Question 1

Three biased coins are tossed.

(a) Write down all the possible outcomes when the three coins are tossed.

[1]

For each coin the probability of getting heads is $\frac{2}{3}$. A random variable, X, is defined as the number of heads when the three coins are tossed.

(b) Complete the following probability distribution table for X:

x	0	1	2	3
$\mathrm{P}(X = x)$				

[3]

(c) Hence, by inserting the relevant probabilities, represent the probability distribution for X as a piecewise function in the form

$$\mathrm{P}(X = x) = f(x) = \begin{cases} & x = 0 \\ & x = 1 \\ & x = 2 \\ & x = 3 \\ 0 & \text{otherwise} \end{cases}$$

[2]

(d) Represent the probability distribution for X as a bar chart.

[2]

H = 'heads' T = 'tails'

There are 8 possible outcomes:

a)

$$\boxed{\begin{array}{lll} \{H, H, H\} & & \\ \{H, H, T\} & \{H, T, H\} & \{T, H, H\} \\ \{H, T, T\} & \{T, H, T\} & \{T, T, H\} \\ \{T, T, T\} & & \end{array}}$$

Three biased coins are tossed.

(a) Write down all the possible outcomes when the three coins are tossed.

[1]

so probability of tails is 1/3

For each coin the probability of getting heads is $\frac{2}{3}$. A random variable, X, is defined as the number of heads when the three coins are tossed.

(b) Complete the following probability distribution table for X:

x	0	1	2	3
$\mathrm{P}(X = x)$	$\frac{1}{27}$	$\frac{6}{27}$	$\frac{12}{27}$	$\frac{8}{27}$

$= \frac{2}{9}$ $= \frac{4}{9}$

[3]

(c) Hence, by inserting the relevant probabilities, represent the probability distribution for X as a piecewise function in the form

$$\mathrm{P}(X = x) = f(x) = \begin{cases} & x = 0 \\ & x = 1 \\ & x = 2 \\ & x = 3 \\ 0 & \text{otherwise} \end{cases}$$

[2]

(d) Represent the probability distribution for X as a bar chart.

[2]

b) $\{H, H, H\}$ $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27} = \mathrm{P}(X = 3)$

$\{H, H, T\}$ $\frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{4}{27}$

$\{H, T, H\}$ $\frac{2}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{4}{27}$

$\{T, H, H\}$ $\frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{4}{27}$

$\frac{4}{27} + \frac{4}{27} + \frac{4}{27} = \frac{12}{27} = \mathrm{P}(X = 2)$

$\{H, T, T\}$ $\frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{2}{27}$

$\{T, H, T\}$ $\frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{2}{27}$

$\{T, T, H\}$ $\frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{2}{27}$

$\frac{2}{27} + \frac{2}{27} + \frac{2}{27} = \frac{6}{27} = \mathrm{P}(X = 1)$

$\{T, T, T\}$ $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27} = \mathrm{P}(X = 0)$

Three biased coins are tossed.

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[1]

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(b) Complete the following probability distribution table for X:

x	0	1	2	3
$P(X = x)$	$\frac{1}{27}$	$\frac{6}{27}$	$\frac{12}{27}$	$\frac{8}{27}$

[3]

(c) Hence, by inserting the relevant probabilities, represent the probability distribution for X as a piecewise function in the form

$$P(X = x) = f(x) = \begin{cases} & x = 0 \\ & x = 1 \\ & x = 2 \\ & x = 3 \\ 0 & \text{otherwise} \end{cases}$$

[2]

(d) Represent the probability distribution for X as a bar chart.

[2]

c)

$$P(X = x) = f(x) = \begin{cases} \frac{1}{27} & x = 0 \\ \frac{6}{27} & x = 1 \\ \frac{12}{27} & x = 2 \\ \frac{8}{27} & x = 3 \\ 0 & \text{otherwise} \end{cases}$$

$\frac{12}{27} = \frac{4}{9}$ $\quad$ $\frac{6}{27} = \frac{2}{9}$

Three biased coins are tossed.

(a) Write down all the possible outcomes when the three coins are tossed.

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For each coin the probability of getting heads is $\frac{2}{3}$. A random variable, X, is defined as the number of heads when the three coins are tossed.

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[3]

(c) Hence, by inserting the relevant probabilities, represent the probability distribution for X as a piecewise function in the form

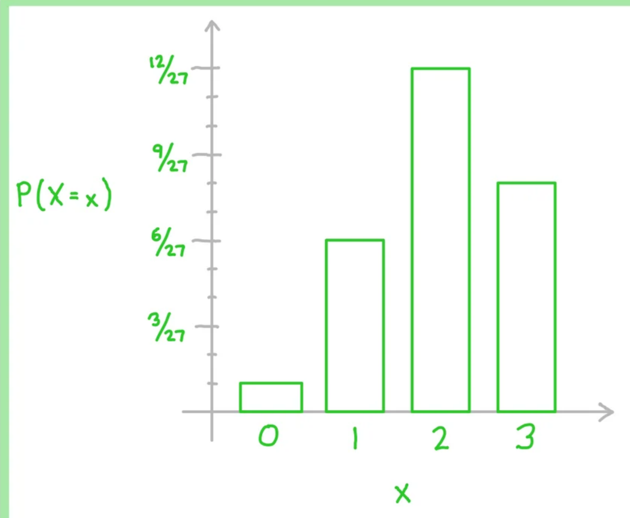
$$P(X = x) = f(x) = \begin{cases} & x = 0 \\ & x = 1 \\ & x = 2 \\ & x = 3 \\ 0 & \text{otherwise} \end{cases}$$

[2]

(d) Represent the probability distribution for X as a bar chart.

[2]

d)



Question 2

The random variable X has the probability function

$$P(X = x) = \begin{cases} \frac{x}{3k} & x = 1, 2, 3, 4, 5 \\ 0 & \text{otherwise} \end{cases}$$

Show that $k = 5$.

[2]

Sum of all probabilities must equal one :

$$\frac{1}{3k} + \frac{2}{3k} + \frac{3}{3k} + \frac{4}{3k} + \frac{5}{3k} = 1$$

$$\frac{1+2+3+4+5}{3k} = 1$$

$$\frac{15}{3k} = 1$$

$$3k = 15$$

$$k = \frac{15}{3}$$

$$\boxed{k = 5}$$

Question 3

The random variable X has the probability function

$$P(X = x) = \begin{cases} kx & x = 1, 3, 5, 7 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of k.

$P(X = 1) = k$

$P(X = 3) = 3k$

[2]

(b) Find $P(X > 3)$.

$P(X = 5) = 5k$

$P(X = 7) = 7k$

[2]

(c) State, with a reason, whether or not X is a discrete random variable.

[1]

a) Sum of all probabilities must equal one :

$$k + 3k + 5k + 7k = 16k = 1$$

$$\boxed{k = \frac{1}{16}}$$

The random variable X has the probability function

$$P(X = x) = \begin{cases} kx & x = 1, 3, 5, 7 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of k.

$P(X=1) = k$

$P(X=3) = 3k$

$P(X=5) = 5k$

$P(X=7) = 7k$

$k = \frac{1}{16}$

[2]

(b) Find $P(X > 3)$.

[2]

(c) State, with a reason, whether or not X is a discrete random variable.

[1]

b) $P(X>3) = P(X = 5 \text{ or } 7) = P(X=5) + P(X=7)$

$P(X>3) = \frac{5}{16} + \frac{7}{16} = \frac{12}{16}$

$$\boxed{P(X>3) = \frac{3}{4}}$$

The random variable X has the probability function

$$P(X = x) = \begin{cases} kx & x = 1, 3, 5, 7 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of k.

[2]

(b) Find $P(X > 3)$.

[2]

(c) State, with a reason, whether or not X is a discrete random variable.

[1]

c) There is a finite number of possible values that X can take, so X is a discrete random variable.

Question 4

The random variable X has the probability function

$$P(X = x) = \begin{cases} 0.23 & x = -1, 4 \\ k & x = 0, 2 \\ 0.13 & x = 1, 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of k.

[2]

(b) Construct a table giving the probability distribution of X.

[2]

(c) Find $P(0 \le X < 3)$.

[1]

a) Sum of all probabilities must equal one :

$0.23 + 0.23 + k + k + 0.13 + 0.13 = 1$

$2k + 0.72 = 1$

$2k = 0.28$

$\boxed{k = 0.14}$

The random variable X has the probability function

$$P(X = x) = \begin{cases} 0.23 & x = -1, 4 \\ k & x = 0, 2 \\ 0.13 & x = 1, 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of k.

$k = 0.14$

[2]

(b) Construct a table giving the probability distribution of X.

[2]

(c) Find $P(0 \le X < 3)$.

[1]

b)

x	-1	0	1	2	3	4
P(X=x)	0.23	0.14	0.13	0.14	0.13	0.23

The random variable X has the probability function

$$P(X = x) = \begin{cases} 0.23 & x = -1, 4 \\ k & x = 0, 2 \\ 0.13 & x = 1, 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of k.

[2]

(b) Construct a table giving the probability distribution of X.

[2]

(c) Find $P(0 \le X < 3)$.

[1]

x	-1	0	1	2	3	4
P(X=x)	0.23	0.14	0.13	0.14	0.13	0.23

c) $P(0 \le X < 3) = P(X = 0 \text{ or } 1 \text{ or } 2)$

$= P(X=0) + P(X=1) + P(X=2)$

$P(0 \le X < 3) = 0.14 + 0.13 + 0.14 = \boxed{0.41}$

Question 5

A discrete random variable X has the probability distribution shown in the following table:

x	0	1	2	3	4
$P(X = x)$	$\frac{5}{24}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{8}$

Find:

(i) $P(X < 4)$

(ii) $P(X > 1)$

(iii) $P(2 < X \leq 4)$

(iv) $P(0 < X < 4)$

[6]

(i) $P(X<4) = P(X \neq 4) = 1 - P(X=4)^{*}$

$P(X<4) = 1 - \frac{1}{8} = \boxed{\frac{7}{8}}$

* This is easier than adding up the probabilities for 0, 1, 2, and 3!

(ii) $P(X>1) = P(X = 2 \text{ or } 3 \text{ or } 4)$

$P(X>1) = \frac{1}{4} + \frac{1}{12} + \frac{1}{8} = \boxed{\frac{11}{24}}$

(iii) $P(2<X \leq 4) = P(X = 3 \text{ or } 4)$

$P(2<X \leq 4) = \frac{1}{12} + \frac{1}{8} = \boxed{\frac{5}{24}}$

(iv) $P(0<X<4) = P(X = 1 \text{ or } 2 \text{ or } 3)$

$P(0<X<4) = \frac{1}{3} + \frac{1}{4} + \frac{1}{12} = \frac{8}{12} = \boxed{\frac{2}{3}}$

EXAM PAPERS PRACTICE

Question 6

Leonardo has constructed a biased spinner with six sectors labelled 0, 1, 1, 2, 3 and 5. The probability of the spinner landing on each of the six sectors is shown in the following table:

number on sector	0	1	1	2	3	5
probability	$\frac{6}{20}$	p	$\frac{3}{20}$	$\frac{5}{20}$	$\frac{3}{20}$	$\frac{1}{20}$

(a) Find the value of p. [1]

Leonardo is playing a game with his biased spinner. The score for the game, X, is the number which the spinner lands on after being spun.

(b) Leonardo plays the game twice and adds the two scores together. Find the probability that Leonardo has a *total* score of 5. [3]

(c) Complete the following cumulative probability function table for X:

Score x	0	1	2	3	5
$P(X \le x)$	$\frac{6}{20}$				1

[2]

(d) Find the probability that X is

(i) no more than 1

(ii) at least 3. [2]

a) Sum of all probabilities must equal one :

$$\frac{6}{20} + p + \frac{3}{20} + \frac{5}{20} + \frac{3}{20} + \frac{1}{20} = 1$$

$$p + \frac{18}{20} = 1$$

$$\boxed{p = \frac{2}{20}} = \frac{1}{10}$$

Leonardo has constructed a biased spinner with six sectors labelled 0, 1, 1, 2, 3 and 5. The probability of the spinner landing on each of the six sectors is shown in the following table:

number on sector	0	1	1	2	3	5
probability	$\frac{6}{20}$	p	$\frac{3}{20}$	$\frac{5}{20}$	$\frac{3}{20}$	$\frac{1}{20}$

(a) Find the value of p.

$p = \frac{2}{20}$

[1]

Leonardo is playing a game with his biased spinner. The score for the game, X, is the number which the spinner lands on after being spun.

(b) Leonardo plays the game twice and adds the two scores together. Find the probability that Leonardo has a *total* score of 5. [3]

(c) Complete the following cumulative probability function table for X:

Score x	0	1	2	3	5
$P(X \le x)$	$\frac{6}{20}$				1

[2]

(d) Find the probability that X is

(i) no more than 1

(ii) at least 3. [2]

b) Possible outcomes that add up to 5 are :

$\{0, 5\}$ $\{5, 0\}$ $\{2, 3\}$ $\{3, 2\}$

$\{0, 5\}$ $\frac{6}{20} \times \frac{1}{20} = \frac{6}{400}$

$\{5, 0\}$ $\frac{1}{20} \times \frac{6}{20} = \frac{6}{400}$

$\{2, 3\}$ $\frac{5}{20} \times \frac{3}{20} = \frac{15}{400}$

$\{3, 2\}$ $\frac{3}{20} \times \frac{5}{20} = \frac{15}{400}$

$$\frac{6}{400} + \frac{6}{400} + \frac{15}{400} + \frac{15}{400} = \frac{42}{400} = \boxed{\frac{21}{200}}$$

$= 0.105$

Leonardo has constructed a biased spinner with six sectors labelled 0, 1, 1, 2, 3 and 5. The probability of the spinner landing on each of the six sectors is shown in the following table:

number on sector	0	1	1	2	3	5
probability	$\frac{6}{20}$	p	$\frac{3}{20}$	$\frac{5}{20}$	$\frac{3}{20}$	$\frac{1}{20}$

(a) Find the value of p.

$p = \frac{2}{20}$

[1]

Leonardo is playing a game with his biased spinner. The score for the game, X, is the number which the spinner lands on after being spun.

(b) Leonardo plays the game twice and adds the two scores together. Find the probability that Leonardo has a *total* score of 5.

[3]

(c) Complete the following cumulative probability function table for X:

Score x	0	1	2	3	5
$P(X \leq x)$	$\frac{6}{20}$	$\frac{11}{20}$	$\frac{16}{20}$	$\frac{19}{20}$	1

[2]

(d) Find the probability that X is

(i) no more than 1

(ii) at least 3.

[2]

c) $\frac{6}{20} + \underbrace{\left(\frac{2}{20} + \frac{3}{20}\right)} = \frac{11}{20}$

Note: Both '1' sectors on the spinner are included in the event $X = 1$.

$\frac{11}{20} + \frac{5}{20} = \frac{16}{20}$

$\frac{16}{20} + \frac{3}{20} = \frac{19}{20}$

Leonardo has constructed a biased spinner with six sectors labelled 0, 1, 1, 2, 3 and 5. The probability of the spinner landing on each of the six sectors is shown in the following table:

number on sector	0	1	1	2	3	5
probability	$\frac{6}{20}$	p	$\frac{3}{20}$	$\frac{5}{20}$	$\frac{3}{20}$	$\frac{1}{20}$

(a) Find the value of p.

$p = \frac{2}{20}$

[1]

Leonardo is playing a game with his biased spinner. The score for the game, X, is the number which the spinner lands on after being spun.

(b) Leonardo plays the game twice and adds the two scores together. Find the probability that Leonardo has a *total* score of 5.

[3]

(c) Complete the following cumulative probability function table for X:

Score x	0	1	2	3	5
$P(X \leq x)$	$\frac{6}{20}$	$\frac{11}{20}$	$\frac{16}{20}$	$\frac{19}{20}$	1

[2]

(d) Find the probability that X is

(i) no more than 1

(ii) at least 3.

[2]

d) (i) $P(X \text{ is no more than } 1) = P(X \leq 1)$

$\boxed{\frac{11}{20}} = 0.55$

(ii) $P(X \text{ is at least } 3) = P(X \geq 3) = 1 - P(X \leq 2)$

$1 - \frac{16}{20} = \frac{4}{20} = \boxed{\frac{1}{5}} = 0.2$

Question 7

A discrete random variable X has the following probability distribution:

x	-3	-1	0	1	3
$P(X = x)$	0.11	k^2	0.1	$2k$	0.1

where k is a positive constant.

(a) Show that $k^2 + 2k - 0.69 = 0$. [2]

(b) Hence find the value of k. [1]

(c) Find $E(X)$. [3]

a) The sum of all probabilities must equal one, so:

$$0.11 + k^2 + 0.1 + 2k + 0.1 = 1$$

$$k^2 + 2k + 0.31 = 1$$

$$k^2 + 2k + 0.31 - 1 = 0$$

$$\boxed{k^2 + 2k - 0.69 = 0}$$

A discrete random variable X has the following probability distribution:

x	-3	-1	0	1	3
$P(X = x)$	0.11	k^2	0.1	$2k$	0.1

where k is a positive constant.

(a) Show that $k^2 + 2k - 0.69 = 0$. [2]

(b) Hence find the value of k. [1]

(c) Find $E(X)$. [3]

b) Solve the quadratic:

$$k^2 + 2k - 0.69 = 0$$

$$(k - 0.3)(k + 2.3) = 0$$

$$k = 0.3 \text{ or } -2.3$$

You can also use your GDC to solve this

But k is positive, so

$$\boxed{k = 0.3}$$

A discrete random variable X has the following probability distribution:

x	-3	-1	0	1	3
$P(X = x)$	0.11	k^2	0.1	$2k$	0.1

0.09   0.6

where k is a positive constant.

(a) Show that $k^2 + 2k - 0.69 = 0$. [2]

(b) Hence find the value of k. [1]

$\boxed{k = 0.3}$

(c) Find $E(X)$. [3]

c) $k^2 = 0.3^2 = 0.09$ $2k = 2(0.3) = 0.6$

$$E(X) = \sum x\,P(X = x)$$ Expected value of a discrete random variable X

$$E(X) = (-3)(0.11) + (-1)(0.09) + (0)(0.1) + (1)(0.6) + (3)(0.1)$$

$$= -0.33 - 0.09 + 0 + 0.6 + 0.3$$

$$\boxed{E(X) = 0.48}$$

Question 8

A spinner is spun on a circle that is divided up into five sections, A, B, C, D and E.

The probability of the spinner landing on each section is given by the following table:

Region	A	B	C	D	E
Probability	0.55	0.15	0.15	0.1	0.05

A person who rotates the spinner scores points depending on which section the spinner lands on. These points are shown below.

Region	A	B	C	D	E
Points	−5	2	3	10	k

Given that the game is fair, find the value of k.

This means $E(X) = 0$

[4]

If X is a person's score, then:

$P(X=-5) = 0.55 \quad P(X=2) = 0.15 \quad P(X=3) = 0.15$

$P(X=10) = 0.1 \quad P(X=k) = 0.05$

And $E(X) = 0$, so:

$E(X) = \sum x\,P(X=x)$ } Expected value of a discrete random variable X

$(-5)(0.55) + (2)(0.15) + (3)(0.15) + (10)(0.1) + k(0.05) = 0$

$-2.75 + 0.3 + 0.45 + 1 + 0.05k = 0$

$0.05k - 1 = 0$

$0.05k = 1$

$k = \frac{1}{0.05}$

$\boxed{k = 20}$

Question 9

A discrete random variable X has the following probability distribution:

x	0	1	2	3	4
$P(X = x)$	0.1	0.05	a	b	0.1

The value of $E(X) = 2.3$.

(a) Show that a and b must satisfy the following two simultaneous equations:

$$a + b = 0.75$$
$$2a + 3b = 1.85$$

[3]

(b) Hence find the value of a and the value of b.

[2]

(c) Find $P(1 \le X < 4)$.

[2]

$E(X) = \sum x\,P(X=x)$ } Expected value of a discrete random variable X

a) The sum of all probabilities must equal one, so:

$0.1 + 0.05 + a + b + 0.1 = 1$

$a + b + 0.25 = 1$

$a + b = 1 - 0.25$

$a + b = 0.75$

And $E(X) = 2.3$, so:

$(0)(0.1) + (1)(0.05) + (2)(a) + (3)(b) + (4)(0.1) = 2.3$

$0 + 0.05 + 2a + 3b + 0.4 = 2.3$

$2a + 3b + 0.45 = 2.3$

$2a + 3b = 2.3 - 0.45$

$2a + 3b = 1.85$

A discrete random variable X has the following probability distribution:

x	0	1	2	3	4
$\mathrm{P}(X = x)$	0.1	0.05	a	b	0.1

The value of $\mathrm{E}(X) = 2.3$.

(a) Show that a and b must satisfy the following two simultaneous equations:

$$a + b = 0.75 \quad (1)$$
$$2a + 3b = 1.85 \quad (2)$$

[3]

(b) Hence find the value of a and the value of b.

[2]

(c) Find $\mathrm{P}(1 \leq X < 4)$.

[2]

b) Solve the simultaneous equations

(2) − 2×(1) :

$$2a + 3b = 1.85$$
$$-(2a + 2b = 1.5)$$
$$b = 0.35$$

Substitute into (1) :

$$a + 0.35 = 0.75$$
$$a = 0.75 - 0.35 = 0.4$$

$$\boxed{a = 0.4 \qquad b = 0.35}$$

You can also use your GDC to solve this

A discrete random variable X has the following probability distribution:

x	0	1	2	3	4
$\mathrm{P}(X = x)$	0.1	0.05	a	b	0.1

The value of $\mathrm{E}(X) = 2.3$.

(a) Show that a and b must satisfy the following two simultaneous equations:

$$a + b = 0.75$$
$$2a + 3b = 1.85$$

[3]

(b) Hence find the value of a and the value of b.

$$\boxed{a = 0.4 \qquad b = 0.35}$$

[2]

(c) Find $\mathrm{P}(1 \leq X < 4)$.

[2]

c) $\mathrm{P}(1 \leq X < 4) = \mathrm{P}(X = 1, 2, \text{ or } 3)$

$$\mathrm{P}(1 \leq X < 4) = 0.05 + a + b$$
$$= 0.05 + 0.4 + 0.35$$

$$\boxed{\mathrm{P}(1 \leq X < 4) = 0.8}$$