4.25x - x^2 - 3$ $\frac{dy}{dx} = 4.25 - 2x$	M1	Finding the derivative

Question			Answer/Indicative content	Marks	Guidance	
			At $(0, 1)$ $\frac{dy}{dx} = 4.25$			
			Gradient of normal is $-\frac{4}{17}$	A1	Finding negative reciprocal of their gradient	
			Equation of normal is $y = -\frac{4}{17}x + 1$	A0	Attempt to solve simultaneous equations	(0, 1) does not lie on C_2
			EITHER point of intersection with C_1 $4x^2 - 4x + 1 = -\frac{4}{17}x + 1$	M1 (M1)	Attempting to find the point on C_1 where tangent parallel to the normal found.	
			OR Attempt to find both coordinates of the point on C_1 with gradient $-\frac{4}{17}$ $\frac{dy}{dx} = 8x - 4 = -\frac{4}{17}$		No further marks are available 4/7 maximum	
					Examiner's Comments There were many good solutions to this question. Some candidates correctly obtained the equation of the normal and the equation of C_2 but then did not know how to proceed. Some incorrectly assumed that the point (0, 1) was a point of intersection of the two curves. Some candidates found the point on C_2 which had the correct gradient but then did not go on to show that the tangent here was the same line as the normal to C_1 and not simply parallel to it.	
			Total	9		

Question			Answer/Indicative content	Marks	Guidance
49		a	The geometric mean cannot be calculated	E1 (AO 2.3) [1]	<u>Examiner's Comments</u> This was answered well by the majority of candidates.
		b	The arithmetic mean will be less than the geometric mean	E1 (AO 2.3) [1]	E.g. The arithmetic mean will be negative <u>Examiner's Comments</u> This was also answered well.
			Total	2	

Question			Answer/Indicative content	Marks	Guidance	
50			$5x - x^2 = x(5 - x)$	M1(AO 3.1a)	DR Factorisation	
			[$x = 0$], $x = 5$	A1(AO 1.1)	Finding 5	
			The line does not go through the origin so $x = 5$	E1(AO 2.4)	Rejection of origin as a point where they cross	May be later
			$y = 4 - kx$ so $0 = 4 - 5k$	M1(AO 3.2a)		
			$k = \frac{4}{5}$	A1(AO 1.1)		
			$4 - \frac{4}{5}x = 5x - x^2$	M1(AO 1.1)		
			$x^2 - 5\frac{4}{5}x + 4 = 0$ **OR** $5x^2 - 29x + 20 = 0$			
			$(5x - 4)(x - 5) = 0$	M1(AO 1.1)		
			$\left(\frac{4}{5}, \frac{84}{25}\right)$ o.e.	A1(AO 2.2a)	$\frac{84}{25} = 3\frac{9}{25} = 3.36$	
				[8]	<u>Examiner's Comments</u> This was a question where candidates generally understood the necessary steps required. Some solutions lost a mark because they did not show why $x = 0$ should be rejected in favour of $x = 5$ being used. Also a few candidates got a little bogged down by focussing on the discriminant.	
			Total	8		