EXAM PAPERS PRACTICE

IB Maths: AA HL

Practice Paper 2

Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AA HL Past Papers

Course	IB Maths
Section	Set C
Topic	Practice Paper 2
Difficulty	Medium

Level: IB Maths

Subject: IB Maths AA HL

Board: IB Maths

Topic: Practice Paper 2

Question 1

The following table shows the mean height, y cm, of primary school children who are age x years old.

Age, x years	6.25	7.35	8.5	9.25	10.75
Mean Height, y cm	115	121	129	136	140

The relationship between x and y can be modelled by the regression line of y on x with equation $y = ax + b$.

(a) (i) Find the value of a and the value of b.

(ii) Write down the value of Pearson's product-moment correlation coefficient, r.

[4 marks]

(b) Use your regression equation from part (a)(i) to estimate the height of a child aged 9 years old.

[2 marks]

(c) Explain why it is not appropriate to use the regression equation to estimate the age of a child who is 133 cm tall.

[1 mark]

Question 2

An arithmetic sequence with a common difference -3.5 has first term 77.

(a) Given that the rth term of the sequence is zero, find the value of r.

[2 marks]

(b) Find the maximum value of the sum of the first n terms of the sequence.

[3 marks]

Question 3

A and B are independent events, such that $\mathrm{P}(A) = 0.25$ and $\mathrm{P}(B) = 0.52$. C is another event, such that B and C are mutually exclusive and $\mathrm{P}(A \cap C) = 0.09$.

Given that $\mathrm{P}(A \cup B \cup C) = 0.95$, find

(i) $\mathrm{P}(A \cap B)$

(ii) $\mathrm{P}(C)$

(iii) $\mathrm{P}(A' \cap B')$

(iv) $\mathrm{P}(A|C')$

[9 marks]

Question 4

Let $f(x) = \frac{5-x^2}{3}$ and $g(x) = 4 - \frac{3}{x}$, where each function has the largest possible valid domain.

(a) Write down the range of f.

[1 mark]

(b) Write down the domain and range of g.

[2 marks]

(c) Find

(i) $(f \circ g)(x)$

(ii) $(g \circ f)(x)$.

[3 marks]

Question 5

The number of bacteria, n, in a dish, after t minutes is given by $n = 5231e^{0.12t}$.

(a) Find the initial amount of bacteria.

[2 marks]

(b) Find the amount of bacteria after 12 minutes. Give your answer in the form $a \times 10^k$, where $1 \leq a < 10, k \in \mathbb{Z}$.

[3 marks]

(c) Find the value of t when $n = 2.7 \times 10^4$.

[2 marks]

Question 6

A UK energy company charges £0.22 per kilowatt hour (kWh) of electricity used.
The amount of energy used per day by the company's customers, X kWh, follows the following probability density function

$$f(x) = \begin{cases} \dfrac{x(k-x)}{972}, & 0 \leq x \leq 18 \\ 0, & \text{otherwise} \end{cases}$$

(a)
Show that $k = 18$.

[2 marks]

(b)
A customer's total daily charge consists of a fixed (standing) charge of £0.38 per day plus the charge for the electricity used.
(i)
Find the expected total daily charge.
(ii)
Find the standard deviation for the total daily charge.

[6 marks]

Question 7

Consider the nine letters in the word MAGNITUDE.

Find the number of ways that the nine letters may be arranged if

(i)
there are no restrictions
(ii)
the four vowels (A, I, U, E) must all be together
(iii)
the arrangement starts with the letter M and ends with the letter E.

[5 marks]

Question 8

Consider $z = \text{cis}\,\theta$ where $z \in \mathbb{C}$, $z \neq 1$.

Show that $\text{Re}\left(\frac{1+z}{1-z}\right) = 0$.

[5 marks]

Question 9

The binomial series expansion for $(1+t)^{-1}$ is given by

$$(1+t)^{-1} = 1 - t + t^2 - \ldots$$

a)
Using the above result and the Maclaurin series for $\cos(2x)$, show that the Maclaurin series for $\sec(2x)$ is

$$1 + 2x^2 + \frac{10}{3}x^4 + \ldots$$

[5 marks]

b)
By using the result from part (a) and the Maclaurin series for $\ln(1+x)$, find the value of the limit

$$\lim_{x \to 0}\left(\frac{x\ln(1+3x)}{\sec(2x)-1}\right)$$

[3 marks]

Question 10

The Strike A Light! matchstick company produces matchsticks with a length, X mm, that is normally distributed with mean 45 and variance σ^2.

The probability that X is greater than 45.37 is 0.1714.

(a) Find $\mathrm{P}(44.63 < X < 45.37)$.

[2 marks]

(b) (i) Find σ, the standard deviation of X.

(ii) Hence, find the probability that a randomly selected matchstick has a length less than 44.5 mm.

[5 marks]

Andrew has a box of Strike A Light! matches with fifteen matchsticks remaining in it. Those matchsticks may be assumed to be a random sample. Let Y represent the number of matchsticks in Andrew's box with lengths less than 44.5 mm.

(c) Find $\mathrm{E}(Y)$.

[3 marks]

(d) Find the probability that exactly one of the matchsticks in Andrew's box has a length less than 44.5 mm.

[2 marks]

A Strike A Light! matchstick is selected at random and is found to have a length greater than 44.5 mm.

(e) Find the probability that the length of the matchstick is between 44.63 mm and 45.37 mm.

[3 marks]

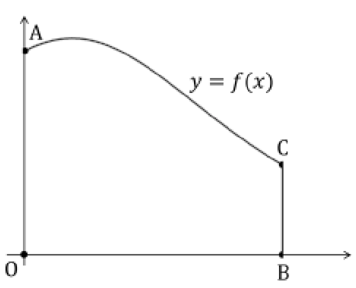
Question 11

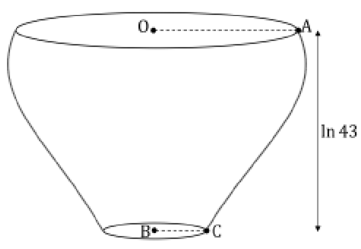
Paola is modelling a small vase from her house for her maths project. To model the edge of the vase in cross-section, she decides to use a function f of the form

$$f(x) = \frac{qe^{\frac{x}{2}}}{2 + e^x}$$

where $x \in \mathbb{R}$, $x \geq 0$ and $q \in \mathbb{R}^+$.

The function and the vase are represented in the diagrams below.





The vertical height of the vase, OB, is measured along the x-axis. The radius of the vase's opening is OA, and its base radius is BC.

To model the vase, she will rotate by 2π radians about the x-axis the region enclosed by the graph of $y = f(x)$, the x-axis, the y-axis, and the line $x = \ln 43$.

a)

Show that the volume of the solid of revolution thus formed is $\frac{14q^2\pi}{45}$ units^3.

[6 marks]

The volume of the actual vase is 100 cm^3.

b)

Use this information to find the value of q.

[2 marks]

c)

Find the cross-sectional radius of the vase

(i) at its base,

(ii) at its widest point.

[4 marks]

Paola wants to investigate how the cross-sectional radius of the vase changes.

d)

Sketch a graph of the derivative of f, and use it to find the value of x at which the cross-sectional radius of the vase is decreasing most rapidly.

[4 marks]

Question 12

A function g is defined by $g(x) = \arccos\left(\frac{x^2 - 1}{x^2 + 1}\right)$, $x \in \mathbb{R}$.

a)
Show that g is an even function.

[1 mark]

b)
By considering the limit of g as x tends to infinity, show that the graph of $y = g(x)$ has a horizontal asymptote and state its equation.

[2 marks]

c)
(i)
Show that $g'(x) = \frac{-2x}{\left(\sqrt{x^2}\right)(x^2 + 1)}$ for $x \in \mathbb{R}$, $x \geq 0$.

(ii)
Considering the fact that $\sqrt{x^2} = |x|$, and also the expression for $g'(x)$ above, show that g is increasing for $x < 0$.

[9 marks]

A new function, h, is created by restricting the domain of g, such that $h(x) = \arccos\left(\frac{x^2 - 1}{x^2 + 1}\right)$, $x \in \mathbb{R}$, $x \geq 0.$, , .

d)
Find an expression for $h^{-1}(x)$, carefully considering the range of h in determining your final answer.

[5 marks]

e)
State the domain of $h^{-1}(x)$.

[2 marks]