Question 1

State which distribution – normal, binomial or Poisson – is likely to be appropriate for calculating the final value of each of the following probabilities. In each case specify any assumptions that would need to be made, and any parameters of which you would need to know the value in order to carry out the calculation.

(a) The probability that the next person to walk through the door of a shop has a height of 1.7 metres or more.

[3]

(b) The probability that exactly 3 people with a height of 1.7 metres or more walk through the door of a shop in the next 20 minutes.

[3]

(c) The probability that of the next 12 people to walk through the door of a shop, exactly three of them have a height of 1.7 metres or more.

[3]

a) Normal distribution

Assumption: Heights of population are normally distributed.

Need to know: mean (μ) and standard deviation (σ) of population.

State which distribution – normal, binomial or Poisson – is likely to be appropriate for calculating the final value of each of the following probabilities. In each case specify any assumptions that would need to be made, and any parameters of which you would need to know the value in order to carry out the calculation.

(a) The probability that the next person to walk through the door of a shop has a height of 1.7 metres or more.

[3]

(b) The probability that exactly 3 people with a height of 1.7 metres or more walk through the door of a shop in the next 20 minutes.

[3]

(c) The probability that of the next 12 people to walk through the door of a shop, exactly three of them have a height of 1.7 metres or more.

[3]

b) Poisson distribution

Assumptions:

① People with a height of 1.7 metres or more enter the shop singly, randomly, and independently of one another.

② The average number of people with a height of 1.7 metres or more who enter the shop in the given interval (here 20 minutes) is uniform and finite.

Need to know: The mean number of people with a height of 1.7 metres or more who enter the shop in a 20 minute interval.

State which distribution – normal, binomial or Poisson – is likely to be appropriate for calculating the final value of each of the following probabilities. In each case specify any assumptions that would need to be made, and any parameters of which you would need to know the value in order to carry out the calculation.

(a) The probability that the next person to walk through the door of a shop has a height of 1.7 metres or more.

[3]

(b) The probability that exactly 3 people with a height of 1.7 metres or more walk through the door of a shop in the next 20 minutes.

[3]

(c) The probability that of the next 12 people to walk through the door of a shop, exactly three of them have a height of 1.7 metres or more.

[3]

c) Binomial distribution

Assumptions:

① People enter the shop independently of one another.

② The probability that a person entering the shop has a height of 1.7 metres or more remains constant.

Need to know: The probability that a person entering the shop has a height of 1.7 metres or more.

Question 2

Amira has a bad Internet connection at her house. Her internet disconnects on average 5 times each day.

(a) Define a suitable distribution to model the number of times the internet at Amira's house disconnects during a day. State any assumptions you make.

[2]

(b) Find the probability that during a random day the internet at Amira's house disconnects:

(i) exactly four times

(ii) at most three times

(iii) no fewer than two times.

[4]

a) Po(5)

Assumptions:

① The disconnects occur singly, randomly, and independently of one another.

② The average number of disconnects in the given interval (here 5 per day) is uniform and finite.

Poisson distribution	$X \sim \text{Po}(m)$
Mean	$\text{E}(X) = m$
Variance	$\text{Var}(X) = m$

Amira has a bad Internet connection at her house. Her internet disconnects on average 5 times each day.

(a) Define a suitable distribution to model the number of times the internet at Amira's house disconnects during a day. State any assumptions you make.

[2]

(b) Find the probability that during a random day the internet at Amira's house disconnects:

(i) exactly four times

(ii) at most three times

(iii) no fewer than two times.

[4]

Poisson distribution	$X \sim \mathrm{Po}(m)$
Mean	$\mathrm{E}(X) = m$
Variance	$\mathrm{Var}(X) = m$

b) Let $X \sim Po(5)$ be the number of disconnects in a day.

(i) $P(X=4) = 0.17546737$ from GDC

$\boxed{P(X=4) = 0.175 \text{ (3 s.f.)}}$

(ii) 'at most 3 times' = '$0 \le X \le 3$' = '$X \le 3$'

$P(X \le 3) = 0.26502591$ from GDC

$\boxed{P(X \le 3) = 0.265 \text{ (3 s.f.)}}$

(iii) 'no fewer than 2 times' = '$X \ge 2$' =

$P(X \ge 2) = 0.95957231$ from GDC

$\boxed{P(X \ge 2) = 0.960 \text{ (3 s.f.)}}$

Question 3

Lucy loves the cinema and goes on average four times a week. The number of times she goes to the cinema in a week can be modelled as a Poisson distribution with a mean of four times.

(a) Find the probability that Lucy goes to the cinema exactly five times in a week.

[2]

(b) Find the probability that Lucy goes to the cinema no more than four times in a fortnight.

[2]

(c) Find the probability that Lucy goes to the cinema at least once in a day.

[3]

Poisson distribution	$X \sim \mathrm{Po}(m)$
Mean	$\mathrm{E}(X) = m$
Variance	$\mathrm{Var}(X) = m$

a) Let $X \sim Po(4)$ be the number of times Lucy goes to the cinema in a week.

$P(X=5) = 0.15629345$ from GDC

$\boxed{0.156 \text{ (3 s.f.)}}$

Lucy loves the cinema and goes on average four times a week. The number of times she goes to the cinema in a week can be modelled as a Poisson distribution with a mean of four times.

(a) Find the probability that Lucy goes to the cinema exactly five times in a week.

[2]

(b) Find the probability that Lucy goes to the cinema no more than four times in a fortnight.

[2]

(c) Find the probability that Lucy goes to the cinema at least once in a day.

[3]

Poisson distribution	$X \sim \mathrm{Po}(m)$
Mean	$\mathrm{E}(X) = m$
Variance	$\mathrm{Var}(X) = m$

b) 4 times per week = 8 times per fortnight

Let $X \sim \mathrm{Po}(8)$ be the number of times Lucy goes to the cinema in a fortnight

'no more than 4 times' = '$0 \le X \le 4$' = '$X \le 4$'

$P(X \le 4) = 0.996324$ from GDC

$\boxed{0.0996 \text{ (3 s.f.)}}$

Lucy loves the cinema and goes on average four times a week. The number of times she goes to the cinema in a week can be modelled as a Poisson distribution with a mean of four times.

(a) Find the probability that Lucy goes to the cinema exactly five times in a week.

[2]

(b) Find the probability that Lucy goes to the cinema no more than four times in a fortnight.

[2]

(c) Find the probability that Lucy goes to the cinema at least once in a day.

[3]

c) 4 times per week = $\frac{4}{7}$ times per day

Let $X \sim \mathrm{Po}(\frac{4}{7})$ be the number of times Lucy goes to the cinema in a day.

'at least once' = '$X \ge 1$'

$P(X \ge 1) = 0.43528187$ from GDC

$\boxed{0.435 \text{ (3 s.f.)}}$

Poisson distribution	$X \sim \mathrm{Po}(m)$
Mean	$\mathrm{E}(X) = m$
Variance	$\mathrm{Var}(X) = m$

Question 4

Comic Stans is a comic book store in the city of Krakoa. Customers enter the store randomly and independently at an average rate of 8 people every 15 minutes.

(a) Find the probability that exactly three people enter the store in a 1-minute period.

[2]

(b) Find the probability that someone enters the store in a 15-second period.

[3]

(c) Find the probability that at most three people enter the store in a 10-minute period.

[3]

(d) Find the variance of the number of people entering the store in a 1-hour period.

[2]

Poisson distribution	$X \sim \mathrm{Po}(m)$
Mean	$\mathrm{E}(X) = m$
Variance	$\mathrm{Var}(X) = m$

a) 8 per 15 minutes $= \frac{8}{15}$ per minute

Let $X \sim \mathrm{Po}(\frac{8}{15})$ be the number of people who enter the store in a minute.

$P(X = 3) = 0.01483273$ from GDC

$\boxed{0.0148 \ (3 \text{ s.f.})}$

Comic Stans is a comic book store in the city of Krakoa. Customers enter the store randomly and independently at an average rate of 8 people every 15 minutes.

(a) Find the probability that exactly three people enter the store in a 1-minute period.

[2]

(b) Find the probability that someone enters the store in a 15-second period.

Be careful – this doesn't mean that exactly one person enters the store!

[3]

(c) Find the probability that at most three people enter the store in a 10-minute period.

[3]

(d) Find the variance of the number of people entering the store in a 1-hour period.

[2]

Poisson distribution	$X \sim \mathrm{Po}(m)$
Mean	$\mathrm{E}(X) = m$
Variance	$\mathrm{Var}(X) = m$

b) 8 per 15 minutes $= \frac{8}{60} = \frac{2}{15}$ per 15 seconds

Let $X \sim \mathrm{Po}(\frac{2}{15})$ be the number of people who enter the store in 15 seconds.

'someone enters the store' = 'the number of people entering the store is not zero' (i.e., at least one person enters the store) = '$X \geq 1$'

$P(X \geq 1) = 0.12482668$ from GDC

$\boxed{0.125 \ (3 \text{ s.f.})}$

Comic Stans is a comic book store in the city of Krakoa. Customers enter the store randomly and independently at an average rate of 8 people every 15 minutes.

(a) Find the probability that exactly three people enter the store in a 1-minute period. [2]

(b) Find the probability that someone enters the store in a 15-second period. [3]

(c) Find the probability that at most three people enter the store in a 10-minute period. [3]

(d) Find the variance of the number of people entering the store in a 1-hour period. [2]

Poisson distribution	$X \sim \text{Po}(m)$
Mean	$\text{E}(X) = m$
Variance	$\text{Var}(X) = m$

c) 8 per 15 minutes $= 8 \times \frac{10}{15} = \frac{16}{3}$ per 10 minutes

Let $X \sim \text{Po}\left(\frac{16}{3}\right)$ be the number of people who enter the store in 10 minutes.

'at most 3 enter' = '$0 \leq X \leq 3$' = '$X \leq 3$'

$P(X \leq 3) = 0.22131084$

$\boxed{0.221 \text{ (3 s.f.)}}$

Comic Stans is a comic book store in the city of Krakoa. Customers enter the store randomly and independently at an average rate of 8 people every 15 minutes.

(a) Find the probability that exactly three people enter the store in a 1-minute period. [2]

(b) Find the probability that someone enters the store in a 15-second period. [3]

(c) Find the probability that at most three people enter the store in a 10-minute period. [3]

(d) Find the variance of the number of people entering the store in a 1-hour period. [2]

Poisson distribution	$X \sim \text{Po}(m)$
Mean	$\text{E}(X) = m$
Variance	$\text{Var}(X) = m$

d) 8 per 15 minutes = 32 per hour

Let $X \sim \text{Po}(32)$ be the number of people who enter the store in an hour.

For the Poisson distribution

$\text{Var}(X) = \text{E}(X) = m$

$\boxed{\text{variance} = 32 \text{ people}^2}$

You don't need the units to get the marks here, but it's good to be aware of what they are.

Question 5

Amber suggests that she can model the number of times that she hiccups using a Poisson distribution.

(a) Write down two conditions that must apply for this model to be applicable.

[2]

The mean number of hiccups in a 30-second period is 2.9.

(b) Assuming a Poisson distribution is applicable, find the probability that

(i) Amber hiccups exactly three times in a 30-second period

(ii) Amber hiccups at least twice but no more than five times in a 15-second period

(iii) Amber hiccups during a one-minute period.

[6]

a) ① The hiccups occur singly, randomly, and independently of one another.

② The average number of hiccups in any given time interval is uniform and finite.

Poisson distribution	$X \sim \mathrm{Po}(m)$
Mean	$\mathrm{E}(X) = m$
Variance	$\mathrm{Var}(X) = m$

Amber suggests that she can model the number of times that she hiccups using a Poisson distribution.

(a) Write down two conditions that must apply for this model to be applicable.

[2]

The mean number of hiccups in a 30-second period is 2.9.

(b) Assuming a Poisson distribution is applicable, find the probability that

(i) Amber hiccups exactly three times in a 30-second period

(ii) Amber hiccups at least twice but no more than five times in a 15-second period

(iii) Amber hiccups during a one-minute period.

[6]

Poisson distribution	$X \sim \mathrm{Po}(m)$
Mean	$\mathrm{E}(X) = m$
Variance	$\mathrm{Var}(X) = m$

b) (i) Let $X \sim \mathrm{Po}(2.9)$ be the number of times Amber hiccups in 30 seconds.

$P(X = 3) = 0.22366021$ from GDC

$\boxed{0.224 \ (3 \text{ s.f.})}$

(ii) 2.9 per 30 seconds $= \frac{2.9}{2} = 1.45$ per 15 seconds

Let $X \sim \mathrm{Po}(1.45)$ be the number of times Amber hiccups in 15 seconds.

'at least twice but no more than 5 times' = '$2 \leq X \leq 5$'

$P(2 \leq X \leq 5) = 0.42151257$ from GDC

$\boxed{0.422 \ (3 \text{ s.f.})}$

(iii) 2.9 per 30 seconds $= 2.9 \times 2 = 5.8$ per minute

Let $X \sim \mathrm{Po}(5.8)$ be the number of times Amber hiccups in a minute.

'hiccups during a one-minute period' = 'doesn't hiccup zero times during a one-minute period' = '$X \geq 1$'

$P(X \geq 1) = 0.99697244$ from GDC

$\boxed{0.997 \ (3 \text{ s.f.})}$

Be careful – this doesn't mean that she hiccups exactly one time!

Question 6

n = 50

(a) The table below shows the data from a sample of 50 observations of a variable x.

(i) Calculate unbiased estimates for the mean and the variance.

(ii) State, with a reason, whether a Poisson distribution could be used to model the population's data.

x	0	1	2	3	4
Frequency	3	5	17	15	10

[5]

(b) The table below shows the data from a sample of 100 observations of a variable y.

(i) Calculate unbiased estimates for the mean and the variance.

(ii) State, with a reason, whether a Poisson distribution could be used to model the population's data.

y	0	1	2	3	4	5
Frequency	15	19	25	21	12	8

[5]

Poisson distribution	$X \sim \mathrm{Po}(m)$
Mean	$\mathrm{E}(X) = m$
Variance	$\mathrm{Var}(X) = m$

Unbiased estimate of population variance s_{n-1}^2	$s_{n-1}^2 = \frac{n}{n-1} s_n^2$

a) (i) Use GDC to calculate $\bar{x}$ and s_{n-1}^2

$\boxed{\bar{x} = 2.48}$ from GDC

$s_{n-1} = 1.11098411$ from GDC

$s_{n-1}^2 = 1.234285...$

$\boxed{s_{n-1}^2 = 1.23 \ (3 \text{ s.f.})}$

(ii)

A Poisson model should not be used, because the mean ($\bar{x}$) and the unbiased variance estimate (s_{n-1}^2) are not close to each other.

(a) The table below shows the data from a sample of 50 observations of a variable x.

(i) Calculate unbiased estimates for the mean and the variance.

(ii) State, with a reason, whether a Poisson distribution could be used to model the population's data.

x	0	1	2	3	4
Frequency	3	5	17	15	10

[5]

n = 100

(b) The table below shows the data from a sample of 100 observations of a variable y.

(i) Calculate unbiased estimates for the mean and the variance.

(ii) State, with a reason, whether a Poisson distribution could be used to model the population's data.

y	0	1	2	3	4	5
Frequency	15	19	25	21	12	8

[5]

Poisson distribution	$X \sim \mathrm{Po}(m)$
Mean	$\mathrm{E}(X) = m$
Variance	$\mathrm{Var}(X) = m$

Unbiased estimate of population variance s_{n-1}^2	$s_{n-1}^2 = \frac{n}{n-1} s_n^2$

b) (i) Use GDC to calculate $\bar{x}$ and s_{n-1}^2

$\boxed{\bar{x} = 2.2}$ from GDC

$s_{n-1} = 1.47709789$ from GDC

$s_{n-1}^2 = 2.181818...$

$\boxed{s_{n-1}^2 = 2.18 \ (3 \text{ s.f.})}$

(ii)

A Poisson model could be used, because the mean ($\bar{x}$) and the unbiased variance estimate (s_{n-1}^2) are very close to one another.

Question 7

Jim is a bird watcher and is trying to model the number of birds that fly past his window. During a 10-minute period he records the number of birds that fly past his window, and he repeats this a total of 120 times to form a sample.

$n=120$

Number of birds	Frequency
0	43
1	44
2	22
3	8
4	3
5 or more	0

(a) Calculate unbiased estimates for the mean and the variance for the number of birds that fly past Jim's window in a ten-minute period.

[4]

(b) Explain why a Poisson distribution would be appropriate to model the number of birds that fly past Jim's window in a 10-minute period.

[1]

Jim uses the distribution Po(1) to model the number of birds that fly past his window in a 10-minute period.

(c) Use Jim's model to calculate the probability that:

(i) exactly two birds fly past Jim's window in a 30-minute period

(ii) fewer than two birds fly past Jim's window in a 1-minute period

(iii) at least four birds fly past Jim's window in a 1-hour period.

[7]

a) Use GDC to calculate $\bar{x}$ and s_{n-1}^2

$\bar{x} = 1.03333333$ from GDC

$\boxed{\bar{x} = 1.03 \quad (3 \text{ s.f.})}$

$s_{n-1} = 1.02024328$ from GDC

$s_{n-1}^2 = 1.040896...$

$\boxed{s_{n-1}^2 = 1.04 \quad (3 \text{ s.f.})}$

Poisson distribution	$X \sim \text{Po}(m)$
Mean	$\text{E}(X) = m$
Variance	$\text{Var}(X) = m$

Unbiased estimate of population variance s_{n-1}^2 $\quad s_{n-1}^2 = \frac{n}{n-1}s_n^2$

Jim is a bird watcher and is trying to model the number of birds that fly past his window. During a 10-minute period he records the number of birds that fly past his window, and he repeats this a total of 120 times to form a sample.

Number of birds	Frequency
0	43
1	44
2	22
3	8
4	3
5 or more	0

(a) Calculate unbiased estimates for the mean and the variance for the number of birds that fly past Jim's window in a ten-minute period.

$\boxed{\bar{x} = 1.03 \quad (3 \text{ s.f.})}$ $\boxed{s_{n-1}^2 = 1.04 \quad (3 \text{ s.f.})}$

[4]

(b) Explain why a Poisson distribution would be appropriate to model the number of birds that fly past Jim's window in a 10-minute period.

[1]

Jim uses the distribution Po(1) to model the number of birds that fly past his window in a 10-minute period.

(c) Use Jim's model to calculate the probability that:

(i) exactly two birds fly past Jim's window in a 30-minute period

(ii) fewer than two birds fly past Jim's window in a 1-minute period

(iii) at least four birds fly past Jim's window in a 1-hour period.

[7]

b) A Poisson distribution is appropriate because the mean ($\bar{x}$) and the unbiased variance estimate (s_{n-1}^2) are approximately equal.

Poisson distribution	$X \sim \text{Po}(m)$
Mean	$\text{E}(X) = m$
Variance	$\text{Var}(X) = m$

Unbiased estimate of population variance s_{n-1}^2 $\quad s_{n-1}^2 = \frac{n}{n-1}s_n^2$

Jim is a bird watcher and is trying to model the number of birds that fly past his window.

Jim uses the distribution Po(1) to model the number of birds that fly past his window in a 10-minute period.

(c) Use Jim's model to calculate the probability that:

(i) exactly two birds fly past Jim's window in a 30-minute period

(ii) fewer than two birds fly past Jim's window in a 1-minute period

(iii) at least four birds fly past Jim's window in a 1-hour period.

[7]

Poisson distribution	$X \sim \mathrm{Po}(m)$
Mean	$\mathrm{E}(X) = m$
Variance	$\mathrm{Var}(X) = m$

c) (i) one per 10 minutes = three per 30 minutes

Let $X \sim \mathrm{Po}(3)$ be the number of birds that fly past Jim's window in 30 minutes.

$P(X=2) = 0.2240418$ from GDC

$\boxed{0.224 \text{ (3 s.f.)}}$

(ii) one per 10 minutes $= \frac{1}{10} = 0.1$ per minute

Let $X \sim \mathrm{Po}(0.1)$ be the number of birds that fly past Jim's window in one minute.

'fewer than 2' = '$X<2$' = '$0 \le X \le 1$' = '$X \le 1$'

$P(X \le 1) = 0.99532116$ from GDC

$\boxed{0.995 \text{ (3 s.f.)}}$

(iii) one per 10 minutes = six per hour

Let $X \sim \mathrm{Po}(6)$ be the number of birds that fly past Jim's window in one hour.

'at least 4' = '$X \ge 4$'

$P(X \ge 4) = 0.84879611$ from GDC

$\boxed{0.849 \text{ (3 s.f.)}}$

Question 8

Roberto orders a pizza from Pizza Palace and asks for two types of meat toppings: ham and salami. It is known that the number of pieces of ham that Pizza Palace put on their pizzas follows a Poisson distribution with a mean of 6.2 pieces per pizza. It is also known that the number of pieces of salami on their pizzas follows a Poisson distribution with a mean of 4.9 pieces per pizza. The ham and salami are put on the pizza independently.

(a) Write down the distribution that can be used to model the total number of pieces of meat on Roberto's pizza.

[1]

(b) Find the probability that Roberto's pizza contains a total of exactly 10 pieces of meat.

[2]

(c) Find the probability that Roberto's pizza contains more than 9 but fewer than 13 pieces of meat.

[3]

Poisson distribution	$X \sim \mathrm{Po}(m)$
Mean	$\mathrm{E}(X) = m$
Variance	$\mathrm{Var}(X) = m$

If $X \sim \mathrm{Po}(m_x)$ and $Y \sim \mathrm{Po}(m_y)$ are independent random variables with the Poisson distribution, then $X+Y$ is a random variable with the $\mathrm{Po}(m_x + m_y)$ distribution.

a) $6.2 + 4.9 = 11.1$

$\boxed{\mathrm{Po}\ (11.1)}$

Roberto orders a pizza from Pizza Palace and asks for two types of meat toppings: ham and salami. It is known that the number of pieces of ham that Pizza Palace put on their pizzas follows a Poisson distribution with a mean of 6.2 pieces per pizza. It is also known that the number of pieces of salami on their pizzas follows a Poisson distribution with a mean of 4.9 pieces per pizza. The ham and salami are put on the pizza independently.

(a) Write down the distribution that can be used to model the total number of pieces of meat on Roberto's pizza.

$\boxed{Po(11.1)}$

[1]

(b) Find the probability that Roberto's pizza contains a total of exactly 10 pieces of meat.

[2]

(c) Find the probability that Roberto's pizza contains more than 9 but fewer than 13 pieces of meat.

[3]

Poisson distribution	$X \sim \text{Po}(m)$
Mean	$\text{E}(X) = m$
Variance	$\text{Var}(X) = m$

b) Let $X \sim Po(11.1)$ be the number of pieces of meat on Roberto's pizza.

$P(X = 10) = 0.11824914$

$\boxed{0.118 \text{ (3 s.f.)}}$

Roberto orders a pizza from Pizza Palace and asks for two types of meat toppings: ham and salami. It is known that the number of pieces of ham that Pizza Palace put on their pizzas follows a Poisson distribution with a mean of 6.2 pieces per pizza. It is also known that the number of pieces of salami on their pizzas follows a Poisson distribution with a mean of 4.9 pieces per pizza. The ham and salami are put on the pizza independently.

(a) Write down the distribution that can be used to model the total number of pieces of meat on Roberto's pizza.

$\boxed{Po(11.1)}$

[1]

(b) Find the probability that Roberto's pizza contains a total of exactly 10 pieces of meat.

[2]

(c) Find the probability that Roberto's pizza contains more than 9 but fewer than 13 pieces of meat.

[3]

Poisson distribution	$X \sim \text{Po}(m)$
Mean	$\text{E}(X) = m$
Variance	$\text{Var}(X) = m$

c) Let $X \sim Po(11.1)$ be the number of pieces of meat on Roberto's pizza.

'more than 9 but fewer than 13' = '$10 \leq X \leq 12$'

$P(10 \leq X \leq 12) = 0.3479481$ from GDC

$\boxed{0.348 \text{ (3 s.f.)}}$

Question 9

André has is a keen amateur astronomer who spends his nights with a telescope trying to discover new comets. Based on his past record of success, the number of times per year that André makes a new discovery may be modelled as a Poisson distribution with mean 0.6.

(a) Use the model to find the probability that André makes exactly one new discovery in any given year.

[1]

(b) Over the course of three consecutive years, find the probability that André

(i) makes exactly two new discoveries

(ii) makes new discoveries in the second and third years only

[5]

André's partner Boglárka is a keen amateur entomologist who spends her spare time trying to discover new species of insects. Based on her past record of success, the number of times per year that Boglárka makes a new discovery may be modelled as a Poisson distribution with mean 1.3.

(c) Find the probability that, between them, André and Boglárka make at least one new discovery over a 3-month period, specifying any assumptions you make.

[5]

(d) Find the probability that over a period of 12 years there will be exactly 4 years during which neither André nor Boglárka make a new discovery.

[4]

a) Let $X \sim Po(0.6)$ be the number of new discoveries that André makes in a year.

$P(X=1) = 0.32928698$ from GDC

$\boxed{0.329 \text{ (3 s.f.)}}$

Poisson distribution	$X \sim \mathrm{Po}(m)$
Mean	$\mathrm{E}(X) = m$
Variance	$\mathrm{Var}(X) = m$

André has is a keen amateur astronomer who spends his nights with a telescope trying to discover new comets. Based on his past record of success, the number of times per year that André makes a new discovery may be modelled as a Poisson distribution with mean 0.6.

(a) Use the model to find the probability that André makes exactly one new discovery in any given year.

[1]

(b) Over the course of three consecutive years, find the probability that André

(i) makes exactly two new discoveries

(ii) makes new discoveries in the second and third years only

[5]

André's partner Boglárka is a keen amateur entomologist who spends her spare time trying to discover new species of insects. Based on her past record of success, the number of times per year that Boglárka makes a new discovery may be modelled as a Poisson distribution with mean 1.3.

(c) Find the probability that, between them, André and Boglárka make at least one new discovery over a 3-month period, specifying any assumptions you make.

[5]

(d) Find the probability that over a period of 12 years there will be exactly 4 years during which neither André nor Boglárka make a new discovery.

[4]

b) (i) $3 \times 0.6 = 1.8$

Let $X \sim Po(1.8)$ be the number of new discoveries that André makes in 3 years.

$P(X=2) = 0.26778419$ from GDC

$\boxed{0.268 \text{ (3 s.f.)}}$

(ii) Let $X \sim Po(0.6)$ be the number of new discoveries that André makes in a year.

'makes new discoveries' = '$X \neq 0$' = '$X \geq 1$'

The probability is:

$P(X=0) \times P(X \geq 1) \times P(X \geq 1)$

$= 0.54881163 \times 0.45118836 \times 0.45118836$ from GDC

$= 0.111722\ldots$ $\boxed{0.112 \text{ (3 s.f.)}}$

Poisson distribution	$X \sim \mathrm{Po}(m)$
Mean	$\mathrm{E}(X) = m$
Variance	$\mathrm{Var}(X) = m$

André has is a keen amateur astronomer who spends his nights with a telescope trying to discover new comets. Based on his past record of success, the number of times per year that André makes a new discovery may be modelled as a Poisson distribution with mean 0.6.

(a) Use the model to find the probability that André makes exactly one new discovery in any given year.

[1]

(b) Over the course of three consecutive years, find the probability that André

(i) makes exactly two new discoveries

(ii) makes new discoveries in the second and third years only

[5]

André's partner Boglárka is a keen amateur entomologist who spends her spare time trying to discover new species of insects. Based on her past record of success, the number of times per year that Boglárka makes a new discovery may be modelled as a Poisson distribution with mean 1.3.

(c) Find the probability that, between them, André and Boglárka make at least one new discovery over a 3-month period, specifying any assumptions you make.

[5]

(d) Find the probability that over a period of 12 years there will be exactly 4 years during which neither André nor Boglárka make a new discovery.

[4]

If $X \sim Po(m_x)$ and $Y \sim Po(m_y)$ are independent random variables with the Poisson distribution, then $X+Y$ is a random variable with the $Po(m_x + m_y)$ distribution.

c) $0.6 + 1.3 = 1.9$ new discoveries per year

$1.9 \times \frac{3}{12} = 0.475$ new discoveries per 3 months

Let $X \sim Po(0.475)$ be the number of new discoveries that André and Boglárka make in 3 months.

$P(X \geq 1) = 0.37811494$ from GDC

$\boxed{0.378 \text{ (3 s.f.)}}$

Assumptions: ① Their discoveries are independent of each other. ② 'Three months' is exactly $\frac{3}{12}$ of a year.

Poisson distribution	$X \sim \text{Po}(m)$
Mean	$\text{E}(X) = m$
Variance	$\text{Var}(X) = m$

André has is a keen amateur astronomer who spends his nights with a telescope trying to discover new comets. Based on his past record of success, the number of times per year that André makes a new discovery may be modelled as a Poisson distribution with mean 0.6.

(a) Use the model to find the probability that André makes exactly one new discovery in any given year.

[1]

(b) Over the course of three consecutive years, find the probability that André

(i) makes exactly two new discoveries

(ii) makes new discoveries in the second and third years only

[5]

André's partner Boglárka is a keen amateur entomologist who spends her spare time trying to discover new species of insects. Based on her past record of success, the number of times per year that Boglárka makes a new discovery may be modelled as a Poisson distribution with mean 1.3.

(c) Find the probability that, between them, André and Boglárka make at least one new discovery over a 3-month period, specifying any assumptions you make.

[5]

(d) Find the probability that over a period of 12 years there will be exactly 4 years during which neither André nor Boglárka make a new discovery.

[4]

If $X \sim Po(m_x)$ and $Y \sim Po(m_y)$ are independent random variables with the Poisson distribution, then $X+Y$ is a random variable with the $Po(m_x + m_y)$ distribution.

d) $0.6 + 1.3 = 1.9$ new discoveries per year

Let $X \sim Po(1.9)$ be the number of new discoveries that André and Boglárka make in a year.

$P(X = 0) = 0.14956861$ from GDC

Let Y be the number of years out of 12 in which they make no new discoveries. Then $Y \sim B(12, 0.14956861)$.

$P(Y = 4) = 0.06777683$ from GDC

$\boxed{0.0678 \text{ (3 s.f.)}}$

Poisson distribution	$X \sim \text{Po}(m)$
Mean	$\text{E}(X) = m$
Variance	$\text{Var}(X) = m$

Binomial distribution	$X \sim \text{B}(n, p)$
Mean	$\text{E}(X) = np$
Variance	$\text{Var}(X) = np(1-p)$