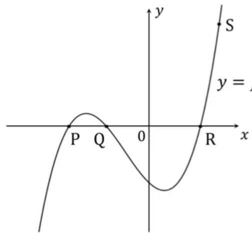
Question 1

Below is the graph of a function $f(x) = ax^3 + bx^2 + cx + d$, passing through the points $P(-3,0)$, $Q(-2,0)$, $R\left(\frac{1}{2},0\right)$ and $S(2,60)$.



(a) Find the values of a, b, c and d.

[4]

The function is translated vertically by the vector $\binom{0}{k}$ so that it passes through the point $(3,190)$.

(b) Find the value of k.

[2]

a) Points P, Q, and R tell us that -3, -2, and $\frac{1}{2}$ are roots of $f(x)$ — therefore $(x+3)$, $(x+2)$, and $(2x-1)$ must be factors.

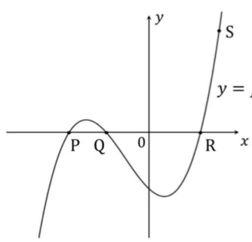
$$f(x) = p(x+3)(x+2)(2x-1)$$
$$= p(x^2+5x+6)(2x-1)$$
$$= p(2x^3+9x^2+7x-6)$$

And point S tells us that $f(2) = 60$

$$f(2) = p\left(2(2)^3 + 9(2)^2 + 7(2) - 6\right) = 60$$
$$\Rightarrow 60p = 60 \Rightarrow p = 1$$
$$\Rightarrow f(x) = 2x^3 + 9x^2 + 7x - 6$$

$$\boxed{a = 2 \quad b = 9 \quad c = 7 \quad d = -6}$$

Below is the graph of a function $f(x) = ax^3 + bx^2 + cx + d$, passing through the points $P(-3,0)$, $Q(-2,0)$, $R\left(\frac{1}{2},0\right)$ and $S(2,60)$.



(a) Find the values of a, b, c and d.

$$\boxed{a = 2 \quad b = 9 \quad c = 7 \quad d = -6}$$

[4]

The function is translated vertically by the vector $\binom{0}{k}$ so that it passes through the point $(3,190)$.

(b) Find the value of k.

[2]

b) $\binom{0}{k}$ is a vertical translation by k units. x-coordinates are unaffected.

$$f(3) = 2(3)^3 + 9(3)^2 + 7(3) - 6 = 150$$

And $f(3) + k = 190$, so:

$$150 + k = 190$$

$$\boxed{k = 40}$$

EXAM PAPERS PRACTICE

Question 2

(a) Given that the equation $2x^2 + 4x - m = 0$ has two real solutions, find the set of possible values of m.

[2]

(b) Given that the function $f(x) = x^2 - 5x + 2c$ has repeated roots, find c.

[2]

(c) Given that the function $g(x) = 2x^2 + 2kx + \left(\frac{3}{2} - k\right)$ has no real roots, find the set of possible values of k.

[4]

a) Two real solutions means the discriminant is greater than zero.

$(4)^2 - 4(2)(-m) > 0$

$8m + 16 > 0$

$8m > -16$

$\boxed{m > -2}$

Discriminant	$\Delta = b^2 - 4ac$

(a) Given that the equation $2x^2 + 4x - m = 0$ has two real solutions, find the set of possible values of m.

[2]

(b) Given that the function $f(x) = x^2 - 5x + 2c$ has repeated roots, find c.

[2]

(c) Given that the function $g(x) = 2x^2 + 2kx + \left(\frac{3}{2} - k\right)$ has no real roots, find the set of possible values of k.

[4]

b) Repeated roots means the discriminant is equal to zero.

$(-5)^2 - 4(1)(2c) = 0$

$25 - 8c = 0$

$8c = 25$

$\boxed{c = \frac{25}{8}}$

Discriminant	$\Delta = b^2 - 4ac$

(a) Given that the equation $2x^2 + 4x - m = 0$ has two real solutions, find the set of possible values of m.

[2]

(b) Given that the function $f(x) = x^2 - 5x + 2c$ has repeated roots, find c.

[2]

(c) Given that the function $g(x) = 2x^2 + 2kx + \left(\frac{3}{2} - k\right)$ has no real roots, find the set of possible values of k.

[4]

Discriminant	$\Delta = b^2 - 4ac$

c) No real roots means the discriminant is less than zero.

$$(2k)^2 - 4(2)\left(\tfrac{3}{2} - k\right) < 0$$

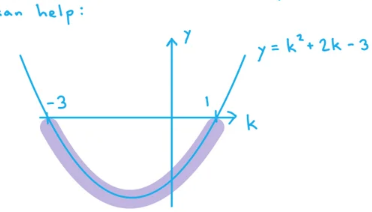
$$4k^2 + 8k - 12 < 0$$

$$k^2 + 2k - 3 < 0$$

$$\underbrace{(k+3)(k-1)}_{} < 0 \quad \text{Factorise}$$

This is equal to zero for $k = -3$ or $k = 1$

To see where it's less than zero, a sketch can help:



$$\boxed{-3 < k < 1}$$

Question 3

Let a function f be defined by $f(x) = 2x^3 + 7x^2 - 3x - 18$.

(i) Show that $(x+3)$ is a factor of $f(x)$.

(ii) Hence factorise $f(x)$ fully.

(iii) Write down all the solutions to $2x^3 + 7x^2 - 3x - 18 = 0$.

[6]

(i) $f(-3) = 2(-3)^3 + 7(-3)^2 - 3(-3) - 18$
$= -54 + 63 + 9 - 18 = 0$

$f(-3) = 0$, so by the factor theorem $(x+3) = (x - (-3))$ is a factor of $f(x)$

(ii) $f(x) = (x+3)(2x^2 + x - 6)$

Factorise by inspection, or by using polynomial or synthetic division

$$\boxed{f(x) = (x+3)(2x-3)(x+2)}$$

Then factorise the quadratic factor

(iii) $\boxed{x = -3,\ -2,\ \text{or}\ \frac{3}{2}}$

EXAM PAPERS PRACTICE

Question 4

(a) Factorise fully $6x^3 + x^2 - 12x + 5$. [4]

(b) $f(x) = ax^3 + (5a-2)x^2 + (4a+2)x - 2a$

(i) Given that $(x+3)$ is a factor of $f(x)$, find a.

(ii) Hence factorise $f(x)$ fully. [7]

a) Note that the sum of the coeffients is zero, i.e. $6 + 1 - 12 + 5 = 0$. This means that 1 is a root of $6x^3 + x^2 - 12x + 5 = 0$.

$6(1)^3 + (1)^2 - 12(1) + 5 = 0$, therefore $(x-1)$ is a factor of $6x^3 + x^2 - 12x + 5$ by the factor theorem

$6x^3 + x^2 - 12x + 5 = (x-1)(6x^2 + 7x - 5)$ Factorise by inspection, or by using polynomial or synthetic division

$\boxed{= (x-1)(3x+5)(2x-1)}$ Then factorise the quadratic factor

(a) Factorise fully $6x^3 + x^2 - 12x + 5$. [4]

(b) $f(x) = ax^3 + (5a-2)x^2 + (4a+2)x - 2a$

(i) Given that $(x+3)$ is a factor of $f(x)$, find a.

(ii) Hence factorise $f(x)$ fully. [7]

b) (i) If $(x+3)$ is a factor, then by the factor theorem we know that $f(-3) = 0$:

$a(-3)^3 + (5a-2)(-3)^2 + (4a+2)(-3) - 2a = 0$

$-27a + 45a - 18 - 12a - 6 - 2a = 0$

$4a - 24 = 0 \implies \boxed{a = 6}$

(ii) $\implies f(x) = 6x^3 + 28x^2 + 26x - 12$

$= (x+3)(6x^2 + 10x - 4)$ Factorise by inspection, or by using polynomial or synthetic division

$= 2(x+3)(3x^2 + 5x - 2)$

$\boxed{f(x) = 2(x+3)(3x-1)(x+2)}$ Then factorise the quadratic factor

Question 5

Consider the polynomial $g(x) = 3x^5 - 25x^4 + 72x^3 - 72x^2 - 16x + 48$.

(a) Show that 2 is a root of $g(x)$.

[2]

(b) Given that 2 is a root of $g(x)$ with multiplicity 3, factorise $g(x)$ fully and hence state the other two roots.

[5]

a) $g(2) = 3(2)^5 - 25(2)^4 + 72(2)^3 - 72(2)^2 - 16(2) + 48$

$= 96 - 400 + 576 - 288 - 32 + 48$

$= 720 - 720 = 0$

$g(2) = 0$, so 2 is a root of $g(x)$

Consider the polynomial $g(x) = 3x^5 - 25x^4 + 72x^3 - 72x^2 - 16x + 48$.

(a) Show that 2 is a root of $g(x)$.

[2]

(b) Given that 2 is a root of $g(x)$ with multiplicity 3, factorise $g(x)$ fully and hence state the other two roots.

[5]

This means that $(x-2)^3$ is a factor of $g(x)$.

b) $(x-2)^3 = (x-2)(x-2)(x-2)$

$= (x^2 - 4x + 4)(x-2)$

$= x^3 - 6x^2 + 12x - 8$

Factorise by inspection, or by using polynomial or synthetic division

So $g(x) = (x^3 - 6x^2 + 12x - 8)(3x^2 - 7x - 6)$

$= (x^3 - 6x^2 + 12x - 8)(3x+2)(x-3)$

Then factorise the quadratic factor

$g(x) = (x-2)^3(3x+2)(x-3)$

The other roots are 3 and $-\frac{2}{3}$

Question 6

Consider the function $f(x) = 4x^3 + 6x^2 - 7x + 2$.

(i) Find the quotient and remainder when $4x^3 + 6x^2 - 7x + 2$ is divided by $(x - 2)$.

(ii) Hence write $4x^3 + 6x^2 - 7x + 2$ in the form $(x - 2)(ax^2 + bx + c) + d$, where a, b, c and d are constants to be determined.

[5]

(i) By the remainder theorem the remainder is

$$f(2) = 4(2)^3 + 6(2)^2 - 7(2) + 2 = 44$$

$$\text{So } f(x) = \underbrace{4x^3 + 6x^2 - 7x - 42}_{(x-2)\text{ times quotient}} + \underbrace{44}_{\text{remainder}}$$

$$4x^3 + 6x^2 - 7x - 42 = (x-2)\underbrace{(4x^2 + 14x + 21)}_{\text{quotient}}$$ Factorise by inspection

$$\boxed{\text{quotient} = 4x^2 + 14x + 21 \qquad \text{remainder} = 44}$$

(ii)

$$\boxed{4x^3 + 6x^2 - 7x - 42 = (x-2)(4x^2 + 14x + 21) + 44}$$

Note: Polynomial division or synthetic division could also be used to answer this question.

Question 7

The function $f(x) = 2x^3 - 5x^2 + ax + b$ has $(2x + 3)$ as a factor, and when $f(x)$ is divided by $(x - 2)$ the remainder is 7.

(a) Show that a and b must satisfy the simultaneous equations:

$$2a + b = 11$$
$$3a - 2b = -36$$

[5]

(b) Hence find a and b.

[2]

a) If $2x + 3 = 2(x + \frac{3}{2})$ is a factor, then by the factor theorem $f(-\frac{3}{2}) = 0$:

$$2(-\tfrac{3}{2})^3 - 5(-\tfrac{3}{2})^2 + a(-\tfrac{3}{2}) + b = 0$$

$$-\frac{27}{4} - \frac{45}{4} - \frac{3}{2}a + b = 0$$

$$\frac{3}{2}a - b = -\frac{36}{2} \implies 3a - 2b = -36$$

If $f(x) \div (x-2)$ has remainder 7, then by the remainder theorem $f(2) = 7$:

$$2(2)^3 - 5(2)^2 + a(2) + b = 7$$

$$16 - 20 + 2a + b = 7$$

$$2a + b - 4 = 7 \implies 2a + b = 11$$

The function $f(x) = 2x^3 - 5x^2 + ax + b$ has $(2x + 3)$ as a factor, and when $f(x)$ is divided by $(x - 2)$ the remainder is 7.

(a) Show that a and b must satisfy the simultaneous equations:

$$2a + b = 11 \quad (1)$$
$$3a - 2b = -36 \quad (2)$$

[5]

(b) Hence find a and b.

[2]

b) $2 \times (1)$ $\quad 4a + 2b = 22$

$+ (2)$ $\quad + (3a - 2b = -36)$

$7a = -14 \implies a = -2$

Substitute value for a into (1):

$2(-2) + b = 11 \implies b = 15$

$$\boxed{a = -2 \quad b = 15}$$

Question 8

Given that $3 + 2\mathrm{i}$ is one of the roots of the equation $x^3 - 3x^2 - 5x + 39 = 0$, find the other two roots.

[5]

$\boxed{3 - 2i \text{ is another root}}$

Complex roots of polynomials always occur in complex conjugate pairs.

Therefore $(x-(3+2i))(x-(3-2i))$ is a factor of $x^3 - 3x^2 - 5x + 39$.

$(x-(3+2i))(x-(3-2i)) = x^2 - (3+2i+3-2i)x + \underbrace{(3+2i)(3-2i)}_{= 9 - 6i + 6i - 4i^2}$

$= x^2 - 6x + 13$

$\implies x^3 - 3x^2 - 5x + 39 = (x^2 - 6x + 13)\underbrace{(x+3)}$

You should be able to find this factor by inspection.

$\boxed{-3 \text{ is the third root}}$

EXAM PAPERS PRACTICE

Question 9

(a) For each of the following polynomials, find the sum of the roots and the product of the roots.

(i) $f(x) = 9x^4 + 7x^3 - 3x + 2$

(ii) $g(x) = 7x^5 - x^4 + 2x^3 + x^2 - 5x + 14$

(iii) $h(x) = 2x^3 - 5x^2 - 3x$

(iv) $j(x) = -3x^4 + 2x^2 + 5x - 3$

[5]

(b) Consider the equation $6x^3 - (4a)x^2 - (a+2)x = 0$.

Given that the sum of the roots is $\frac{8}{3}$, find the three roots of the equation.

[5]

Sum & product of the roots of polynomial equations of the form $\sum_{r=0}^{n} a_r x^r = 0$	Sum is $\frac{-a_{n-1}}{a_n}$; product is $\frac{(-1)^n a_0}{a_n}$

a) (i) $\boxed{\text{sum} = -\frac{7}{9} \qquad \text{product} = \frac{(-1)^4 \cdot 2}{9} = \frac{2}{9}}$

(ii) $\boxed{\text{sum} = \frac{-(-1)}{7} = \frac{1}{7} \qquad \text{product} = \frac{(-1)^5 \cdot 14}{7} = -2}$

$a_0 = 0$ here – be careful!

(iii) $\boxed{\text{sum} = \frac{-(-5)}{2} = \frac{5}{2} \qquad \text{product} = \frac{(-1)^3 \cdot 0}{2} = 0}$

$a_3 = 0$ here – be careful!

(iv) $\boxed{\text{sum} = \frac{-0}{(-3)} = 0 \qquad \text{product} = \frac{(-1)^4 \cdot (-3)}{-3} = 1}$

(a) For each of the following polynomials, find the sum of the roots and the product of the roots.

(i) $f(x) = 9x^4 + 7x^3 - 3x + 2$

(ii) $g(x) = 7x^5 - x^4 + 2x^3 + x^2 - 5x + 14$

(iii) $h(x) = 2x^3 - 5x^2 - 3x$

(iv) $j(x) = -3x^4 + 2x^2 + 5x - 3$

[5]

(b) Consider the equation $6x^3 - (4a)x^2 - (a+2)x = 0$.

Given that the sum of the roots is $\frac{8}{3}$, find the three roots of the equation.

[5]

b) $\frac{-(-4a)}{6} = \frac{2a}{3} = \frac{8}{3} \Rightarrow a = 4$ Use sum of roots formula to find a

$6x^3 - 16x^2 - 6x = 0$

$3x^3 - 8x^2 - 3x = 0$

$x(3x^2 - 8x - 3) = 0$

$x(3x+1)(x-3) = 0$

$\boxed{x = 0, -\frac{1}{3}, 3}$

Note that $0 + (-\frac{1}{3}) + 3 = \frac{8}{3}$ as expected

Sum & product of the roots of polynomial equations of the form $\sum_{r=0}^{n} a_r x^r = 0$	Sum is $\frac{-a_{n-1}}{a_n}$; product is $\frac{(-1)^n a_0}{a_n}$

Question 10

For the function $f(x) = ax^4 + bx^3 - x^2 - 24x - (5b+1)$, the sum of the roots is $\frac{-7}{2}$ and the product of the roots is -18. Find the values of a and b.

[4]

Use sum and product formulae to find simultaneous equations in a and b:

$\frac{-b}{a} = \frac{-7}{2} \Rightarrow 7a - 2b = 0$ ①

$\frac{(-1)^4 \cdot (-(5b+1))}{a} = -\frac{5b+1}{a} = -18 \Rightarrow 18a - 5b = 1$ ②

$2 \times$② $36a - 10b = 2$

$-5 \times$① $-(35a - 10b = 0)$

$a = 2 \Rightarrow b = 7$

Or solve using GDC

$\boxed{a = 2 \quad b = 7}$

Sum & product of the roots of polynomial equations of the form $\sum_{r=0}^{n} a_r x^r = 0$	Sum is $\frac{-a_{n-1}}{a_n}$; product is $\frac{(-1)^n a_0}{a_n}$

Question 11

The function $f(x) = (x-3)(x^2+3x-4)(ax^2+bx+c)$ has three real and two complex roots.

(a) Find the three real roots.

[2]

It is given for $f(x)$ that the sum of the roots is $-\frac{3}{2}$ and the product of the roots is -60.

(b) Find the two complex roots, giving your answers in exact form.

[5]

(c) Given that $f(2) = -144$, find the values of a, b and c.

[4]

a) $x^2 + 3x - 4 = (x-1)(x+4)$, so

$f(x) = (x-3)(x-1)(x+4)(ax^2+bx+c)$

$\boxed{\text{The real roots are } 3,\ 1, \text{ and } -4.}$

The function $f(x) = (x-3)(x^2+3x-4)(ax^2+bx+c)$ has three real and two complex roots.

(a) Find the three real roots. $\boxed{3,\ 1, \text{ and } -4.}$

[2]

It is given for $f(x)$ that the sum of the roots is $-\frac{3}{2}$ and the product of the roots is -60.

(b) Find the two complex roots, giving your answers in exact form.

[5]

(c) Given that $f(2) = -144$, find the values of a, b and c.

[4]

Solutions of a quadratic equation	$ax^2+bx+c=0 \Rightarrow x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$, $a \neq 0$

b) Let the two roots of ax^2+bx+c be α and β.

Then

$3 + 1 + (-4) + \alpha + \beta = -\frac{3}{2} \Rightarrow \alpha + \beta = -\frac{3}{2}$ ①

and

$(3)(1)(-4)(\alpha)(\beta) = -60 \Rightarrow \alpha\beta = 5 \Rightarrow \beta = \frac{5}{\alpha}$ ②

Substitute ② into ①:

$\Rightarrow \alpha + \frac{5}{\alpha} = -\frac{3}{2} \Rightarrow 2\alpha^2 + 3\alpha + 10 = 0$

$\Rightarrow \alpha = \dfrac{-3 \pm \sqrt{3^2 - 4(2)(10)}}{2(2)} = -\frac{3}{4} \pm \frac{\sqrt{71}}{4}i$

If we choose the 'plus' version of that for α, then we get the 'minus' version for β (and vice versa). This is expected, as α and β must be a complex conjugate pair.

$\boxed{-\frac{3}{4} + \frac{\sqrt{71}}{4}i \quad \text{and} \quad -\frac{3}{4} - \frac{\sqrt{71}}{4}i}$

The function $f(x) = (x-3)(x^2+3x-4)(ax^2+bx+c)$ has three real and two complex roots.

(a) Find the three real roots.

3, 1, and − 4.

[2]

It is given for $f(x)$ that the sum of the roots is $-\frac{3}{2}$ and the product of the roots is -60.

(b) Find the two complex roots, giving your answers in exact form.

$-\frac{3}{4}+\frac{\sqrt{71}}{4}i$ and $-\frac{3}{4}-\frac{\sqrt{71}}{4}i$

[5]

(c) Given that $f(2) = -144$, find the values of a, b and c.

[4]

c) $ax^2+bx+c = a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right)$

$= a\left(x-\left(-\frac{3}{4}+\frac{\sqrt{71}}{4}i\right)\right)\left(x-\left(-\frac{3}{4}-\frac{\sqrt{71}}{4}i\right)\right)$ Use roots from part (b)

$= a\left(\underbrace{x^2+\frac{3}{2}x+5}\right)$ Expand brackets and simplify

So $\frac{b}{a}=\frac{3}{2}$, $\frac{c}{a}=5$

$f(2) = (2-3)\left((2)^2+3(2)-4\right)\left(a\left((2)^2+\frac{3}{2}(2)+5\right)\right) = -144$

$\Rightarrow (-1)(6)(12a) = -72a = -144$

$\Rightarrow a = 2 \Rightarrow \frac{b}{2}=\frac{3}{2}, \frac{c}{2}=5$

$\boxed{a=2 \quad b=3 \quad c=10}$

Question 12

α and β are non-real roots of the equation $x^2+3kx+2k+1=0$, where $k>0$ is a constant.

(a) Find $\alpha+\beta$ and $\alpha\beta$, in terms of k.

[2]

(b) Given that $\alpha^2+\beta^2=3$, show that $(\alpha+\beta)^2 = 4k+5$.

[2]

(c) Hence find the value of k.

[3]

a) Use sum and product of roots formulae:

$\alpha+\beta = \frac{-3k}{1} = -3k$

$\alpha\beta = \frac{(-1)^2\cdot(2k+1)}{1} = 2k+1$

Sum & product of the roots of polynomial equations of the form $\sum_{r=0}^{n} a_r x^r = 0$	Sum is $\frac{-a_{n-1}}{a_n}$; product is $\frac{(-1)^n a_0}{a_n}$

α and β are non-real roots of the equation $x^2 + 3kx + 2k + 1 = 0$, where $k > 0$ is a constant.

(a) Find $\alpha + \beta$ and $\alpha\beta$, in terms of k.

$\boxed{\alpha + \beta = -3k \qquad \alpha\beta = 2k + 1}$

[2]

(b) Given that $\alpha^2 + \beta^2 = 3$, show that $(\alpha + \beta)^2 = 4k + 5$.

[2]

(c) Hence find the value of k.

[3]

b)

$$(\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$$
$$= 2(\alpha\beta) + (\alpha^2 + \beta^2)$$
$$= 2(2k+1) + 3$$
$$= 4k + 2 + 3$$
$$= 4k + 5$$

α and β are non-real roots of the equation $x^2 + 3kx + 2k + 1 = 0$, where $k > 0$ is a constant.

(a) Find $\alpha + \beta$ and $\alpha\beta$, in terms of k.

$\boxed{\alpha + \beta = -3k \qquad \alpha\beta = 2k + 1}$

[2]

(b) Given that $\alpha^2 + \beta^2 = 3$, show that $(\alpha + \beta)^2 = 4k + 5$.

[2]

(c) Hence find the value of k.

[3]

c) Combine results from parts (a) and (b):

$$(\alpha + \beta)^2 = 4k + 5$$
$$\Longrightarrow (-3k)^2 = 4k + 5$$
$$\Longrightarrow 9k^2 - 4k - 5 = 0$$
$$\Longrightarrow (9k + 5)(k - 1) = 0 \quad \text{Factorise}$$
$$\Longrightarrow k = -\tfrac{5}{9} \text{ or } k = 1$$

But $k > 0$, so

$$\boxed{k = 1}$$

Question 13

Consider the function $f(x) = kx^3 + 3x^2 + 11x + 3k$, where k is a constant.

It is given that $(2x-1)$ is a factor of $f(x)$.

(a) Find the value of k.

[2]

(b) Fully factorise $f(x)$.

[3]

(c) Hence sketch the graph of $y = f(x)$. Clearly label the coordinates of any points where the graph intersects the coordinate axes.

[3]

a) If $(2x-1)$ is a factor, then by the factor theorem we know that $f(\frac{1}{2}) = 0$:

$2x-1=0 \Rightarrow x=\frac{1}{2}$

$$k\left(\tfrac{1}{2}\right)^3 + 3\left(\tfrac{1}{2}\right)^2 + 11\left(\tfrac{1}{2}\right) + 3k = 0$$

$$\tfrac{1}{8}k + \tfrac{3}{4} + \tfrac{11}{2} + 3k = 0$$

$$k + 6 + 44 + 24k = 0$$

$$25k + 50 = 0$$

$$\boxed{k = -2}$$

Consider the function $f(x) = kx^3 + 3x^2 + 11x + 3k$, where k is a constant.

It is given that $(2x-1)$ is a factor of $f(x)$.

(a) Find the value of k. $\boxed{k = -2}$

[2]

(b) Fully factorise $f(x)$.

[3]

(c) Hence sketch the graph of $y = f(x)$. Clearly label the coordinates of any points where the graph intersects the coordinate axes.

[3]

b) $-2x^3 + 3x^2 + 11x - 6 = (2x-1)(ax^2+bx+c)$

$$= 2ax^3 + (2b-a)x^2 + (2c-b)x - c$$

$$\Rightarrow a=-1, \quad b=1, \quad c=6$$

$$f(x) = (2x-1)(-x^2+x+6)$$

$$= -(2x-1)(x^2-x-6)$$

$$\boxed{f(x) = -(2x-1)(x-3)(x+2)}$$

Note: You could also use polynomial division to find the $-x^2+x+6$ factor.

Consider the function $f(x) = kx^3 + 3x^2 + 11x + 3k$, where k is a constant.

It is given that $(2x - 1)$ is a factor of $f(x)$.

(a) Find the value of k. $\boxed{k = -2}$

[2]

(b) Fully factorise $f(x)$. $\boxed{f(x) = -(2x-1)(x-3)(x+2)}$

[3]

(c) Hence sketch the graph of $y = f(x)$. Clearly label the coordinates of any points where the graph intersects the coordinate axes.

[3]

c) $f(x) = -2x^3 + 3x^2 + 11x - 6 = -(2x-1)(x-3)(x+2)$

$\Longrightarrow$ x-intercepts at $x = \frac{1}{2}$, 3 and -2

$\Longrightarrow$ y-intercept at $y = f(0) = -6$

The $-2x^3$ term tells us that $f(x)$ will be:

large and positive when x is large and negative

large and negative when x is large and positive