GCSE Edexcel Math
1MA1
Proof/Reasoning

Answers

"We will help you to achieve A Star "

Answer 1

FACTOR OF 2

Show that $(n + 3)^2 - (n - 3)^2$ is an even number for all positive integer values of n.

F O I L

$$(n+3)^2 = (n+3)(n+3) = n^2 + 3n + 3n + 9$$
$$= n^2 + 6n + 9$$

F O I L

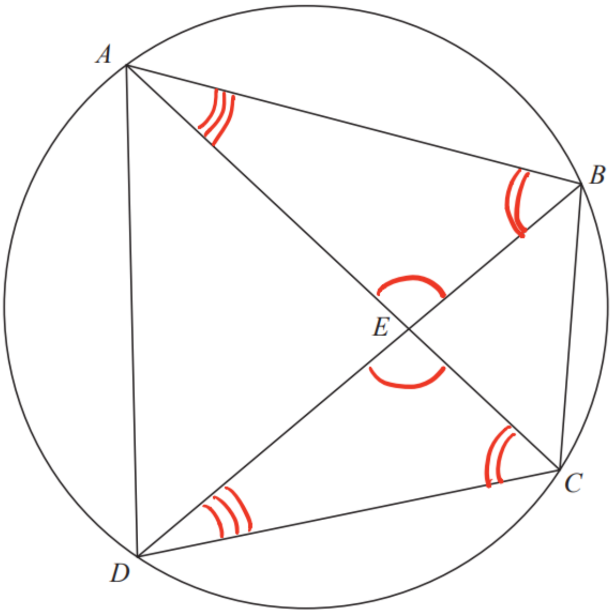
$$(n-3)^2 = (n-3)(n-3) = n^2 - 3n - 3n + 9$$
$$= n^2 - 6n + 9$$

$$(n+3)^2 - (n-3)^2 = n^2 + 6n + 9 - (n^2 - 6n + 9)$$
$$= \cancel{n^2} + 6n \cancel{+9} \cancel{-n^2} + 6n \cancel{-9}$$
$$= 6n + 6n$$
$$= 12n$$
$$= 2 \times 6n$$

FACTOR OF 2 SO EVEN.

Answer 2

A, B, C and D are four points on the circumference of a circle.



CIRCLE THEOREMS

AEC and BED are straight lines.

Prove that triangle ABE and triangle DCE are similar. → SAME ANGLES
You must give reasons for each stage of your working.

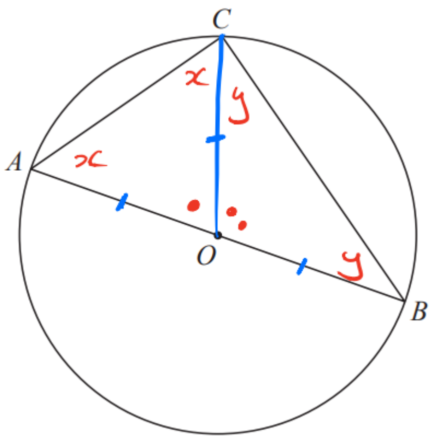
$A\hat{E}B = D\hat{E}C$ (OPPOSITE ANGLES)

$A\hat{B}E = D\hat{C}A$ (ANGLES IN THE SAME SEGMENT)

$B\hat{A}E = C\hat{D}E$ (ANGLES IN THE SAME SEGMENT)

SO SINCE 3 PAIRS OF ANGLES ARE EQUAL THE $\triangle$s ARE SIMILAR

Answer 3



A, B and C are points on the circumference of a circle, centre O.
AOB is a diameter of the circle.

Prove that angle ACB is 90°
You must **not** use any circle theorems in your proof.

SINCE $OA = OC = OB$

$\triangle OAC$ AND $\triangle OBC$ ARE ISOSCELES

LET $O\hat{A}C = x = O\hat{C}A$ AND $O\hat{B}C = y = O\hat{C}B$

SO $A\hat{O}C = 180 - 2x$ AND $B\hat{O}C = 180 - 2y$

SINCE $A\hat{O}B$ IS A STRAIGHT LINE

$A\hat{O}C + B\hat{O}C = 180$

$\cancel{180} - 2x + 180 - 2y = \cancel{180}\,0$

$+2x \quad +2y \quad +2x + 2y$

$180 = 2x + 2y$

$\div 2$

$90 = x + y$

$90 = A\hat{C}B$

Answer 4

The product of two consecutive positive integers is added to the larger of the two integers.

Prove that the result is always a square number.

WHOLE NUMBERS

TWO CONSECUTIVE INTEGERS: n, $n+1$

PRODUCT + LARGER $= n(n+1) + n+1$

$= n^2 + n + n + 1$

$= n^2 + 2n + 1$

$= (n+1)^2$

WHICH IS A SQUARE NUMBER.

$(n+1)^2 = (n+1)(n+1)$

F O I L

$= n^2 + n + n + 1$

$= n^2 + 2n + 1$

EXAM PAPERS PRACTICE

Answer 5

"WHOLE NUMBER"

n is an integer greater than 1

"MULTIPLE OF 2"

Prove algebraically that $n^2 - 2 - (n-2)^2$ is always an even number.

$$n^2 - 2 - (n-2)^2 = n^2 - 2 - (n-2)(n-2)$$

$$= n^2 - 2 - [\overset{F}{n^2} \overset{O}{-2n} \overset{I}{-2n} \overset{L}{+4}]$$

$$= \cancel{n^2} - 2 - \cancel{n^2} + 2n + 2n - 4$$

$$= 4n - 6$$

$$= 2 \times 2n - 2 \times 3$$

$$= 2(2n - 3)$$

THIS HAS A FACTOR OF 2 AND SO IS AN EVEN NUMBER

Answer 6

Prove that

$$(2n + 3)^2 - (2n - 3)^2 \text{ is a multiple of } 8$$

for all positive integer values of n.

$$(2n+3)^2 - (2n-3)^2$$

$$= (2n+3)(2n+3) - (2n-3)(2n-3)$$

F O I L F O I L

$$= 4n^2 + 6n + 6n + 9 - (4n^2 - 6n - 6n + 9)$$

$$= \cancel{4n^2} + 12n \cancel{+9} \cancel{-4n^2} + 12n \cancel{-9}$$

$$= 24n$$

$$= 8 \times 3n$$ WHICH IS A MULTIPLE OF 8.

OR

$$\boxed{\text{D.O.T.S.} \quad a^2 - b^2 = (a+b)(a-b)}$$

Let $a = 2n+3$, $b = 2n-3$

$$a^2 - b^2 = (2n+3+2n-3)(2n+3-(2n-3))$$

$$= 4n \times 6$$

$$= 24n$$

$$= \underline{\underline{8 \times 3n}}$$

EXAM PAPERS PRACTICE

Answer 7

Prove algebraically that

$$(2n + 1)^2 - (2n + 1) \quad \text{is an even number}$$

HAS A FACTOR OF 2

for all positive integer values of n.

$= (2n+1)(2n+1) - (2n+1)$

F O I L

$= 4n^2 + 2n + 2n + 1 - 2n - 1$

$= 4n^2 + 2n + 0$

$= 2n(2n+1)$

OR

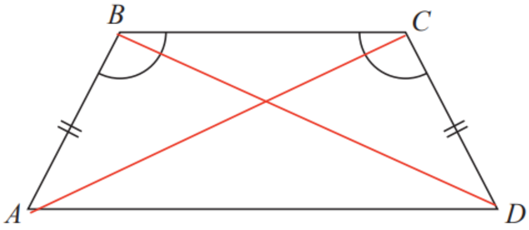
$= (2n+1)[(2n+1) - 1]$

$= (2n+1)[2n]$

THIS HAS A FACTOR OF 2 AND IS THEREFORE EVEN.

Answer 8

ABCD is a quadrilateral.



$AB = CD$.
Angle ABC = angle BCD.

Prove that $AC = BD$.

GIVEN $AB = CD$

AND $A\hat{B}C = B\hat{C}D$

ALSO BC IS COMMON SIDE

SO "SAS" MEANS THAT

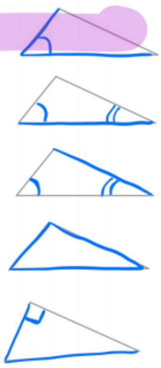
ΔABC IS CONGRUENT ΔBCD

SO $\underline{AC = BD}$

CONGRUENT TRIANGLES

NEED TO USE ONE OF THE "STANDARD" TESTS: IE SHOW THAT THREE PARTICULAR THINGS ARE THE SAME IN THE TWO TRIANGLES:

1. SAS
2. ASA
3. AAS
4. SSS
5. RHS



Answer 9

Prove that the square of an odd number is always 1 more than a multiple of 4

IF n IS AN INTEGER (WHOLE NUMBER)

THEN $2n$ IS AN EVEN NUMBER

SO $2n+1$ IS AN ODD NUMBER

$$(2n+1)^2 = (2n+1)(2n+1)$$

F O I L

$$= 4n^2 + 2n + 2n + 1$$

$$= 4n^2 + 4n + 1$$

$$= \underbrace{4(n^2+n)}_{\text{MULTIPLE OF 4}} + 1$$

"1 MORE THAN"

EXAM PAPERS PRACTICE

Answer 10

(i) Factorise $2t^2 + 5t + 2$ → 1×2

$(2t+1)(t+2)$

CHECK ($= 2t^2 + 4t + t + 2$)

F O I L

$= 2t^2 + 5t + 2$ ✓

(ii) t is a positive whole number.

The expression $2t^2 + 5t + 2$ can never have a value that is a prime number.

Explain why.

HAS FACTORS OF 1 AND ITSELF ONLY

$2t^2 + 5t + 2 = (2t+1)(t+2)$

SINCE $2t+1 > 1$

AND $t+2 > 1$

$2t^2+5t+2$ IS THE PRODUCT OF TWO NUMBERS BIGGER THAN 1 SO IS NOT PRIME

Answer 11

Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

"WHOLE NUMBERS"

TWO CONSECUTIVE INTEGERS: n AND $n+1$

SUM $= n + n + 1 = \underline{2n+1}$

DIFFERENCE BETWEEN SQUARES $= (n+1)^2 - n^2$

$= (n+1)(n+1) - n^2$

F O I L

$= n^2 + n + n + 1 - n^2$

$= \cancel{n^2} + 2n + 1 - n^2$

$= \underline{2n+1}$

SO DIFF BETWEEN SQUARES = SUM

Answer 12

Here are the first five terms of an arithmetic sequence.

7 $\xrightarrow{+6}$ 13 $\xrightarrow{+6}$ 19 $\xrightarrow{+6}$ 25 $\xrightarrow{+6}$ 31

Prove that the difference between the squares of any two terms of the sequence is always a multiple of 24

n^{TH} TERM = $6n + 1$ → MAKES 1^{ST} TERM WORK

p^{TH} TERM = $6p + 1$

DIFFERENCE OF TWO SQUARES
$a^2 - b^2 = (a-b)(a+b)$

USE:
$a = 6n+1$
$b = 6p+1$

$$\text{DIFFERENCE} = (6n+1)^2 - (6p+1)^2$$
$$= ((6n+1) - (6p+1))((6n+1) + (6p+1))$$
$$= (6n\cancel{+1} - 6p \cancel{-1})(6n+1+6p+1)$$
$$= (6n-6p)\times(6n+6p+2)$$
$$= 6(n-p) \times 2(3n+3p+1)$$
$$= 12(n-p)(3(n+p)+1)$$

"MULTIPLE OF 12"

→ IF n AND p ARE BOTH ODD OR BOTH EVEN, $(n-p)$ IS EVEN SO MULTIPLE OF 2

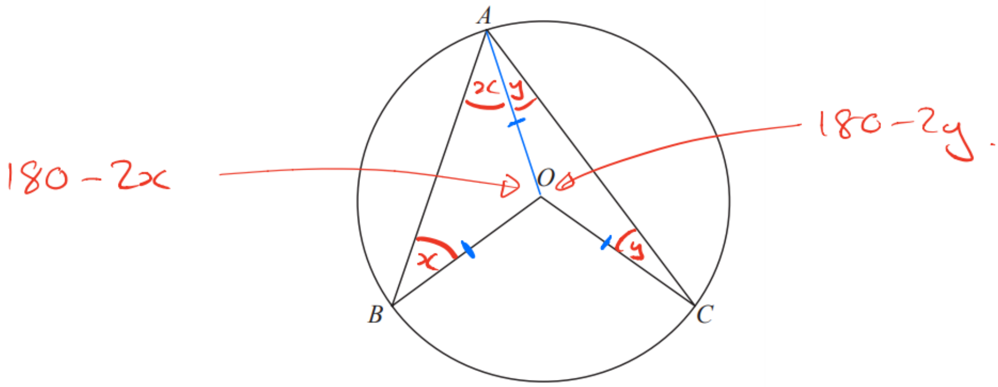
→ IF n AND p ARE ONE ODD, ONE EVEN $(n+p)$ IS ODD SO $3(n+p)+1$ IS EVEN SO MULTIPLE OF 2

SO WE ALWAYS HAVE A MULTIPLE OF 2 (AS WELL AS 12) AND SO WE HAVE A MULTIPLE OF 24...

EXAM PAPERS PRACTICE

Answer 13

A, B and C are points on the circumference of a circle centre O.



Prove that angle BOC is twice the size of angle BAC.

AOB AND AOC ARE ISOSCELES TRIANGLES

LET $A\hat{B}O = x$ AND $A\hat{C}O = y$

SO $B\hat{A}O = x$ AND $C\hat{A}O = y$.

SO $\underline{B\hat{A}C = x + y}$

$A\hat{O}B = 180 - 2x$ AND $A\hat{O}C = 180 - 2y$

$B\hat{O}C + A\hat{O}B + A\hat{O}C = 360°$

$$B\hat{O}C + \underset{-180}{180} - 2x + \underset{-180}{180} - 2y = \underset{-180\ -180}{360}$$

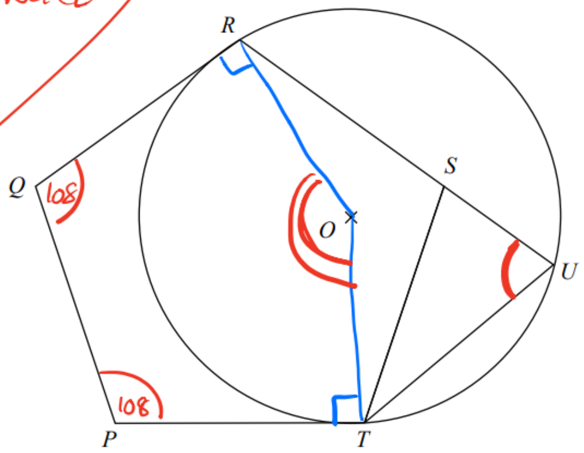
$$B\hat{O}C \underset{+2x}{-2x} \underset{+2y}{-2y} = \underset{+2x\ +2y}{0}$$

$$B\hat{O}C = 2x + 2y$$

$$\underline{B\hat{O}C = 2(x+y) = 2 \times B\hat{A}C}$$

Answer 14

TOTAL INTERNAL ANGLE OF A PENTAGON
= 5 × 108
= 540°



PQRST is a regular pentagon.
R, *U* and *T* are points on a circle, centre *O*.
QR and *PT* are tangents to the circle. → TANGENT AND RADIUS MEET AT 90°
RSU is a straight line.

Prove that $ST = UT$.

REGULAR POLYGONS (n SIDES)
EXTERIOR ANGLE = $\frac{360}{n}$
INT EXT
INTERIOR ANGLE = 180 − EXTERIOR ANGLE

+ CIRCLE THEOREMS

IE. PROVE THAT Δ TSU IS ISOSCELES
IE. PROVE THAT $T\hat{S}U = S\hat{U}T$.

$T\hat{S}U$ IS THE EXT ANGLE OF THE PENTAGON, SO $T\hat{S}U = \frac{360}{5} = 72°$

ANGLE AT CENTRE IS TWICE THE ANGLE AT CIRCUM.

$R\hat{O}T = 2 \times S\hat{U}T$

NB. $R\hat{Q}P = Q\hat{P}T$ = INT ANGLE OF REG PENTAGON
$= 180 - 72 = 108°$

LOOKING AT PQROT:
$R\hat{O}T = 540 - (90 + 90 + 108 + 108) = 144°$
SO $S\hat{U}T = \frac{1}{2} \times 144 = 72°$
AND $S\hat{U}T = T\hat{S}U$ AND SO $ST = UT$

EXAM PAPERS PRACTICE

Answer 15

Prove that, for all positive values of n,

$$\frac{(n+2)^2-(n+1)^2}{2n^2+3n}=\frac{1}{n}$$

F O I L

$$(n+2)^2 = (n+2)(n+2) = n^2 + 2n + 2n + 4$$
$$= \underline{n^2 + 4n + 4}$$

F O I L

$$(n+1)^2 = (n+1)(n+1) = n^2 + n + n + 1$$
$$= \underline{n^2 + 2n + 1}$$

$$\frac{(n+2)^2-(n+1)^2}{2n^2+3n} = \frac{n^2+4n+4-(n^2+2n+1)}{2n^2+3n}$$

$$= \frac{\cancel{n^2}+4n+4\cancel{-n^2}-2n-1}{2n^2+3n}$$

$$= \frac{2n+3}{2n^2+3n}$$

$$= \frac{1\times\cancel{(2n+3)}}{n\cancel{(2n+3)}}$$

$$= \underline{\underline{\frac{1}{n}}}$$