Question 1

The lengths, in cm, of 120 adult platypuses are recorded in the following table:

Length, l (cm)	Frequency (female)	Frequency (male)
$39 \le l < 42$	14	0
$42 \le l < 45$	29	0
$45 \le l < 48$	12	7
$48 \le l < 51$	6	21
$51 \le l < 54$	3	19
$54 \le l < 57$	1	5
$57 \le l < 60$	0	2
$60 \le l < 63$	0	1

One platypus is chosen at random. Find the probability that the platypus is:

(i) male

(ii) less than 51 cm long

(iii) a male less than 45 cm long

(iv) a female between 45 and 54 cm long.

[4]

(i) SUM OF MALES $7+21+19+5+2+1=55$

$P(\text{MALE}) = \frac{55}{120} = \frac{11}{24}$

$\boxed{P(\text{MALE}) = \frac{11}{24}}$

(ii) SUM <51 (FIRST 4 ROWS)

$14+29+12+6+7+21 = 89$

$\boxed{P(<51) = \frac{89}{120}}$

(iii) NO MALES ARE <45 cm

$\boxed{P(<45) = 0}$

(iv) SUM $45 \le L < 54$ FOR FEMALES ONLY

$12+6+3 = 21$

$P(45 \le F < 54) = \frac{21}{120} = \frac{7}{40}$

$\boxed{P(45 \le F < 54) = \frac{7}{40}}$

Question 2

Two fair spinners each have three sectors numbered 1 to 3. The two spinners are spun together and then the product of the numbers indicated on each spinner is recorded.

Find the probability of the product indicated by the spinners being

(i) exactly 6

(ii) less than 4

(iii) an odd number.

[4]

DRAW A SAMPLE SPACE DIAGRAM TO SHOW ALL POSSIBILITIES

×	1	2	3
1	1	2	3
2	2	4	6
3	3	6	9

USE THE SAMPLE SPACE DIAGRAM TO FIND REQUIRED PROBABILITIES

(i) $\boxed{P(6) = \frac{2}{9}}$

(ii) $\boxed{P(<4) = \frac{5}{9}}$

(iii) $\boxed{P(\text{ODD}) = \frac{4}{9}}$

EXAM PAPERS PRACTICE

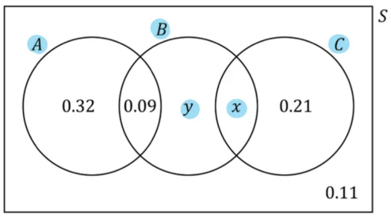
Question 3

The Venn diagram below shows the probabilities of members of an exotic sports society participating in various activities.

A represents the event that the member participates in aerial yoga.

B represents the event that the member participates in bog snorkelling.

C represents the event that the member participates in cheese rolling.



Given that the probability of a member participating in cheese rolling is 0.44,

(a) determine the values of

(i) x

(ii) y.

[3]

(b) Determine the probability that a member of the society

(i) participates in at least one of the three activities

(ii) participates in exactly one of the three activities.

[2]

(a)

(i) IF P(CHEESE ROLLING) = 0.44

$x = 0.44 - 0.21 = 0.23$

$\boxed{x = 0.23}$

(ii) PROBABILITIES SHOULD TOTAL 1

$y = 1 - (0.32 + 0.09 + 0.23 + 0.21 + 0.11)$

$= 1 - 0.96 = 0.04$

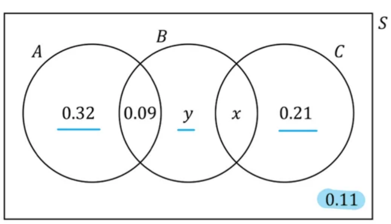
$\boxed{y = 0.04}$

The Venn diagram below shows the probabilities of members of an exotic sports society participating in various activities.

A represents the event that the member participates in aerial yoga.

B represents the event that the member participates in bog snorkelling.

C represents the event that the member participates in cheese rolling.



Given that the probability of a member participating in cheese rolling is 0.44,

(a) determine the values of

(i) x

(ii) y.

$\boxed{x = 0.23 \quad y = 0.04}$

[3]

(b) Determine the probability that a member of the society

(i) participates in at least one of the three activities

(ii) participates in exactly one of the three activities.

[2]

(b)

(i) ATLEAST ONE = EVERYTHING INSIDE CIRCLES

$1 - 0.11 = 0.89$ (COULD ADD ALL INSIDE)

$\boxed{P(\text{ATLEAST ONE}) = 0.89}$

(ii) EXACTLY ONE = INSIDE INDIVIDUAL CIRCLES (NOT OVERLAPS)

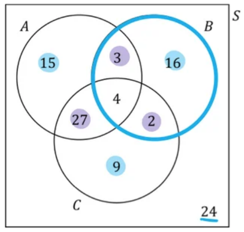
$0.32 + 0.04 + 0.21 = 0.57$

$\boxed{P(\text{EXACTLY ONE}) = 0.57}$

EXAM PAPERS PRACTICE

Question 4

The following Venn diagram shows the number of adults in a poll who said they enjoy watching action films (A), Bollywood musicals (B), and crime thrillers (C). 100 adults were polled in total.



(a) One of the adults who was polled is selected at random. Given that the adult chosen enjoys watching at least one of those three genres of film, find the probability that the adult enjoys watching:

(i) Bollywood musicals

(ii) only one of the three genres of film

(iii) exactly two of the three genres of film.

[3]

(b) Find the following probabilities:

(i) $P(A \cap C)$

(ii) $P(A \cup C)$

(iii) $P(C|B)$

(iv) $P(B')$

[4]

(a) 'GIVEN THEY ENJOY ATLEAST ONE GENRE' IGNORE VALUES OUTSIDE CIRCLES (24)

$15+3+16+27+4+2+9 = 76$

(i) $P(\text{BOLLYWOOD MUSICALS}) = \frac{16+3+4+2}{76}$ ← VALUES IN B

$$\boxed{P(\text{BOLLYWOOD}) = \frac{25}{76}}$$

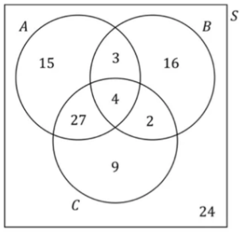
(ii) $P(\text{ONLY ONE GENRE}) = \frac{15+16+9}{76} = \frac{40}{76}$ ← VALUES NOT IN INTERSECTIONS

$$\boxed{P(\text{ONLY ONE GENRE}) = \frac{10}{19}}$$

(iii) $P(\text{EXACTLY TWO GENRES}) = \frac{3+27+2}{76} = \frac{32}{76}$ ← VALUES IN DUAL INTERSECTIONS

$$\boxed{P(\text{EXACTLY TWO GENRES}) = \frac{8}{19}}$$

The following Venn diagram shows the number of adults in a poll who said they enjoy watching action films (A), Bollywood musicals (B), and crime thrillers (C). 100 adults were polled in total.



(a) One of the adults who was polled is selected at random. Given that the adult chosen enjoys watching at least one of those three genres of film, find the probability that the adult enjoys watching:

(i) Bollywood musicals

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(iii) exactly two of the three genres of film.

[3]

(b) Find the following probabilities:

(i) $P(A \cap C)$

(ii) $P(A \cup C)$

(iii) $P(C|B)$

(iv) $P(B')$

[4]

(b) (i) INTERSECTION

$$P(A \cap C) = \frac{27+4}{100} = \frac{31}{100} = 0.31$$

$$\boxed{P(A \cap C) = \frac{31}{100}}$$ OR 0.31

(ii) $P(A \cup C)$ UNION

COULD ADD ALL VALUES IN A OR C OR TAKE VALUES NOT IN EITHER

$$\frac{15+3+4+27+9+2}{100} = \frac{60}{100} = \frac{3}{5} = 0.6$$

OR

$$\frac{100-(16+24)}{100} = \frac{60}{100} = \frac{3}{5} = 0.6$$

$$\boxed{P(A \cup C) = \frac{3}{5}}$$ OR 0.6

(iii) $P(C|B)$ = CONDITIONAL C GIVEN B

$n(B) = 3+4+2+16 = 25$ TOTAL IN B

$n(C \cap B) = 4+2 = 6$ TOTAL IN INTERSECTION

$$P(C|B) = \frac{6}{25} = 0.24$$

$$\boxed{P(C|B) = \frac{6}{25}}$$ OR 0.24

(iv) $P(B') = 1 - P(B)$ NOT IN B

$$P(B) = \frac{3+4+2+16}{100} = \frac{25}{100}$$

$$P(B') = 1 - \frac{25}{100} = \frac{75}{100} = \frac{3}{4} = 0.75$$

$$\boxed{P(B') = \frac{3}{4}}$$ OR 0.75

EXAM PAPERS PRACTICE

Question 5

On any given day the probability that Radigast has a lichen smoothie with his lunch is 0.4, and the probability that he has a wild mushroom wrap is 0.8. Given that the probability of him having both those items is 0.35, find the probability that Radigast has:

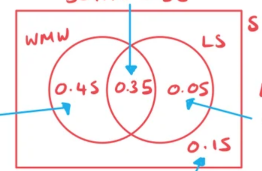
(i) a wild mushroom wrap but not a lichen smoothie

(ii) neither a wild mushroom wrap nor a lichen smoothie.

[4]

DRAW A VENN DIAGRAM TO ORGANISE OPTIONS

BOTH = 0.35



LS ONLY
0.4 − 0.35 = 0.05

(i) WMW ONLY
0.8 − 0.35 = $\boxed{0.45}$

(ii) NEITHER
1 − (0.8 + 0.4 − 0.35) = $\boxed{0.15}$
OR
1 − (0.45 + 0.35 + 0.05)

OR USE JUST CALCULATIONS

(i) WMW ONLY 0.8 − 0.35 = 0.45

$\boxed{P(\text{WMW ONLY}) = 0.45}$

(ii) NEITHER 1 − (0.8 + 0.4 − 0.35) = 0.15

$\boxed{P(\text{NEITHER}) = 0.15}$

Question 6

(a) A and B are two events such that $P(A) = 0.35$, $P(B) = 0.25$ and $P(A \cup B) = 0.6$. State, with a reason, whether A and B are mutually exclusive.

[2]

(b) C and D are two events such that $P(C) = 0.2$, $P(D) = 0.4$ and $P(C \cap D) = 0.18$. State, with a reason, whether C and D are independent.

[2]

(a) '∪' = 'OR' = ADD PROBABILITIES

MUTUALLY EXCLUSIVE = NO COMMON OUTCOMES
(NO OVERLAP IN VENN DIAGRAM)

$P(A) + P(B) = 0.35 + 0.25 = 0.6 = P(A \cup B)$

$P(A) + P(B) = P(A \cup B)$

A AND B ARE MUTUALLY EXCLUSIVE

(a) A and B are two events such that $P(A) = 0.35$, $P(B) = 0.25$ and $P(A \cup B) = 0.6$. State, with a reason, whether A and B are mutually exclusive.

[2]

(b) C and D are two events such that $P(C) = 0.2$, $P(D) = 0.4$ and $P(C \cap D) = 0.18$. State, with a reason, whether C and D are independent.

[2]

(b) '$\cap$' = 'AND' = MULTIPLY

INDEPENDENT = NO EFFECT ON EACH OTHER

$P(C) \times P(D) = 0.2 \times 0.4 = 0.08 \neq 0.18 = P(C \cap D)$

$P(C) \times P(D) \neq P(C \cap D)$

C AND D ARE NOT INDEPENDENT

Question 7

A bag contains 13 yellow tokens and 7 green tokens. Two tokens are drawn from the bag without replacement.

(a) Draw a tree diagram to represent this experiment.

[3]

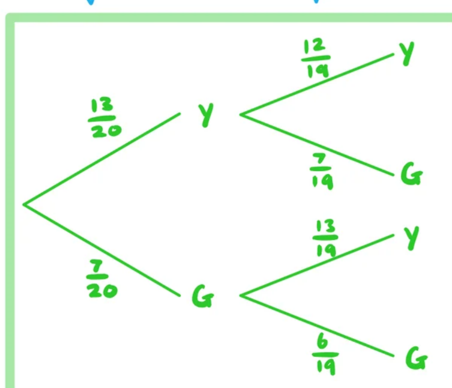
(b) Find the probability that the two tokens drawn are the same colour.

[3]

(a)

INITIAL TOTAL $7+13=20$

WITHOUT REPLACEMENT TOTALS ARE NOW $20-1=19$



YELLOW REDUCED BY 1 $13-1=12$

GREEN STAYS THE SAME

YELLOW STAYS THE SAME

GREEN REDUCED BY 1 $7-1=6$

CHECK EACH BRANCH PAIR = 1

YOU CAN THEN CALCULATE ALL POSSIBLE OUTCOMES BY MULTIPLYING ALONG EACH BRANCH

A bag contains 13 yellow tokens and 7 green tokens. Two tokens are drawn from the bag without replacement.

(a) Draw a tree diagram to represent this experiment.

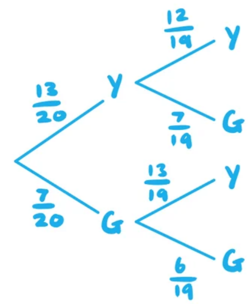
[3]

(b) Find the probability that the two tokens drawn are the same colour.

[3]

(b) USING TREE DIAGRAM FROM PART (a)

MULTIPLY ALONG BRANCHES WITH SAME COLOUR



$P(Y \text{ AND } Y) = \frac{13}{20} \times \frac{12}{19} = \frac{156}{380}$

$P(G \text{ AND } G) = \frac{7}{20} \times \frac{6}{19} = \frac{42}{380}$

NOT SIMPLIFYING MAKES PROBABILITIES EASIER TO ADD

$P(Y \text{ AND } Y) + P(G \text{ AND } G) = P(\text{SAME COLOUR})$

$$\left(\frac{13}{20} \times \frac{12}{19}\right) + \left(\frac{7}{20} \times \frac{6}{19}\right) = \frac{156}{380} + \frac{42}{380} = \frac{198}{380} = \frac{99}{190}$$

$$\boxed{P(\text{SAME COLOUR}) = \frac{99}{190}}$$

Question 8

A, B and C are three events with $P(A) = 0.2$, $P(B) = 0.25$, $P(C) = 0.6$ and $P(B \cap C) = 0.08$.

(a) Given that events A and C are mutually exclusive, and that events A and B are independent, draw a Venn diagram to illustrate the probabilities.

[4]

(b) Find:

(i) $P(A' \cap C')$

(ii) $P((A \cap B') \cup C)$

(iii) $P(A' \cup (B \cap C)')$

[6]

(a) A AND C MUTUALLY EXCLUSIVE $\Rightarrow$ DO NOT OVERLAP

A AND B INDEPENDENT $\Rightarrow$

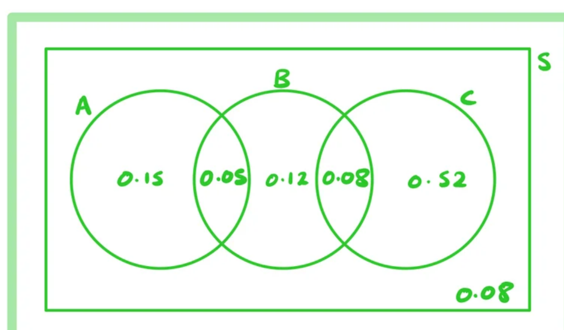
$P(A \cap B) = P(A)P(B) = 0.2 \times 0.25 = 0.05$

A ONLY $= 0.2 - 0.05 = 0.15$

B ONLY $= 0.25 - (0.05 + 0.08) = 0.12$

C ONLY $= 0.6 - 0.08 = 0.52$

OUTSIDE $= 1 - (0.15 + 0.05 + 0.12 + 0.08 + 0.52) = 0.08$



A, B and C are three events with $P(A) = 0.2$, $P(B) = 0.25$, $P(C) = 0.6$ and $P(B \cap C) = 0.08$.

(a) Given that events A and C are mutually exclusive, and that events A and B are independent, draw a Venn diagram to illustrate the probabilities.

[4]

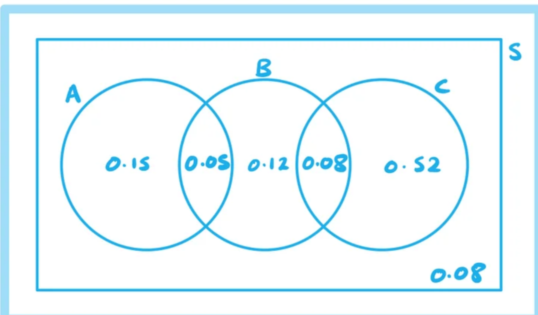
(b) Find:

(i) $P(A' \cap C')$

(ii) $P((A \cap B') \cup C)$

(iii) $P(A' \cup (B \cap C)')$

[6]



(b) A' = NOT IN A $\cup$ = OR $\cap$ = AND

(i) NOT IN A AND NOT IN C

$0.12 + 0.08 = 0.2$

$\boxed{P(A' \cap C') = 0.2}$

(ii) (IN A AND NOT IN B) OR IN C

$0.15 + 0.6 = 0.75$

$\boxed{P((A \cap B') \cup C) = 0.75}$

(iii) NOT IN A OR (NOT IN INTERSECTION OF B AND C)

EVERYTHING = 1

$\boxed{P(A' \cup (B \cap C)') = 1}$

Question 9

Three events, A, B and C, are such that B and C are mutually exclusive and A and C are independent. $P(A) = 0.3$, $P(B) = 0.45$ and $P(C) = 0.1$.

(a) Given that $P((A \cup B \cup C)') = 0.43$, draw a Venn diagram to show the probabilities for events A, B and C.

[4]

(b) Find:

(i) $P(B|A)$

(ii) $P(A|B')$

(iii) $P(A|(B \cup C))$

[6]

(a) $P((A \cup B \cup C)') = 0.43$ NOT IN A OR B OR C

B AND C MUTUALLY EXCLUSIVE $\Rightarrow$ DO NOT OVERLAP

A AND C INDEPENDENT $\Rightarrow$

$P(A \cap C) = P(A)P(C) = 0.3 \times 0.1 = 0.03$

ONLY C: $0.1 - 0.03 = 0.07$

$A \cup B = 1 - 0.43 - 0.07 = 0.5$

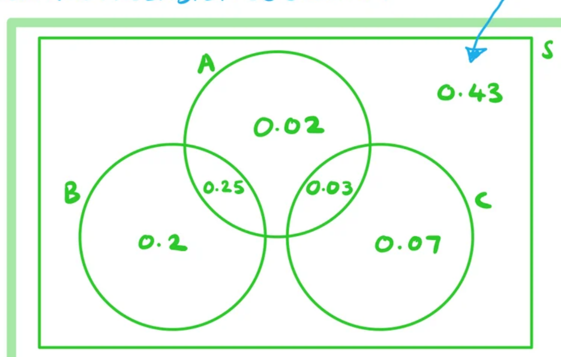
$A + B = 0.3 + 0.45 = 0.75$

$A \cap B = 0.75 - 0.5 = 0.25$

ONLY B $= 0.45 - 0.25 = 0.2$

ONLY A $= 0.3 - 0.25 - 0.03 = 0.02$

CHECK ALL PROBABILITIES SUM TO 1



Three events, A, B and C, are such that B and C are mutually exclusive and A and C are independent. $P(A) = 0.3$, $P(B) = 0.45$ and $P(C) = 0.1$.

(a) Given that $P((A \cup B \cup C)') = 0.43$, draw a Venn diagram to show the probabilities for events A, B and C.

[4]

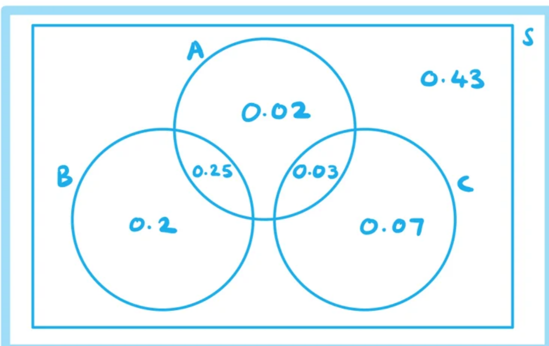
(b) Find:

(i) $P(B|A)$

(ii) $P(A|B')$

(iii) $P(A|(B \cup C))$

[6]



(b) (i) $P(B|A)$ = IN B GIVEN IN A

$$P(B|A) = \frac{\overset{P(B \cap A)}{0.25}}{\underset{P(A)}{0.3}} = \frac{5}{6}$$

$$\boxed{P(B|A) = \frac{5}{6}}$$

(ii) $P(A|B')$ = IN A GIVEN NOT IN B

$$P(A|B') = \frac{\overset{P(A \cap B')}{0.02 + 0.03}}{\underset{P(B')}{1 - 0.45}} = \frac{0.05}{0.55} = \frac{1}{11}$$

$$\boxed{P(A|B') = \frac{1}{11}}$$

(iii) $P(A|(B \cup C))$ = IN A GIVEN IN B OR C

$$P(A|(B \cup C)) = \frac{\overset{P(A \cap B) + P(A \cap C)}{0.25 + 0.03}}{\underset{P(B \cup C)}{0.45 + 0.1}} = \frac{0.28}{0.55} = \frac{28}{55}$$

$$\boxed{P(A|(B \cup C)) = \frac{28}{55}}$$

Question 10

Given that $P(A) = 0.27$, $P(B) = 0.39$ and $P(A \cap B) = 0.21$, find:

(a) (i) $P(A \cup B)$ IN A OR B

(ii) $P(B|A)$ IN B GIVEN IN A

[4]

The event C has $P(C) = 0.19$. The events A and C are mutually exclusive.

(b) Given that $P(B \cap C) = 0.04$, find $P(A \cup B \cup C)$.

[2]

(a)

(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0.27 + 0.39 - 0.21 = 0.45$

$$\boxed{P(A \cup B) = 0.45}$$

(ii) $P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)}$

$P(B|A) = \frac{0.21}{0.27} = \frac{7}{9}$

$$\boxed{P(B|A) = \frac{7}{9}}$$

EXAM PAPERS PRACTICE

Given that $P(A) = 0.27$, $P(B) = 0.39$ and $P(A \cap B) = 0.21$, find:

(a) (i) $P(A \cup B)$

(ii) $P(B|A)$

$\boxed{P(A \cup B) = 0.45}$

[4]

The event C has $P(C) = 0.19$. The events A and C are mutually exclusive.

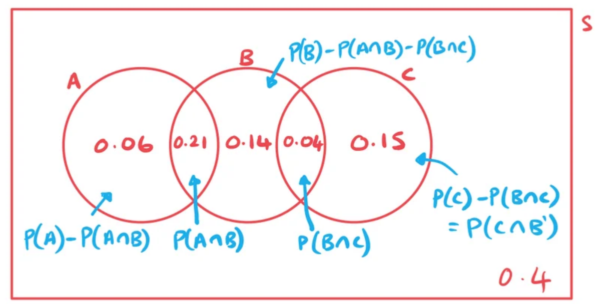
(b) Given that $P(B \cap C) = 0.04$, find $P(A \cup B \cup C)$.

IN A OR B OR C

[2]

(b) DRAWING A VENN DIAGRAM MAY HELP

A AND C = MUTUALLY EXCLUSIVE ⇒ DONT OVERLAP



$P(A \cup B \cup C) = P(A \cup B) + P(C \cap B')$

A OR B (FROM PART (a)) + C AND NOT IN B

$P(A \cup B \cup C) = 0.45 + 0.15 = 0.6$

$\boxed{P(A \cup B \cup C) = 0.6}$

Question 11

Ichabod is a keen chess player who plays one game of chess online every night before going to bed. In any one of those games, the probabilities of Ichabod winning, drawing, or losing are 0.4, 0.27 and 0.33 respectively. Following each game, the probabilities of Ichabod sleeping well after winning, drawing or losing are 0.7, 0.9 and 0.2 respectively.

(a) Draw a tree diagram to represent this information.

[3]

(b) Find the probability that on a randomly chosen night

(i) Ichabod loses his chess game and sleeps well

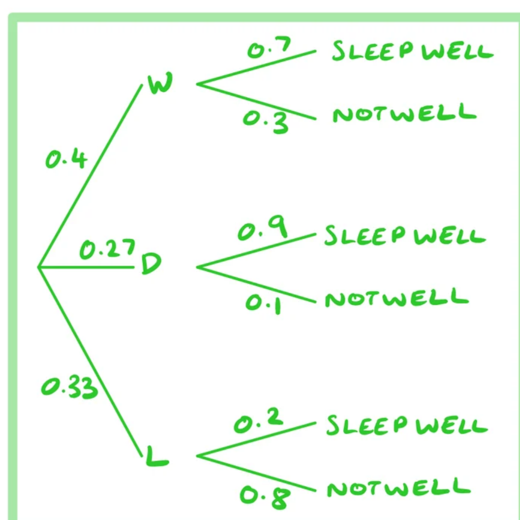
(ii) Ichabod sleeps well.

[4]

(c) Given that Ichabod sleeps well, find the probability that his chess game did not end in a draw.

[4]

(a)



CHECK EACH BRANCH PAIR = 1

YOU CAN THEN CALCULATE ALL POSSIBLE OUTCOMES BY MULTIPLYING ALONG EACH BRANCH

Ichabod is a keen chess player who plays one game of chess online every night before going to bed. In any one of those games, the probabilities of Ichabod winning, drawing, or losing are 0.4, 0.27 and 0.33 respectively. Following each game, the probabilities of Ichabod sleeping well after winning, drawing or losing are 0.7, 0.9 and 0.2 respectively.

(a) Draw a tree diagram to represent this information.

[3]

(b) Find the probability that on a randomly chosen night

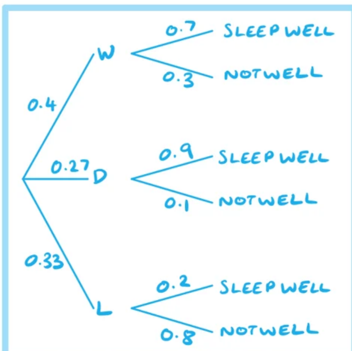
(i) Ichabod loses his chess game and sleeps well

(ii) Ichabod sleeps well.

[4]

(c) Given that Ichabod sleeps well, find the probability that his chess game did not end in a draw.

[4]



(b)

(i) $P(\text{LOSE AND SLEEP WELL}) = 0.33 \times 0.2 = 0.066$

$\boxed{P(\text{LOSE AND SLEEP WELL}) = 0.066}$

(ii) $P(\text{SLEEPS WELL})$ = SLEEP WELL AFTER WIN, DRAW AND LOSE

$0.4 \times 0.7 + 0.27 \times 0.9 + 0.33 \times 0.2$

$0.28 + 0.243 + 0.066$

$\boxed{P(\text{SLEEPS WELL}) = 0.589}$

Ichabod is a keen chess player who plays one game of chess online every night before going to bed. In any one of those games, the probabilities of Ichabod winning, drawing, or losing are 0.4, 0.27 and 0.33 respectively. Following each game, the probabilities of Ichabod sleeping well after winning, drawing or losing are 0.7, 0.9 and 0.2 respectively.

(a) Draw a tree diagram to represent this information.

[3]

(b) Find the probability that on a randomly chosen night

(i) Ichabod loses his chess game and sleeps well

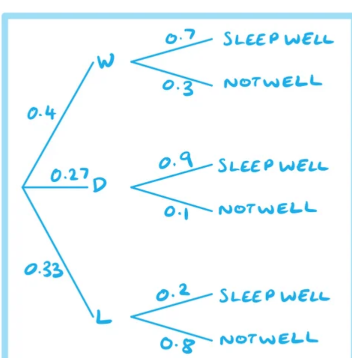
(ii) Ichabod sleeps well.

$\boxed{P(\text{SLEEPS WELL}) = 0.589}$

[4]

(c) Given that Ichabod sleeps well, find the probability that his chess game did not end in a draw.

[4]



(c)

$$P(\text{DRAW}' \mid \text{SLEEP WELL}) = \frac{P(\text{WIN} \ {}^{\text{SLEEP}}_{\text{WELL}}) + P(\text{LOSE} \ {}^{\text{SLEEP}}_{\text{WELL}})}{P(\text{SLEEP WELL})}$$

FROM (b)(ii)

$$\frac{(0.4 \times 0.7) + (0.33 \times 0.2)}{0.589} = \frac{0.346}{0.589}$$

$$\boxed{P(\text{DRAW}' \mid \text{SLEEP WELL}) = \frac{346}{589}}$$

EXAM PAPERS PRACTICE

Question 12

Two siblings, Percy and Cathy, have been taking driving lessons and are on a reserve list to take their test. It is equally likely that either of them will be called to take their test first. The probabilities that Percy and Cathy will pass their test if they are called first are 0.35 and 0.8 respectively.

(a) (i) Draw a tree diagram to represent this information.

(ii) Given that it was Cathy that took the test first, find the probability that the test was failed.

(iii) Given that it was not Cathy that took the test first, find the probability that the test was failed.

[4]

(b) Hence, find the probability that the person taking the test was Cathy given that they failed.

[3]

(a)(i) CONSIDER OPTIONS TO CONSTRUCT DIAGRAM

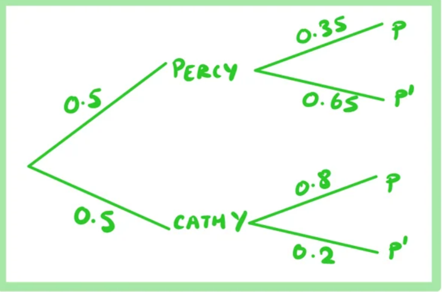
① PERCY OR CATHY

② EACH PASS OR FAIL

FAIL CAN BE WRITTEN AS NOT PASS P' OR $\overline{P}$

PERCY PASS $= 0.35 \Rightarrow$ FAIL $= P' = 1 - 0.35 = 0.65$

CATHY PASS $= 0.8 \Rightarrow$ FAIL $= P' = 1 - 0.8 = 0.2$



(ii) GIVEN EVENTS ARE INDEPENDENT $P(A|B) = P(A) = P(A|B')$

P(CATHY FAILS) $\boxed{P(P'|\text{CATHY}) = 0.2}$

(iii) P(PERCY FAILS) $\boxed{P(P'|\text{CATHY}') = 0.65}$

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[3]

AS THERE ARE TWO OPTIONS OF FAIL (P') USE BAYES THEOREM FROM FORMULA BOOKLET

$$P(B|A) = \frac{P(B)\,P(A|B)}{P(B)\,P(A|B) + P(B')\,P(A|B')}$$

(b) REWRITE GIVEN B = CATHY A = PASS' OR $\overline{\text{PASS}}$

THIS CAN BE EASIER TO SPOT!

$$P(\text{CATHY}|\overline{\text{PASS}}) = \frac{P(\text{CATHY})\,P(\overline{\text{PASS}}|\text{CATHY})}{P(\text{CATHY})\,P(\overline{\text{PASS}}|\text{CATHY})\,P(\overline{\text{CATHY}})\,P(\overline{\text{PASS}}|\overline{\text{CATHY}})}$$

SUBSTITUTE APPROPRIATE VALUES FROM (a)

$$P(\text{CATHY}|\overline{\text{PASS}}) = \frac{0.5 \times 0.2}{0.5 \times 0.2 + 0.5 \times 0.65}$$

$$= \frac{4}{17} = 0.235294117 6$$

$$\boxed{P(\text{CATHY}|\overline{\text{PASS}}) = 0.235 \text{ (3sf)}}$$