

IB Maths: AA HL

Practice Paper 1

Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AA HL Past Papers

Course	IB Maths
Section	Set A
Topic	Practice Paper 1
Difficulty	Medium

Level: IB Maths

Subject: IB Maths AA HL

Board: IB Maths

Topic: Practice Paper 1

Question 1

a)

(i)

Expand $(2k - 1)^3$

(ii)

Hence, or otherwise, show that $(2k - 1)^3 - (2k - 1) = 8k^3 - 12k^2 + 4k$

[2 marks]

b)

Thus prove, given $k > 1$, $k \in \mathbb{N}$, that the difference between an odd natural number greater than 1 and its cube is always even.

[3 marks]

Question 2

Let A and B be events such that $P(A) = 0.3$, $P(B) = 0.75$ and $P(A \cup B) = 0.9$.

Find $P(B|A)$.

[5 marks]

Question 3

The functions f and g are defined such that $f(x) = 6x + 7$ and $g(x) = \frac{x - 5}{3}$.

a)

Show that $(f \circ g)(x) = 2x - 3$.

[2 marks]

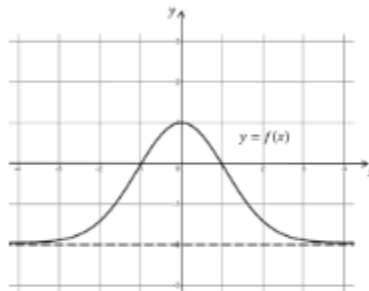
b)

Given that $(f \circ g)^{-1}(a) = 6$, find the value of a .

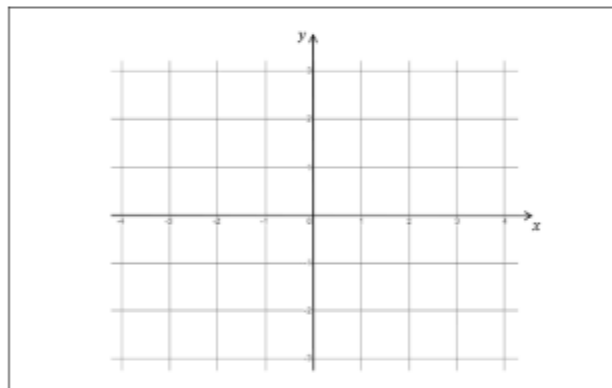
[3 marks]

Question 4

The following diagram shows the graph of $y = f(x)$. The graph has a horizontal asymptote at $y = -2$. The graph crosses the x -axis at $x = -1$ and $x = 1$, and the y -axis at $y = 1$.



On the following set of axes, sketch the graph of $y = [f(x)]^2 - 2$, clearly showing any asymptotes with their equations along with the coordinates of any local maxima or minima.



[5 marks]

Question 5

Given that $\frac{dy}{dx} = 3x^2 \cos\left(3x^3 + \frac{\pi}{2}\right)$ and that the graph of y passes through the point $(0, -1)$, find an expression for y in terms of x .

[5 marks]

Question 6

The plane Π has the Cartesian equation $5x - y - z = 15$.

The line L has the vector equation $r = \begin{pmatrix} -4 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} k \\ -2 \\ 2 \end{pmatrix}$, $\lambda, k \in \mathbb{R}$. The acute angle between the line L and the plane Π is 60° .

Find the possible values of k .

[7 marks]

Question 7

a)

Show that $\log_4(\cos 2x + 13) = \log_2 \sqrt{\cos 2x + 13}$

[3 marks]

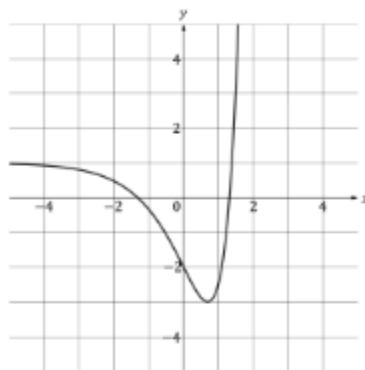
b)

Hence or otherwise $\log_2(3\sqrt{2} \cos x) = \log_4(\cos 2x + 13)$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

[5 marks]

Question 8

The function f is defined by $f(x) = e^{2x} - 4e^x + 1$, $x \in \mathbb{R}$, $x \leq a$. The graph of $y = f(x)$ is shown in the following diagram.



a)

Find the largest value of a such that f has an inverse function.

[3 marks]

b)

For this value of a , find an expression for $f^{-1}(x)$, stating its domain.

[5 marks]

Question 9

A continuous random variable X has the probability density function f given by

$$f(x) = \begin{cases} \frac{\pi x}{81} \sin\left(\frac{\pi x}{9}\right), & 0 \leq x \leq 9 \\ 0, & \text{otherwise} \end{cases}$$

Find $P(3 \leq X \leq 6)$.

[7 marks]

Question 10

a)

Express $-2 - 2\sqrt{3}i$ in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$.

[5 marks]

Let the roots of the equation $z^3 = -2 - 2\sqrt{3}i$ be u , v and w .

b)

Find u , v and w expressing your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$.

[5 marks]

On an Argand diagram u , v and w are represented by the points U , V and W respectively.

c)

Find the area of triangle UVW .

[4 marks]

d)

By considering the sum of the roots u, v and w , show that

$$\cos\left(\frac{2\pi}{9}\right) + \cos\left(\frac{4\pi}{9}\right) + \cos\left(\frac{8\pi}{9}\right) = 0$$

[4 marks]

Question 11

The function f is defined by $f(x) = 2e^{\sin 2x}$.

a)

Find the first two derivatives of $f(x)$ and hence find the Maclaurin series for $f(x)$ up to and including the x^2 term.

[8 marks]

b)

Show that the coefficient of x^3 in the Maclaurin series for $f(x)$ is zero.

[4 marks]

c)

Using the Maclaurin series for $\arctan x$ and $e^x - 1$, find the Maclaurin series for $\arctan(e^{2x} - 1)$ up to and including the x^3 term.

[6 marks]

d)

Hence, or otherwise, find $\lim_{x \rightarrow 0} \frac{f(x) - 2}{\arctan(e^{2x} - 1)}$

[3 marks]

Question 12

Let $f(x) = \frac{\ln px}{qx}$ where $x > 0$, $p, q \in \mathbb{R}^+$.

a)

Show that $f'(x) = \frac{1 - \ln px}{qx^2}$.

[3 marks]

The graph of f has exactly one maximum point A.

b)

Find the x -coordinate of A.

[3 marks]

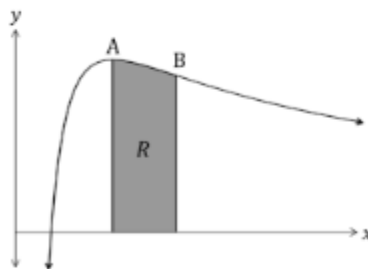
The second derivative of f is given by $f''(x) = \frac{2\ln px - 3}{qx^3}$. The graph of f has exactly one point of inflexion B.

c)

Show that the x -coordinate of B is $\frac{e^{\frac{3}{2}}}{p}$.

[3 marks]

The region R is enclosed by the graph of f , the x -axis, and the vertical lines through the maximum point A and the point of inflexion B.



d)

Calculate the area of R in terms of q and show that the value of the area is independent of p .

[7 marks]