

# Probability Distributions

# Mark Schemes

## Question 1

Three biased coins are tossed.

(a) Write down all the possible outcomes when the three coins are tossed.

[1]

For each coin the probability of getting heads is  $\frac{2}{3}$ . A random variable,  $X$ , is defined as the number of heads when the three coins are tossed.

(b) Complete the following probability distribution table for  $X$ :

$x$	0	1	2	3
$P(X = x)$				

[3]

(c) Hence, by inserting the relevant probabilities, represent the probability distribution for  $X$  as a piecewise function in the form

$$P(X = x) = f(x) = \begin{cases} & x = 0 \\ & x = 1 \\ & x = 2 \\ & x = 3 \\ 0 & \text{otherwise} \end{cases}$$

[2]

(d) Represent the probability distribution for  $X$  as a bar chart.

[2]

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(b) Complete the following probability distribution table for  $X$ :

$x$	0	1	2	3
$P(X = x)$	$\frac{1}{27}$	$\frac{6}{27}$	$\frac{12}{27}$	$\frac{8}{27}$

$$= \frac{2}{9} = \frac{4}{9}$$

[3]

(c) Hence, by inserting the relevant probabilities, represent the probability distribution for  $X$  as a piecewise function in the form

$$P(X = x) = f(x) = \begin{cases} & x = 0 \\ & x = 1 \\ & x = 2 \\ & x = 3 \\ 0 & \text{otherwise} \end{cases}$$

[2]

(d) Represent the probability distribution for  $X$  as a bar chart.

[2]

$H = \text{'heads'}$     $T = \text{'tails'}$

There are 8 possible outcomes:

a)

$\{H, H, H\}$

$\{H, H, T\}$     $\{H, T, H\}$     $\{T, H, H\}$

$\{H, T, T\}$     $\{T, H, T\}$     $\{T, T, H\}$

$\{T, T, T\}$

b)  $\{H, H, H\} \quad \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27} = P(X=3)$

$\{H, H, T\} \quad \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{4}{27}$

$\{H, T, H\} \quad \frac{2}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{4}{27}$

$\{T, H, H\} \quad \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{4}{27}$

$\frac{4}{27} + \frac{4}{27} + \frac{4}{27} = \frac{12}{27} = P(X=2)$

$\{H, T, T\} \quad \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{2}{27}$

$\{T, H, T\} \quad \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{2}{27}$

$\{T, T, H\} \quad \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{2}{27}$

$\frac{2}{27} + \frac{2}{27} + \frac{2}{27} = \frac{6}{27} = P(X=1)$

$\{T, T, T\} \quad \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27} = P(X=0)$

Three biased coins are tossed.

(a) Write down all the possible outcomes when the three coins are tossed.

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$$P(X = x) = f(x) = \begin{cases} & x = 0 \\ & x = 1 \\ & x = 2 \\ & x = 3 \\ 0 & \text{otherwise} \end{cases}$$

[2]

(d) Represent the probability distribution for  $X$  as a bar chart.

[2]

c)

$$P(X = x) = f(x) = \begin{cases} \frac{1}{27} & x = 0 \\ \frac{6}{27} & x = 1 \\ \frac{12}{27} & x = 2 \\ \frac{8}{27} & x = 3 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{12}{27} = \frac{4}{9} \qquad \frac{6}{27} = \frac{2}{9}$$

Three biased coins are tossed.

(a) Write down all the possible outcomes when the three coins are tossed.

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For each coin the probability of getting heads is  $\frac{2}{3}$ . A random variable,  $X$ , is defined as the number of heads when the three coins are tossed.

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[3]

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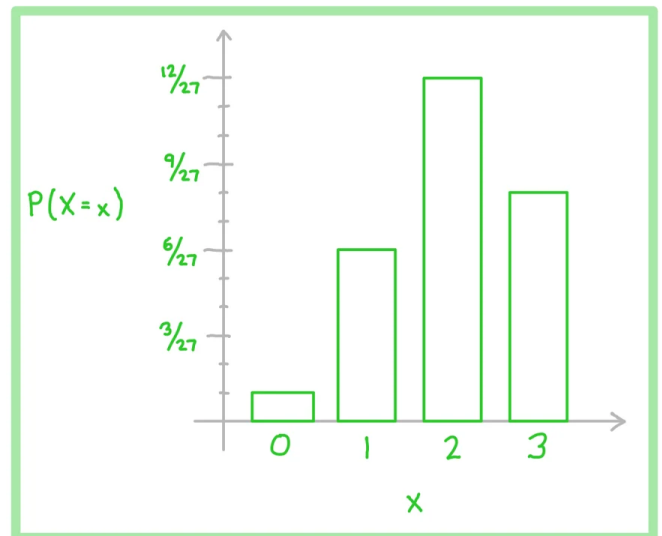
$$P(X = x) = f(x) = \begin{cases} & x = 0 \\ & x = 1 \\ & x = 2 \\ & x = 3 \\ 0 & \text{otherwise} \end{cases}$$

[2]

(d) Represent the probability distribution for  $X$  as a bar chart.

[2]

d)



## Question 2

The random variable  $X$  has the probability function

$$P(X = x) = \begin{cases} \frac{x}{3k} & x = 1, 2, 3, 4, 5 \\ 0 & \text{otherwise} \end{cases}$$

Show that  $k = 5$ .

[2]

Sum of all probabilities must equal one :

$$\begin{aligned} \frac{1}{3k} + \frac{2}{3k} + \frac{3}{3k} + \frac{4}{3k} + \frac{5}{3k} &= 1 \\ \frac{1+2+3+4+5}{3k} &= 1 \\ \frac{15}{3k} &= 1 \\ 3k &= 15 \\ k &= \frac{15}{3} \\ k &= 5 \end{aligned}$$

## Question 3

The random variable  $X$  has the probability function

$$P(X = x) = \begin{cases} kx & x = 1, 3, 5, 7 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of  $k$ .

$$P(X = 1) = k$$

$$P(X = 3) = 3k$$

$$P(X = 5) = 5k$$

$$P(X = 7) = 7k$$

[2]

(b) Find  $P(X > 3)$ .

[2]

(c) State, with a reason, whether or not  $X$  is a discrete random variable.

[1]

a) Sum of all probabilities must equal one :

$$k + 3k + 5k + 7k = 16k = 1$$

$$k = \frac{1}{16}$$

The random variable  $X$  has the probability function

$$P(X = x) = \begin{cases} kx & x = 1, 3, 5, 7 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of  $k$ .

$$k = \frac{1}{16}$$

$$P(X = 1) = k$$

$$P(X = 3) = 3k \quad [2]$$

$$P(X = 5) = 5k$$

$$P(X = 7) = 7k \quad [2]$$

(c) State, with a reason, whether or not  $X$  is a discrete random variable.

[1]

$$b) P(X > 3) = P(X = 5 \text{ or } 7) = P(X = 5) + P(X = 7)$$

$$P(X > 3) = \frac{5}{16} + \frac{7}{16} = \frac{12}{16}$$

$$P(X > 3) = \frac{3}{4}$$

The random variable  $X$  has the probability function

$$P(X = x) = \begin{cases} kx & x = 1, 3, 5, 7 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of  $k$ .

[2]

(b) Find  $P(X > 3)$ .

[2]

(c) State, with a reason, whether or not  $X$  is a discrete random variable.

[1]

c) There is a finite number of possible values that  $X$  can take, so  $X$  is a discrete random variable.

### Question 4

The random variable  $X$  has the probability function

$$P(X = x) = \begin{cases} 0.23 & x = -1, 4 \\ k & x = 0, 2 \\ 0.13 & x = 1, 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of  $k$ .

[2]

(b) Construct a table giving the probability distribution of  $X$ .

[2]

(c) Find  $P(0 \leq X < 3)$ .

[1]

a) Sum of all probabilities must equal one:

$$0.23 + 0.23 + k + k + 0.13 + 0.13 = 1$$

$$2k + 0.72 = 1$$

$$2k = 0.28$$

$$k = 0.14$$

The random variable  $X$  has the probability function

$$P(X = x) = \begin{cases} 0.23 & x = -1, 4 \\ k & x = 0, 2 \\ 0.13 & x = 1, 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of  $k$ .

$$k = 0.14$$

[2]

(b) Construct a table giving the probability distribution of  $X$ .

[2]

(c) Find  $P(0 \leq X < 3)$ .

[1]

b)

$x$	-1	0	1	2	3	4
$P(X=x)$	0.23	0.14	0.13	0.14	0.13	0.23

The random variable  $X$  has the probability function

$$P(X = x) = \begin{cases} 0.23 & x = -1, 4 \\ k & x = 0, 2 \\ 0.13 & x = 1, 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of  $k$ .

[2]

(b) Construct a table giving the probability distribution of  $X$ .

[2]

(c) Find  $P(0 \leq X < 3)$ .

[1]

$x$	-1	0	1	2	3	4
$P(X=x)$	0.23	0.14	0.13	0.14	0.13	0.23

$$\begin{aligned}
 c) P(0 \leq X < 3) &= P(X = 0 \text{ or } 1 \text{ or } 2) \\
 &= P(X=0) + P(X=1) + P(X=2)
 \end{aligned}$$

$$P(0 \leq X < 3) = 0.14 + 0.13 + 0.14 = 0.41$$

### Question 5

A discrete random variable  $X$  has the probability distribution shown in the following table:

$x$	0	1	2	3	4
$P(X = x)$	$\frac{5}{24}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{8}$

Find:

- (i)  $P(X < 4)$
- (ii)  $P(X > 1)$
- (iii)  $P(2 < X \leq 4)$
- (iv)  $P(0 < X < 4)$

(i)  $P(X < 4) = P(X \neq 4) = 1 - P(X = 4)^*$

$$P(X < 4) = 1 - \frac{1}{8} = \frac{7}{8}$$

\* This is easier than adding up the probabilities for 0, 1, 2, and 3!

(ii)  $P(X > 1) = P(X = 2 \text{ or } 3 \text{ or } 4)$

[6]  $P(X > 1) = \frac{1}{4} + \frac{1}{12} + \frac{1}{8} = \frac{11}{24}$

(iii)  $P(2 < X \leq 4) = P(X = 3 \text{ or } 4)$

$$P(2 < X \leq 4) = \frac{1}{12} + \frac{1}{8} = \frac{5}{24}$$

(iv)  $P(0 < X < 4) = P(X = 1 \text{ or } 2 \text{ or } 3)$

$$P(0 < X < 4) = \frac{1}{3} + \frac{1}{4} + \frac{1}{12} = \frac{8}{12} = \frac{2}{3}$$

### Question 6

Leonardo has constructed a biased spinner with six sectors labelled 0, 1, 1, 2, 3 and 5. The probability of the spinner landing on each of the six sectors is shown in the following table:

<b>number on sector</b>	0	1	1	2	3	5
<b>probability</b>	$\frac{6}{20}$	$p$	$\frac{3}{20}$	$\frac{5}{20}$	$\frac{3}{20}$	$\frac{1}{20}$

- (a) Find the value of  $p$ .

[1]

Leonardo is playing a game with his biased spinner. The score for the game,  $X$ , is the number which the spinner lands on after being spun.

- (b) Leonardo plays the game twice and adds the two scores together. Find the probability that Leonardo has a *total* score of 5.

[3]

- (c) Complete the following cumulative probability function table for  $X$ :

<b>Score <math>x</math></b>	0	1	2	3	5
<b><math>P(X \leq x)</math></b>	$\frac{6}{20}$				1

[2]

- (d) Find the probability that  $X$  is

- (i) no more than 1
- (ii) at least 3.

[2]

a) Sum of all probabilities must equal one:

$$\frac{6}{20} + p + \frac{3}{20} + \frac{5}{20} + \frac{3}{20} + \frac{1}{20} = 1$$

$$p + \frac{18}{20} = 1$$

$$p = \frac{2}{20} = \frac{1}{10}$$

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number on sector	0	1	1	2	3	5
probability	$\frac{6}{20}$	$p$	$\frac{3}{20}$	$\frac{5}{20}$	$\frac{3}{20}$	$\frac{1}{20}$

(a) Find the value of  $p$ .

$$p = \frac{2}{20}$$

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[3]

(c) Complete the following cumulative probability function table for  $X$ :

Score $x$	0	1	2	3	5
$P(X \leq x)$	$\frac{6}{20}$	$\frac{11}{20}$	$\frac{16}{20}$	$\frac{19}{20}$	1

[2]

(d) Find the probability that  $X$  is

- (i) no more than 1
- (ii) at least 3.

[2]

b) Possible outcomes that add up to 5 are :

$$\{0, 5\} \quad \{5, 0\} \quad \{2, 3\} \quad \{3, 2\}$$

$$\{0, 5\} \quad \frac{6}{20} \times \frac{1}{20} = \frac{6}{400}$$

$$\{5, 0\} \quad \frac{1}{20} \times \frac{6}{20} = \frac{6}{400}$$

$$\{2, 3\} \quad \frac{5}{20} \times \frac{3}{20} = \frac{15}{400}$$

$$\{3, 2\} \quad \frac{3}{20} \times \frac{5}{20} = \frac{15}{400}$$

$$\frac{6}{400} + \frac{6}{400} + \frac{15}{400} + \frac{15}{400} = \frac{42}{400} = \frac{21}{200} = 0.105$$

$$c) \quad \frac{6}{20} + \left( \frac{2}{20} + \frac{3}{20} \right) = \frac{11}{20}$$

Note: Both '1' sectors on the spinner are included in the event  $X = 1$ .

$$\frac{11}{20} + \frac{5}{20} = \frac{16}{20}$$

$$\frac{16}{20} + \frac{3}{20} = \frac{19}{20}$$

Leonardo has constructed a biased spinner with six sectors labelled 0, 1, 1, 2, 3 and 5. The probability of the spinner landing on each of the six sectors is shown in the following table:

number on sector	0	1	1	2	3	5
probability	$\frac{6}{20}$	$p$	$\frac{3}{20}$	$\frac{5}{20}$	$\frac{3}{20}$	$\frac{1}{20}$

(a) Find the value of  $p$ .

$$p = \frac{2}{20}$$

[1]

Leonardo is playing a game with his biased spinner. The score for the game,  $X$ , is the number which the spinner lands on after being spun.

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(c) Complete the following cumulative probability function table for  $X$ :

Score $x$	0	1	2	3	5
$P(X \leq x)$	$\frac{6}{20}$	$\frac{11}{20}$	$\frac{16}{20}$	$\frac{19}{20}$	1

[2]

(d) Find the probability that  $X$  is

- (i) no more than 1
- (ii) at least 3.

[2]

d) (i)  $P(X \text{ is no more than } 1) = P(X \leq 1)$

$$\frac{11}{20} = 0.55$$

(ii)  $P(X \text{ is at least } 3) = P(X \geq 3) = 1 - P(X \leq 2)$

$$1 - \frac{16}{20} = \frac{4}{20} = \frac{1}{5} = 0.2$$

## Question 7

A discrete random variable  $X$  has the following probability distribution:

$x$	-3	-1	0	1	3
$P(X = x)$	0.11	$k^2$	0.1	$2k$	0.1

where  $k$  is a positive constant.

(a) Show that  $k^2 + 2k - 0.69 = 0$ .

[2]

(b) Hence find the value of  $k$ .

[1]

(c) Find  $E(X)$ .

[3]

a) The sum of all probabilities must equal one, so:

$$0.11 + k^2 + 0.1 + 2k + 0.1 = 1$$

$$k^2 + 2k + 0.31 = 1$$

$$k^2 + 2k + 0.31 - 1 = 0$$

$$k^2 + 2k - 0.69 = 0$$



A discrete random variable  $X$  has the following probability distribution:

$x$	-3	-1	0	1	3
$P(X = x)$	0.11	$k^2$	0.1	$2k$	0.1

where  $k$  is a positive constant.

(a) Show that  $k^2 + 2k - 0.69 = 0$ .

(b) Hence find the value of  $k$ .

(c) Find  $E(X)$ .

[2]

[1]

[3]

b) Solve the quadratic:

$$k^2 + 2k - 0.69 = 0$$

$$(k - 0.3)(k + 2.3) = 0$$

$$k = 0.3 \text{ or } -2.3 \quad \text{You can also use your GDC to solve this}$$

But  $k$  is positive, so

$$k = 0.3$$

A discrete random variable  $X$  has the following probability distribution:

$x$	-3	-1	0	1	3
$P(X = x)$	0.11	$k^2$	0.1	$2k$	0.1

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(a) Show that  $k^2 + 2k - 0.69 = 0$ .

(b) Hence find the value of  $k$ .

$$k = 0.3$$

(c) Find  $E(X)$ .

[2]

[1]

[3]

c)  $k^2 = 0.3^2 = 0.09$       $2k = 2(0.3) = 0.6$

$$E(X) = \sum x P(X=x) \quad \left. \vphantom{\sum} \right\} \text{Expected value of a discrete random variable } X$$

$$E(X) = (-3)(0.11) + (-1)(0.09) + (0)(0.1) + (1)(0.6) + (3)(0.1)$$

$$= -0.33 - 0.09 + 0 + 0.6 + 0.3$$

$$E(X) = 0.48$$

### Question 8

A spinner is spun on a circle that is divided up into five sections, A, B, C, D and E.

The probability of the spinner landing on each section is given by the following table:

Region	A	B	C	D	E
Probability	0.55	0.15	0.15	0.1	0.05

A person who rotates the spinner scores points depending on which section the spinner lands on. These points are shown below.

Region	A	B	C	D	E
Points	-5	2	3	10	$k$

Given that the game is fair, find the value of  $k$ .

→ This means  $E(X) = 0$

If  $X$  is a person's score, then :

$$P(X = -5) = 0.55 \quad P(X = 2) = 0.15 \quad P(X = 3) = 0.15$$

$$P(X = 10) = 0.1 \quad P(X = k) = 0.05$$

And  $E(X) = 0$ , so :

$$E(X) = \sum x P(X=x) \quad \left. \vphantom{E(X)} \right\} \text{Expected value of a discrete random variable } X$$

[4]  $(-5)(0.55) + (2)(0.15) + (3)(0.15) + (10)(0.1) + k(0.05) = 0$

$$-2.75 + 0.3 + 0.45 + 1 + 0.05k = 0$$

$$0.05k - 1 = 0$$

$$0.05k = 1$$

$$k = \frac{1}{0.05}$$

$$k = 20$$

### Question 9

A discrete random variable  $X$  has the following probability distribution:

$x$	0	1	2	3	4
$P(X = x)$	0.1	0.05	$a$	$b$	0.1

The value of  $E(X) = 2.3$ .

(a) Show that  $a$  and  $b$  must satisfy the following two simultaneous equations:

$$\begin{aligned} a + b &= 0.75 \\ 2a + 3b &= 1.85 \end{aligned}$$

(b) Hence find the value of  $a$  and the value of  $b$ .

(c) Find  $P(1 \leq X < 4)$ .

$$E(X) = \sum x P(X=x) \quad \left. \vphantom{E(X)} \right\} \text{Expected value of a discrete random variable } X$$

a) The sum of all probabilities must equal one, so :

$$0.1 + 0.05 + a + b + 0.1 = 1$$

$$a + b + 0.25 = 1$$

$$a + b = 1 - 0.25$$

$$a + b = 0.75$$

[3]

[2]

And  $E(X) = 2.3$ , so :

[2]

$$(0)(0.1) + (1)(0.05) + (2)(a) + (3)(b) + (4)(0.1) = 2.3$$

$$0 + 0.05 + 2a + 3b + 0.4 = 2.3$$

$$2a + 3b + 0.45 = 2.3$$

$$2a + 3b = 2.3 - 0.45$$

$$2a + 3b = 1.85$$

A discrete random variable  $X$  has the following probability distribution:

$x$	0	1	2	3	4
$P(X = x)$	0.1	0.05	$a$	$b$	0.1

The value of  $E(X) = 2.3$ .

(a) Show that  $a$  and  $b$  must satisfy the following two simultaneous equations:

$$\begin{aligned} a + b &= 0.75 & \textcircled{1} \\ 2a + 3b &= 1.85 & \textcircled{2} \end{aligned}$$

(b) Hence find the value of  $a$  and the value of  $b$ .

(c) Find  $P(1 \leq X < 4)$ .

[3]

[2]

[2]

b) Solve the simultaneous equations

$$\begin{aligned} \textcircled{2} - 2 \times \textcircled{1} : & \quad 2a + 3b = 1.85 \\ & \quad -(2a + 2b = 1.5) \\ & \quad \hline & \quad \quad b = 0.35 \end{aligned}$$

Substitute into  $\textcircled{1}$ :

$$a + 0.35 = 0.75$$

$$a = 0.75 - 0.35 = 0.4$$

$$a = 0.4 \quad b = 0.35$$

You can also use your GDC to solve this

A discrete random variable  $X$  has the following probability distribution:

$x$	0	1	2	3	4
$P(X = x)$	0.1	0.05	$a$	$b$	0.1

The value of  $E(X) = 2.3$ .

(a) Show that  $a$  and  $b$  must satisfy the following two simultaneous equations:

$$\begin{aligned} a + b &= 0.75 \\ 2a + 3b &= 1.85 \end{aligned}$$

(b) Hence find the value of  $a$  and the value of  $b$ .

$$a = 0.4 \quad b = 0.35$$

(c) Find  $P(1 \leq X < 4)$ .

[3]

[2]

[2]

$$c) P(1 \leq X < 4) = P(X = 1, 2, \text{ or } 3)$$

$$\begin{aligned} P(1 \leq X < 4) &= 0.05 + a + b \\ &= 0.05 + 0.4 + 0.35 \end{aligned}$$

$$P(1 \leq X < 4) = 0.8$$

