

# IB Maths: AI HL

## Probability Distributions

### Topic Questions

**These practice questions can be used by students and teachers and is Suitable for IB Maths AI HL Topic Questions**

Course	IB Maths
Section	4. Statistics & Probability
Topic	4.5 Probability Distributions
Difficulty	Medium

**Level: IB Maths**

**Subject: IB Maths AI HL**

**Board: IB Maths**

**Topic: Probability Distributions**

### Question 1

Three biased coins are tossed.

(a) Write down all the possible outcomes when the three coins are tossed.

[1 mark]

For each coin the probability of getting heads is  $\frac{2}{3}$ . A random variable,  $X$ , is defined as the number of heads when the three coins are tossed.

(b) Complete the following probability distribution table for  $X$ :

$x$	0	1	2	3
$P(X = x)$				

[3 marks]

(c) Hence, by inserting the relevant probabilities, represent the probability distribution for  $X$  as a piecewise function in the form

$$P(X = x) = f(x) = \begin{cases} & x = 0 \\ & x = 1 \\ & x = 2 \\ & x = 3 \\ 0 & \text{otherwise} \end{cases}$$

[2 marks]

(d) Represent the probability distribution for  $X$  as a bar chart.

[2 marks]

### Question 2

The random variable  $X$  has the probability function

$$P(X = x) = \begin{cases} \frac{x}{3k} & x = 1, 2, 3, 4, 5 \\ 0 & \text{otherwise} \end{cases}$$

Show that  $k = 5$ .

[2 marks]

### Question 3

The random variable  $X$  has the probability function

$$P(X = x) = \begin{cases} kx & x = 1, 3, 5, 7 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of  $k$ .

[2 marks]

(b) Find  $P(X > 3)$ .

[2 marks]

(c) State, with a reason, whether or not  $X$  is a discrete random variable.

[1 mark]

#### Question 4

The random variable  $X$  has the probability function

$$P(X = x) = \begin{cases} 0.23 & x = -1, 4 \\ k & x = 0, 2 \\ 0.13 & x = 1, 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of  $k$ .

[2 marks]

(b) Construct a table giving the probability distribution of  $X$ .

[2 marks]

(c) Find  $P(0 \leq X < 3)$ .

[1 mark]

### Question 5

A discrete random variable  $X$  has the probability distribution shown in the following table:

$x$	0	1	2	3	4
$P(X = x)$	$\frac{5}{24}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{8}$

Find:

- (i)  $P(X < 4)$
- (ii)  $P(X > 1)$
- (iii)  $P(2 < X \leq 4)$
- (iv)  $P(0 < X < 4)$

[6 marks]

### Question 6

Leonardo has constructed a biased spinner with six sectors labelled 0, 1, 1, 2, 3 and 5. The probability of the spinner landing on each of the six sectors is shown in the following table:

<b>number on sector</b>	0	1	1	2	3	5
<b>probability</b>	$\frac{6}{20}$	$p$	$\frac{3}{20}$	$\frac{5}{20}$	$\frac{3}{20}$	$\frac{1}{20}$

- (a) Find the value of  $p$ .

[1 mark]

Leonardo is playing a game with his biased spinner. The score for the game,  $X$ , is the number which the spinner lands on after being spun.

(b) Leonardo plays the game twice and adds the two scores together. Find the probability that Leonardo has a *total* score of 5.

[3 marks]

(c) Complete the following cumulative probability function table for  $X$ :

<b>Score <math>x</math></b>	0	1	2	3	5
<b><math>P(X \leq x)</math></b>	$\frac{6}{20}$				1

[2 marks]

(d) Find the probability that  $X$  is

- (i) no more than 1
- (ii) at least 3.

[2 marks]

### Question 7

A discrete random variable  $X$  has the following probability distribution:

$x$	-3	-1	0	1	3
$P(X = x)$	0.11	$k^2$	0.1	$2k$	0.1

where  $k$  is a positive constant.

(a) Show that  $k^2 + 2k - 0.69 = 0$ .

[2 marks]

(b) Hence find the value of  $k$ .

[1 mark]

(c) Find  $E(X)$ .

[3 marks]

### Question 8

A spinner is spun on a circle that is divided up into five sections, A, B, C, D and E.

The probability of the spinner landing on each section is given by the following table:

<b>Region</b>	A	B	C	D	E
<b>Probability</b>	0.55	0.15	0.15	0.1	0.05

A person who rotates the spinner scores points depending on which section the spinner lands on. These points are shown below.

<b>Region</b>	A	B	C	D	E
<b>Points</b>	-5	2	3	10	$k$

Given that the game is fair, find the value of  $k$ .

[4 marks]

### Question 9

A discrete random variable  $X$  has the following probability distribution:

$x$	0	1	2	3	4
$P(X = x)$	0.1	0.05	$a$	$b$	0.1

The value of  $E(X) = 2.3$ .

(a) Show that  $a$  and  $b$  must satisfy the following two simultaneous equations:

$$a + b = 0.75$$

$$2a + 3b = 1.85$$

[3 marks]



(b) Hence find the value of  $a$  and the value of  $b$ .

[2 marks]

(c) Find  $P(1 \leq X < 4)$ .

[2 marks]