## Question 1

The lengths, in cm, of 120 adult platypuses are recorded in the following table:

| Length, $l$ (cm) | Frequency (female) | Frequency (male) |
|---|---|---|
| $39 \le l < 42$ | 14 | 0 |
| $42 \le l < 45$ | 29 | 0 |
| $45 \le l < 48$ | 12 | 7 |
| $48 \le l < 51$ | 6 | 21 |
| $51 \le l < 54$ | 3 | 19 |
| $54 \le l < 57$ | 1 | 5 |
| $57 \le l < 60$ | 0 | 2 |
| $60 \le l < 63$ | 0 | 1 |

One platypus is chosen at random. Find the probability that the platypus is:

(i) male

(ii) less than 51 cm long

(iii) a male less than 45 cm long

(iv) a female between 45 and 54 cm long.

[4]

(i) SUM OF MALES $7+21+19+5+2+1=55$

$P(\text{MALE}) = \frac{55}{120} = \frac{11}{24}$

$\boxed{P(\text{MALE}) = \frac{11}{24}}$

(ii) SUM $<51$ (FIRST 4 ROWS)

$14+29+12+6+7+21 = 89$

$\boxed{P(<51) = \frac{89}{120}}$

(iii) NO MALES ARE $<45$ cm

$\boxed{P(<45) = 0}$

(iv) SUM $45 \le L < 54$ FOR FEMALES ONLY

$12+6+3 = 21$

$P(45 \le F < 54) = \frac{21}{120} = \frac{7}{40}$

$\boxed{P(45 \le F < 54) = \frac{7}{40}}$

## Question 2

Two fair spinners each have three sectors numbered 1 to 3. The two spinners are spun together and then the product of the numbers indicated on each spinner is recorded.

Find the probability of the product indicated by the spinners being

(i) exactly 6

(ii) less than 4

(iii) an odd number.

[4]

DRAW A SAMPLE SPACE DIAGRAM TO SHOW ALL POSSIBILITIES

| × | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 1 | 2 | 3 |
| 2 | 2 | 4 | 6 |
| 3 | 3 | 6 | 9 |

USE THE SAMPLE SPACE DIAGRAM TO FIND REQUIRED PROBABILITIES

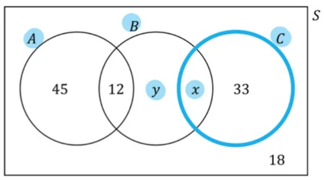
(i) $\boxed{P(6) = \frac{2}{9}}$

(ii) $\boxed{P(<4) = \frac{5}{9}}$

(iii) $\boxed{P(\text{ODD}) = \frac{4}{9}}$

## Question 3

The Venn diagram below shows the number of members of an amateur Elizabethan dramatic society who have been involved with productions of the following three plays by Ben Jonson: *The Alchemist* ($A$), *Bartholomew Fayre* ($B$) and *Chloridia* ($C$).




There are 150 members of the society in total.

Given that the probability of a member having been involved with a production of *Chloridia* is $\frac{8}{25}$,

(a) determine the values of

(i) $x$

(ii) $y$.

[4]

(b) Determine the probability that a member of the society

(i) has been involved with a production of at least one of the three plays

(ii) has been involved with a production of exactly one of the three plays.

[2]

(a) (i) $P(\text{Chloridia}) = \frac{8}{25} \xrightarrow{\times 6} \frac{48}{150}$ OR $\frac{8}{25} \times 150 = 48$

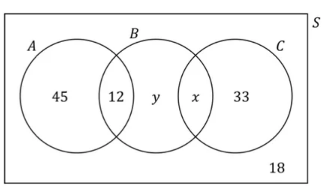
$x = 48 - 33 = 15$

$\boxed{x = 15}$

(ii) $y = 150 - (45 + 12 + \overset{x}{15} + 33 + 18)$

$\boxed{y = 27}$

The Venn diagram below shows the number of members of an amateur Elizabethan dramatic society who have been involved with productions of the following three plays by Ben Jonson: *The Alchemist* ($A$), *Bartholomew Fayre* ($B$) and *Chloridia* ($C$).




There are 150 members of the society in total.

Given that the probability of a member having been involved with a production of *Chloridia* is $\frac{8}{25}$,

(a) determine the values of

(i) $x$

(ii) $y$.

$\boxed{x = 15}$ $\boxed{y = 27}$

[4]

(b) Determine the probability that a member of the society

(i) has been involved with a production of at least one of the three plays

(ii) has been involved with a production of exactly one of the three plays.

[2]

(b)

(i) P(AT LEAST ONE) = 1 − P(NONE)

$\frac{150 - 18}{150} = \frac{132}{150} = \frac{22}{25} = 0.88$

$\boxed{P(\text{AT LEAST ONE}) = \frac{22}{25}}$ OR 0.88

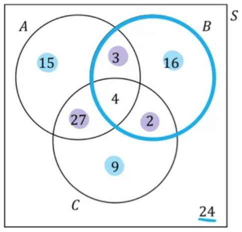
(ii) P(EXACTLY ONE) = INSIDE VENN, NOT IN INTERSECTIONS

$\frac{45 + \overset{y}{27} + 33}{150} = \frac{105}{150} = \frac{7}{10} = 0.7$

$\boxed{P(\text{EXACTLY ONE}) = \frac{7}{10}}$ OR 0.7

## Question 4

The following Venn diagram shows the number of adults in a poll who said they enjoy watching action films ($A$), Bollywood musicals ($B$), and crime thrillers ($C$). 100 adults were polled in total.




(a) One of the adults who was polled is selected at random. Given that the adult chosen enjoys watching at least one of those three genres of film, find the probability that the adult enjoys watching:

(i) Bollywood musicals

(ii) only one of the three genres of film

(iii) exactly two of the three genres of film.

[3]

(b) Find the following probabilities:

(i) $P(A \cap C)$

(ii) $P(A \cup C)$

(iii) $P(C|B)$

(iv) $P(B')$

[4]

(a) 'GIVEN THEY ENJOY AT LEAST ONE GENRE' IGNORE VALUES OUTSIDE CIRCLES (24)

$15+3+16+27+4+2+9=76$

(i) $P(\text{BOLLYWOOD MUSICALS}) = \dfrac{16+3+4+2}{76}$ VALUES IN B

$\boxed{P(\text{BOLLYWOOD}) = \dfrac{25}{76}}$

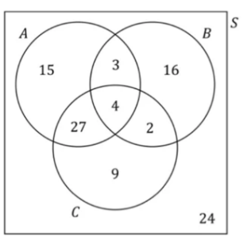
(ii) $P(\text{ONLY ONE GENRE}) = \dfrac{15+16+9}{76} = \dfrac{40}{76}$ VALUES NOT IN INTERSECTIONS

$\boxed{P(\text{ONLY ONE GENRE}) = \dfrac{10}{19}}$

(iii) $P(\text{EXACTLY TWO GENRES}) = \dfrac{3+27+2}{76} = \dfrac{32}{76}$ VALUES IN DUAL INTERSECTIONS

$\boxed{P(\text{EXACTLY TWO GENRES}) = \dfrac{8}{19}}$


The following Venn diagram shows the number of adults in a poll who said they enjoy watching action films ($A$), Bollywood musicals ($B$), and crime thrillers ($C$). 100 adults were polled in total.



(a) One of the adults who was polled is selected at random. Given that the adult chosen enjoys watching at least one of those three genres of film, find the probability that the adult enjoys watching:

(i) Bollywood musicals

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(iii) exactly two of the three genres of film.

[3]

(b) Find the following probabilities:

(i) $P(A \cap C)$

(ii) $P(A \cup C)$

(iii) $P(C|B)$

(iv) $P(B')$

[4]


(b) (i) INTERSECTION $P(A \cap C) = \dfrac{27+4}{100} = \dfrac{31}{100} = 0.31$

$\boxed{P(A \cap C) = \dfrac{31}{100}}$ OR 0.31

(ii) $P(A \cup C)$ UNION

COULD ADD ALL VALUES IN A OR C OR TAKE VALUES NOT IN EITHER

$\dfrac{15+3+4+27+9+2}{100} = \dfrac{60}{100} = \dfrac{3}{5} = 0.6$

OR $\dfrac{100-(16+24)}{100} = \dfrac{60}{100} = \dfrac{3}{5} = 0.6$

$\boxed{P(A \cup C) = \dfrac{3}{5}}$ OR 0.6

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EXAM PAPERS PRACTICE

(iii) $P(C \mid B)$ = CONDITIONAL C GIVEN B

$n(B) = 3+4+2+16 = 25$ TOTAL IN B

$n(C \cap B) = 4+2 = 6$ TOTAL IN INTERSECTION

$P(C \mid B) = \frac{6}{25} = 0.24$

$\boxed{P(C \mid B) = \frac{6}{25}}$ OR 0.24

(iv) $P(B') = 1 - P(B)$ NOT IN B

$P(B) = \frac{3+4+2+16}{100} = \frac{25}{100}$

$P(B') = 1 - \frac{25}{100} = \frac{75}{100} = \frac{3}{4} = 0.75$

$\boxed{P(B') = \frac{3}{4}}$ OR 0.75

## Question 5

On any given day the probability that Radigast has a lichen smoothie with his lunch is 0.4, and the probability that he has a wild mushroom wrap is 0.8. Given that the probability of him having both those items is 0.35, find the probability that Radigast has:

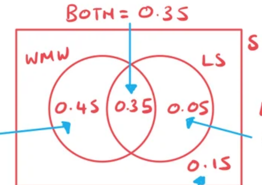
(i) a wild mushroom wrap but not a lichen smoothie

(ii) neither a wild mushroom wrap nor a lichen smoothie.

[4]

DRAW A VENN DIAGRAM TO ORGANISE OPTIONS

BOTH = 0.35




LS ONLY
$0.4 - 0.35 = 0.05$

(i) WMW ONLY
$0.8 - 0.35 = \boxed{0.45}$

(ii) NEITHER
$1-(0.8+0.4-0.35) = \boxed{0.15}$
OR
$1-(0.45+0.35+0.05)$

OR USE JUST CALCULATIONS

(i) WMW ONLY $0.8-0.35 = 0.45$

$\boxed{P(\text{WMW ONLY}) = 0.45}$

(ii) NEITHER $1-(0.8+0.4-0.35) = 0.15$

$\boxed{P(\text{NEITHER}) = 0.15}$

## Question 6

(a) $A$ and $B$ are two events such that $P(A) = 0.35$, $P(B) = 0.25$ and $P(A \cup B) = 0.6$. State, with a reason, whether $A$ and $B$ are mutually exclusive.

[2]

(b) $C$ and $D$ are two events such that $P(C) = 0.2$, $P(D) = 0.4$ and $P(C \cap D) = 0.18$. State, with a reason, whether $C$ and $D$ are independent.

[2]

(a) '$\cup$' = 'OR' = ADD PROBABILITIES

MUTUALLY EXCLUSIVE = NO COMMON OUTCOMES (NO OVERLAP IN VENN DIAGRAM)

$P(A) + P(B) = 0.35 + 0.25 = 0.6 = P(A \cup B)$

$P(A) + P(B) = P(A \cup B)$

A AND B ARE MUTUALLY EXCLUSIVE

(a) $A$ and $B$ are two events such that $P(A) = 0.35$, $P(B) = 0.25$ and $P(A \cup B) = 0.6$. State, with a reason, whether $A$ and $B$ are mutually exclusive.

[2]

(b) $C$ and $D$ are two events such that $P(C) = 0.2$, $P(D) = 0.4$ and $P(C \cap D) = 0.18$. State, with a reason, whether $C$ and $D$ are independent.

[2]

(b) '$\cap$' = 'AND' = MULTIPLY

INDEPENDENT = NO EFFECT ON EACH OTHER

$P(C) \times P(D) = 0.2 \times 0.4 = 0.08 \neq 0.18 = P(C \cap D)$

$P(C) \times P(D) \neq P(C \cap D)$

C AND D ARE NOT INDEPENDENT

## Question 7

A bag contains 13 yellow tokens and 7 green tokens. Two tokens are drawn from the bag without replacement.

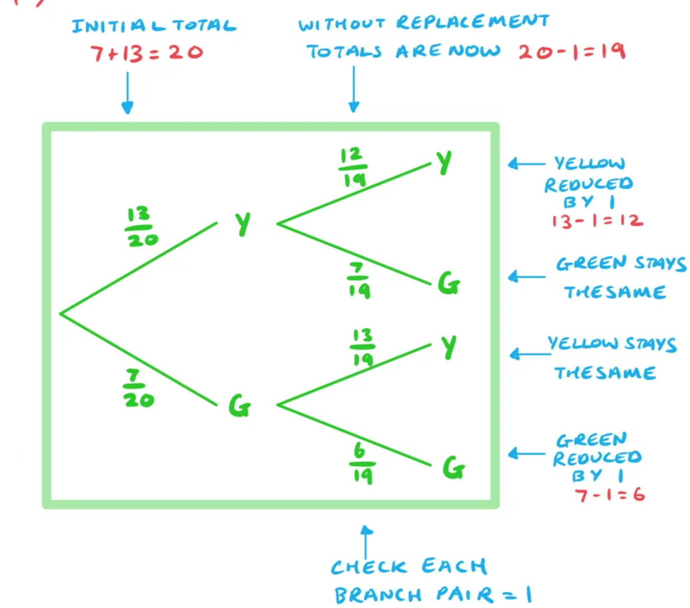
(a) Draw a tree diagram to represent this experiment.

[3]

(b) Find the probability that the two tokens drawn are the same colour.

[3]

(a)




YOU CAN THEN CALCULATE ALL POSSIBLE OUTCOMES BY MULTIPLYING ALONG EACH BRANCH

A bag contains 13 yellow tokens and 7 green tokens. Two tokens are drawn from the bag without replacement.

(a) Draw a tree diagram to represent this experiment.

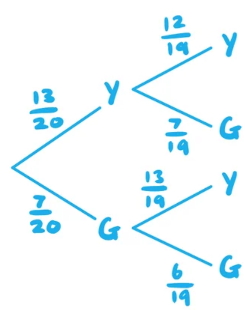
[3]

(b) Find the probability that the two tokens drawn are the same colour.

[3]

(b) USING TREE DIAGRAM FROM PART (a)

MULTIPLY ALONG BRANCHES WITH SAME COLOUR




$P(Y \text{ AND } Y) = \frac{13}{20} \times \frac{12}{19} = \frac{156}{380}$

$P(G \text{ AND } G) = \frac{7}{20} \times \frac{6}{19} = \frac{42}{380}$

NOT SIMPLIFYING MAKES PROBABILITIES EASIER TO ADD

$P(Y \text{ AND } Y) + P(G \text{ AND } G) = P(\text{SAME COLOUR})$

$$\left(\frac{13}{20} \times \frac{12}{19}\right) + \left(\frac{7}{20} \times \frac{6}{19}\right) = \frac{156}{380} + \frac{42}{380} = \frac{198}{380} = \frac{99}{190}$$

$$\boxed{P(\text{SAME COLOUR}) = \frac{99}{190}}$$

## Question 8

Ichabod is a keen chess player who plays one game of chess online every night before going to bed. In any one of those games, the probabilities of Ichabod winning, drawing, or losing are 0.4, 0.27 and 0.33 respectively. Following each game, the probabilities of Ichabod sleeping well after winning, drawing or losing are 0.7, 0.9 and 0.2 respectively.

(a) Draw a tree diagram to represent this information.

[3]

(b) Find the probability that on a randomly chosen night

(i) Ichabod loses his chess game and sleeps well

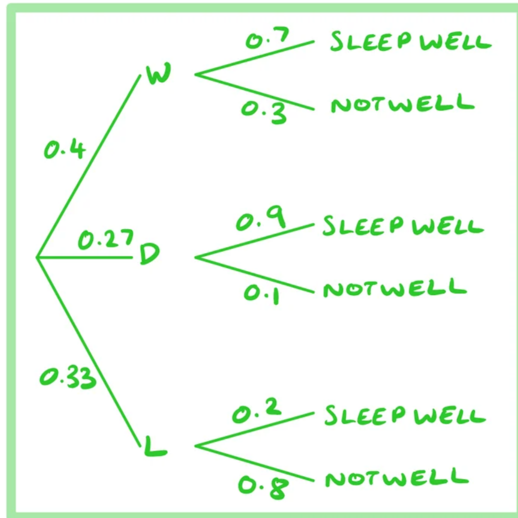
(ii) Ichabod sleeps well.

[4]

(c) Given that Ichabod sleeps well, find the probability that his chess game did not end in a draw.

[4]

(a)




CHECK EACH BRANCH PAIR = 1

YOU CAN THEN CALCULATE ALL POSSIBLE OUTCOMES BY MULTIPLYING ALONG EACH BRANCH

Ichabod is a keen chess player who plays one game of chess online every night before going to bed. In any one of those games, the probabilities of Ichabod winning, drawing, or losing are 0.4, 0.27 and 0.33 respectively. Following each game, the probabilities of Ichabod sleeping well after winning, drawing or losing are 0.7, 0.9 and 0.2 respectively.

(a) Draw a tree diagram to represent this information.

[3]

(b) Find the probability that on a randomly chosen night

(i) Ichabod loses his chess game and sleeps well

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[4]

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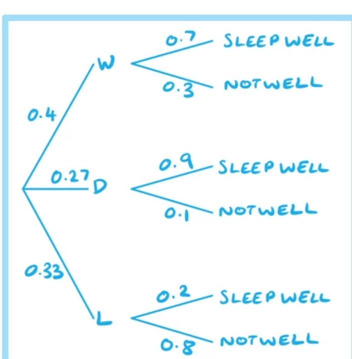
[4]




(b)

(i) P(LOSE AND SLEEP WELL) = $0.33 \times 0.2 = 0.066$

$\boxed{\text{P(LOSE AND SLEEP WELL)} = 0.066}$

(ii) P(SLEEPS WELL) = SLEEP WELL AFTER WIN, DRAW AND LOSE

$0.4 \times 0.7 + 0.27 \times 0.9 + 0.33 \times 0.2$

$0.28 + 0.243 + 0.066$

$\boxed{\text{P(SLEEPS WELL)} = 0.589}$

Ichabod is a keen chess player who plays one game of chess online every night before going to bed. In any one of those games, the probabilities of Ichabod winning, drawing, or losing are 0.4, 0.27 and 0.33 respectively. Following each game, the probabilities of Ichabod sleeping well after winning, drawing or losing are 0.7, 0.9 and 0.2 respectively.

(a) Draw a tree diagram to represent this information.

[3]

(b) Find the probability that on a randomly chosen night

(i) Ichabod loses his chess game and sleeps well

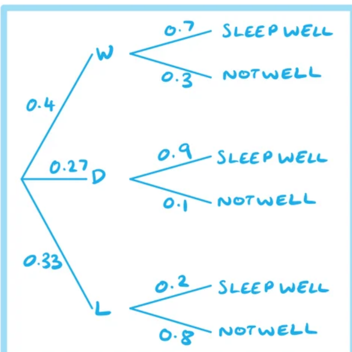
(ii) Ichabod sleeps well.

$$\boxed{P(\text{SLEEPS WELL}) = 0.589}$$

[4]

(c) Given that Ichabod sleeps well, find the probability that his chess game did not end in a draw.

[4]




(c) $$P(\text{DRAW}' \mid \text{SLEEP WELL}) = \frac{P(\text{WIN SLEEP WELL}) + P(\text{LOSE SLEEP WELL})}{P(\text{SLEEP WELL})}$$

FROM (b)(ii)

$$\frac{(0.4 \times 0.7) + (0.33 \times 0.2)}{0.589} = \frac{0.346}{0.589}$$

$$\boxed{P(\text{DRAW}' \mid \text{SLEEP WELL}) = \frac{346}{589}}$$