

IB Maths: AA HL

Polynomial Functions

Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AA HL Topic Questions

Course	IB Maths
Section	2. Functions
Topic	2.7 Polynomial Functions
Difficulty	Medium

Level: IB Maths

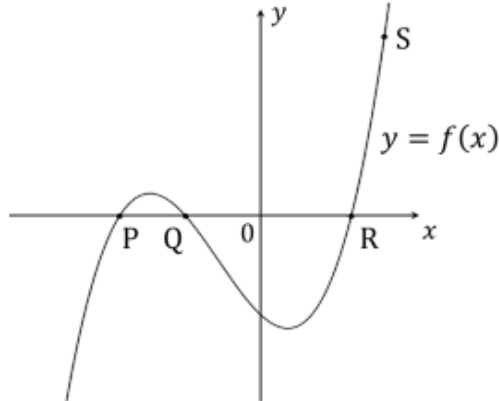
Subject: IB Maths AA HL

Board: IB Maths

Topic: Polynomial Functions

Question 1

Below is the graph of a function $f(x) = ax^3 + bx^2 + cx + d$, passing through the points $P(-3,0)$, $Q(-2,0)$, $R\left(\frac{1}{2}, 0\right)$ and $S(2, 60)$.



- a)
Find the values of a , b , c and d .

[2 marks]

The function is translated vertically by the vector $\begin{pmatrix} 0 \\ k \end{pmatrix}$ so that it passes through the point $(3, 190)$.

- b)
Find the value of k .

[4 marks]

Question 2

- a)
Given that the equation $2x^2 + 4x - m = 0$ has two real solutions, find the set of possible values of m .

[2 marks]

- b)
Given that the function $f(x) = x^2 - 5x + 2c$ has repeated roots, find c .

[2 marks]

- c)
Given that the function $g(x) = 2x^2 + 2kx + \left(\frac{3}{2} - k\right)$ has no real roots, find the set of possible values of k .

[4 marks]

Question 3

Let a function f be defined by $f(x) = 2x^3 + 7x^2 - 3x - 18$.

(i)

Show that $(x+3)$ is a factor of $f(x)$.

(ii)

Hence factorise $f(x)$ fully.

(iii)

Write down all the solutions to $2x^3 + 7x^2 - 3x - 18 = 0$.

[6 marks]

Question 4

a)

Factorise fully $6x^3 + x^2 - 12x + 5$.

[4 marks]

b)

$f(x) = ax^3 + (5a-2)x^2 + (4a+2)x - 2a$

(i)

Given that $(x+3)$ is a factor of $f(x)$, find a .

(ii)

Hence factorise $f(x)$ fully.

[7 marks]

Question 5

Consider the polynomial $g(x) = 3x^5 - 25x^4 + 72x^3 - 72x^2 - 16x + 48$.

a)

Show that 2 is a root of $g(x)$.

[2 marks]

- b)
Given that 2 is a root of $g(x)$ with multiplicity 3, factorise $g(x)$ fully and hence state the other two roots.

[5 marks]

Question 6

Consider the function $f(x) = 4x^3 + 6x^2 - 7x + 2$.

- (i)
Find the quotient and remainder when $4x^3 + 6x^2 - 7x + 2$ is divided by $(x - 2)$.

- (ii)
Hence write $4x^3 + 6x^2 - 7x + 2$ in the form $(x - 2)(ax^2 + bx + c) + d$, where a , b , c and d are constants to be determined.

[5 marks]

Question 7

The function $f(x) = 2x^3 - 5x^2 + ax + b$ has $(2x + 3)$ as a factor, and when $f(x)$ is divided by $(x - 2)$ the remainder is 7.

- a)
Show that a and b must satisfy the simultaneous equations:

$$2a + b = 11$$

$$3a - 2b = -36$$

[5 marks]

- b)
Hence find a and b .

[2 marks]

Question 8

Given that $3 + 2i$ is one of the roots of the equation $x^3 - 3x^2 - 5x + 39 = 0$, find the other two roots.

[5 marks]

Question 9

a)

For each of the following polynomials, find the sum of the roots and the product of the roots.

(i)

$$f(x) = 9x^4 + 7x^3 - 3x + 2$$

(ii)

$$g(x) = 7x^5 - x^4 + 2x^3 + x^2 - 5x + 14$$

(iii)

$$h(x) = 2x^3 - 5x^2 - 3x$$

(iv)

$$j(x) = -3x^4 + 2x^2 + 5x - 3$$

[5 marks]

b)

Consider the equation $6x^3 - (4a)x^2 - (a+2)x = 0$.

Given that the sum of the roots is $\frac{8}{3}$, find the three roots of the equation.

[5 marks]

Question 10

For the function $f(x) = ax^4 + bx^3 - x^2 - 24x - (5b + 1)$, the sum of the roots is $\frac{-7}{2}$ and the product of the roots is -18 . Find the values of a and b .

[4 marks]

Question 11

The function $f(x) = (x-3)(x^2+3x-4)(ax^2+bx+c)$ has three real and two complex roots.

a)

Find the three real roots.

[2 marks]

It is given for $f(x)$ that the sum of the roots is $-\frac{3}{2}$ and the product of the roots is -60 .

b)

Find the two complex roots, giving your answers in exact form.

[5 marks]

c)

Given that $f(2) = -144$, find the values of a , b and c .

[4 marks]

Question 12

α and β are non-real roots of the equation $x^2+3kx+2k+1=0$, where $k>0$ is a constant.

a)

Find $\alpha+\beta$ and $\alpha\beta$, in terms of k .

[2 marks]

b)

Given that $\alpha^2+\beta^2=3$, show that $(\alpha+\beta)^2=4k+5$.

[2 marks]

c)

Hence find the value of k .

[3 marks]

Question 13

Consider the function $f(x) = kx^3 + 3x^2 + 11x + 3k$, where k is a constant.

It is given that $(2x - 1)$ is a factor of $f(x)$.

(a)

Find the value of k .

[2 marks]

(b)

Fully factorise $f(x)$.

[3 marks]

(c)

Hence sketch the graph of $y = f(x)$. Clearly label the coordinates of any points where the graph intersects the coordinate axes.

[3 marks]