EXAM PAPERS PRACTICE

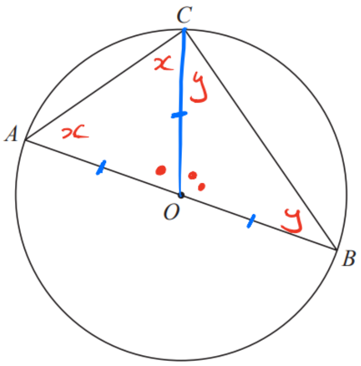
GCSE OCR Math J560

Proof/Reasoning

Answers

"We will help you to achieve A Star "

Answer 1



A, B and C are points on the circumference of a circle, centre O.
AOB is a diameter of the circle.

Prove that angle ACB is 90°
You must **not** use any circle theorems in your proof.

SINCE $OA = OC = OB$

ΔOAC AND ΔOBC ARE ISOSCELES

LET $O\hat{A}C = x = O\hat{C}A$ AND $O\hat{B}C = y = O\hat{C}B$

SO $A\hat{O}C = 180 - 2x$ AND $B\hat{O}C = 180 - 2y$

SINCE $A\hat{O}B$ IS A STRAIGHT LINE

$$A\hat{O}C + B\hat{O}C = 180$$

$$\cancel{180} - 2x + 180 - 2y = \cancel{180}^{0}$$
$$+2x \quad +2y \quad +2x + 2y$$

$$180 = 2x + 2y$$

$\div 2$

$$90 = x + y$$

$$\underline{90 = A\hat{C}B}$$

Answer 2

FACTOR OF 2

Show that $(n + 3)^2 – (n – 3)^2$ is an even number for all positive integer values of n.

F O I L

$(n+3)^2 = (n+3)(n+3) = n^2 + 3n + 3n + 9$

$= n^2 + 6n + 9$

F O I L

$(n-3)^2 = (n-3)(n-3) = n^2 - 3n - 3n + 9$

$= n^2 - 6n + 9$

$(n+3)^2 - (n-3)^2 = n^2 + 6n + 9 - (n^2 - 6n + 9)$

$= \cancel{n^2} + 6n \cancel{+9} \cancel{-n^2} + 6n \cancel{-9}$

$= 6n + 6n$

$= 12n$

$= 2 \times 6n$

FACTOR OF 2 SO EVEN.

Answer 3

n is an integer. "WHOLE NUMBER"

Prove algebraically that the sum of $\frac{1}{2}n(n+1)$ and $\frac{1}{2}(n+1)(n+2)$ is always a square number.

"ADD THEM"

$$\text{SUM} = \frac{1}{2}n(n+1) + \frac{1}{2}(n+1)(n+2)$$

$$= \frac{1}{2}n^2 + \frac{1}{2}n + \frac{1}{2}[\overset{F}{n^2} + \overset{O}{2n} + \overset{I}{n} + \overset{L}{2}]$$

$$= \frac{1}{2}n^2 + \frac{1}{2}n + \frac{1}{2}n^2 + \frac{3}{2}n + 1$$

$$= n^2 + 2n + 1$$

$$= (n+1)^2$$

$$(n+1)(n+1)$$

$$= n^2 + n + n + 1$$

$$= n^2 + 2n + 1$$

WHICH IS ALWAYS A SQUARE NUMBER

Answer 4

Prove algebraically that

$(2n + 1)^2 - (2n + 1)$ is an even number

for all positive integer values of n.

"MULTIPLE OF 2"

$$(2n+1)^2 - (2n+1)$$

$$= (2n+1)(2n+1) - 1(2n+1)$$

F O I L

$$= 4n^2 + 2n + 2n \cancel{+1} - 2n \cancel{-1}$$

$$= 4n^2 + 2n$$

SIMPLE FACTORISATION

$$= 2n(2n+1)$$

THIS HAS A FACTOR OF 2 AND SO IS AN EVEN NUMBER

Answer 5

"WHOLE NUMBER"

n is an integer greater than 1

"MULTIPLE OF 2"

Prove algebraically that $n^2 - 2 - (n - 2)^2$ is always an even number.

$$n^2 - 2 - (n-2)^2 = n^2 - 2 - (n-2)(n-2)$$

F O I L

$$= n^2 - 2 - [n^2 - 2n - 2n + 4]$$

$$= \cancel{n^2} - 2 - \cancel{n^2} + 2n + 2n - 4$$

$$= 4n - 6$$

$$= 2 \times 2n - 2 \times 3$$

$$= 2(2n - 3)$$

THIS HAS A FACTOR OF 2 AND SO IS AN EVEN NUMBER

EXAM PAPERS PRACTICE

Answer 6

The product of two consecutive positive integers is added to the larger of the two integers.

WHOLE NUMBERS

Prove that the result is always a square number.

TWO CONSECUTIVE INTEGERS: n , $n+1$

PRODUCT + LARGER $= n(n+1) + n+1$

$= n^2 + n + n + 1$

$= n^2 + 2n + 1$

$= (n+1)^2$

WHICH IS A SQUARE NUMBER.

$(n+1)^2 = (n+1)(n+1)$

F O I L

$= n^2 + n + n + 1$

$= n^2 + 2n + 1$

EXAM PAPERS PRACTICE

Answer 7

Prove that

$$(2n + 3)^2 - (2n - 3)^2 \text{ is a multiple of } 8$$

for all positive integer values of n.

$$(2n+3)^2 - (2n-3)^2$$

$$= (2n+3)(2n+3) - (2n-3)(2n-3)$$

F O I L F O I L

$$= 4n^2 + 6n + 6n + 9 - (4n^2 - 6n - 6n + 9)$$

$$= \cancel{4n^2} + 12n \cancel{+9} \cancel{-4n^2} + 12n \cancel{-9}$$

$$= 24n$$

$$= 8 \times 3n \quad \text{WHICH IS A MULTIPLE OF 8.}$$

OR

$$\boxed{\text{DOTS.} \quad a^2 - b^2 = (a+b)(a-b)}$$

Let $a = 2n+3$, $b = 2n-3$

$$a^2 - b^2 = (2n+3+2n-3)(2n+3-(2n-3))$$

$$= 4n \times 6$$

$$= 24n$$

$$= \underline{\underline{8 \times 3n}}$$

EXAM PAPERS PRACTICE

Answer 8

Prove algebraically that

$$(2n + 1)^2 - (2n + 1) \quad \text{is an even number}$$

HAS A FACTOR OF 2

for all positive integer values of n.

$= (2n+1)(2n+1) - (2n+1)$

F O I L

$= 4n^2 + 2n + 2n + 1 - 2n - 1$

$= 4n^2 + 2n + 0$

$= 2n(2n+1)$

OR

$= (2n+1)[(2n+1) - 1]$

$= (2n+1)[2n]$

THIS HAS A FACTOR OF 2 AND IS THEREFORE EVEN.

Answer 9

(i) Factorise $2t^2 + 5t + 2$ $\rightarrow$ 1×2

$\underline{(2t+1)(t+2)}$

HECK $\left(= \overset{F}{2t^2} \overset{O}{+4t} \overset{I}{+t} \overset{L}{+2} \right)$

$= 2t^2 + 5t + 2$ ✓

(ii) t is a positive whole number.

The expression $2t^2 + 5t + 2$ can never have a value that is a prime number.

Explain why.

HAS FACTORS OF 1 AND ITSELF <u>ONLY</u>

C $2t^2 + 5t + 2 = (2t+1)(t+2)$

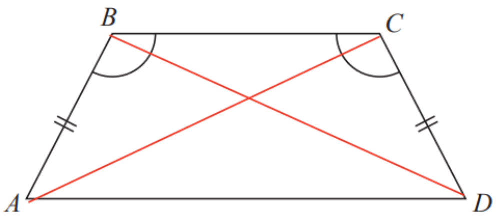
SINCE $2t+1 > 1$

AND $t+2 > 1$

$2t^2+5t+2$ IS THE PRODUCT OF TWO NUMBERS BIGGER THAN 1 SO IS NOT PRIME

Answer 10

ABCD is a quadrilateral.



$AB = CD$.
Angle ABC = angle BCD.

Prove that $AC = BD$.

GIVEN $AB = CD$

AND $A\hat{B}C = B\hat{C}D$

ALSO BC IS COMMON SIDE

SO "SAS" MEANS THAT

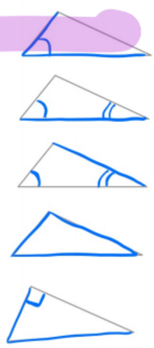
ΔABC IS CONGRUENT ΔBCD

SO $\underline{AC = BD}$

CONGRUENT TRIANGLES

NEED TO USE ONE OF THE "STANDARD" TESTS: IE SHOW THAT THREE PARTICULAR THINGS ARE THE SAME IN THE TWO TRIANGLES:

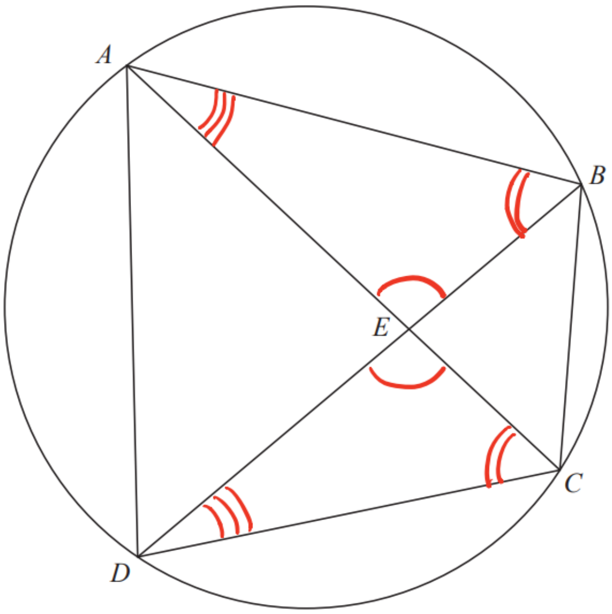
1. SAS
2. ASA
3. AAS
4. SSS
5. RHS



EXAM PAPERS PRACTICE

Answer 11

A, B, C and D are four points on the circumference of a circle.



CIRCLE THEOREMS

AEC and BED are straight lines.

Prove that triangle ABE and triangle DCE are similar. → SAME ANGLES
You must give reasons for each stage of your working.

$A\hat{E}B = D\hat{E}C$ (OPPOSITE ANGLES)

$A\hat{B}E = D\hat{C}A$ (ANGLES IN THE SAME SEGMENT)

$B\hat{A}E = C\hat{D}E$ (ANGLES IN THE SAME SEGMENT)

SO SINCE 3 PAIRS OF ANGLES ARE EQUAL THE Δs ARE SIMILAR

Answer 12

Prove that the square of an odd number is always 1 more than a multiple of 4

IF n IS AN INTEGER (WHOLE NUMBER)

THEN $2n$ IS AN EVEN NUMBER

SO $2n+1$ IS AN ODD NUMBER

$$(2n+1)^2 = (2n+1)(2n+1)$$

F O I L

$$= 4n^2 + 2n + 2n + 1$$

$$= 4n^2 + 4n + 1$$

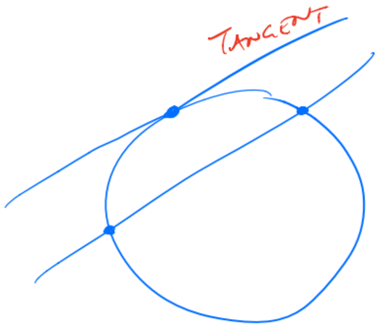
$$= \underbrace{4(n^2+n)}_{\text{MULTIPLE OF 4}} + 1$$

"1 MORE THAN"

Answer 13

Prove algebraically that the straight line with equation $x - 2y = 10$ (A) is a tangent to the circle with equation $x^2 + y^2 = 20$ (B)

$$\boxed{\begin{array}{l}\text{QUADRATIC SIMULTANEOUS EQUATIONS} \\ \text{USE "SUBSTITUTION" METHOD}\end{array}}$$



SHOW THAT THERE IS ONE POINT OF INTERSECTION ONLY

(A) : $x - 2y = 10$

$+2y \qquad +2y$

$x = 2y + 10$

→ (B) $x^2 + y^2 = 20$

$(2y+10)^2 + y^2 = 20$

$(2y+10)(2y+10) + y^2 = 20$

F O I L

$4y^2 + 20y + 20y + 100 + y^2 = 20$

$-20 \qquad -20$

$5y^2 + 40y + 80 = 0$

(÷5) $y^2 + 8y + 16 = 0$

$(y+4)(y+4) = 0$

$(y+4)^2 = 0$

SO $y = -4$ IS THE ONLY SOLUTION

SO $x - 2y = 10$ IS A TANGENT TO THE CIRCLE $x^2 + y^2 = 20$.

Answer 14

Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

"WHOLE NUMBERS"

TWO CONSECUTIVE INTEGERS: n AND $n+1$

SUM $= n + n + 1 = \underline{2n+1}$

DIFFERENCE BETWEEN SQUARES $= (n+1)^2 - n^2$

$= (n+1)(n+1) - n^2$

F O I L

$= n^2 + n + n + 1 - n^2$

$= \cancel{n^2} + 2n + 1 - n^2$

$= \underline{2n+1}$

SO DIFF BETWEEN SQUARES = SUM