

Markscheme

November 2023

Physics

Higher level

Paper 3

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Subject Details: Physics HL Paper 3 Markscheme

Candidates are required to answer **all** questions in Section A and **all** questions from **one** option in Section B. Maximum total = **45 marks**.

1. Each row in the “Question” column relates to the smallest subpart of the question.
2. The maximum mark for each question subpart is indicated in the “Total” column.
3. Each marking point in the “Answers” column is shown by means of a tick (✓) at the end of the marking point.
4. A question subpart may have more marking points than the total allows. This will be indicated by “**max**” written after the mark in the “Total” column. The related rubric, if necessary, will be outlined in the “Notes” column.
5. An alternative wording is indicated in the “Answers” column by a slash (/). Either wording can be accepted.
6. An alternative answer is indicated in the “Answers” column by “**OR**”. Either answer can be accepted.
7. An alternative markscheme is indicated in the “Answers” column under heading **ALTERNATIVE 1** etc. Either alternative can be accepted.
8. Words inside chevrons « » in the “Answers” column are not necessary to gain the mark.
9. Words that are underlined are essential for the mark.
10. The order of marking points does not have to be as in the “Answers” column, unless stated otherwise in the “Notes” column.
11. If the candidate’s answer has the same “meaning” or can be clearly interpreted as being of equivalent significance, detail and validity as that in the “Answers” column then award the mark. Where this point is considered to be particularly relevant in a question it is emphasized by **OWTTE** (or words to that effect) in the “Notes” column.
12. Remember that many candidates are writing in a second language. Effective communication is more important than grammatical accuracy.
13. Occasionally, a part of a question may require an answer that is required for subsequent marking points. If an error is made in the first marking point then it should be penalized. However, if the incorrect answer is used correctly in subsequent marking points then **follow through** marks should be awarded. When marking, indicate this by adding **ECF** (error carried forward) on the script. “ECF acceptable” will be displayed in the “Notes” column.
14. Do **not** penalize candidates for errors in units or significant figures, **unless** it is specifically referred to in the “Notes” column.

Section A

Question			Answers	Notes	Total
1.	a	i	smooth curve through all error bars ✓	<i>Curve can start at $t = 0.5$ min or before.</i>	1
1.	a	ii	17 (± 1.5) «°C» ✓	<i>If candidates did not score in a) i) but showed a y-intercept, allow ECF from their graph.</i>	1
1.	b		there are heat losses OR block radiates/loses thermal energy ✓ at a rate that equals the power of the heater ✓	<i>Do not accept any reference to thermal equilibrium, unless clearly explained as the balance between energy coming in and out of the block.</i>	2
1.	c		evidence of tangent line at $t = 0$ OR use of the curve in its initial part of up to 0.75 min ✓ correct slope calculation ✓	<i>Ignore the sign.</i> <i>For MP2, allow the use of any two points on the curve other than the ones indicated above, for [1] max.</i>	2

1.	d		$52 = 0.85 \times c \times 4 \checkmark$ Converts to seconds to get $c = \llcorner 917.6 = \gg 920 \llcorner \text{J kg}^{-1} \text{K}^{-1} \gg \checkmark$	Allow ECF from c) Allow [1] max for a calculation as in MP1 using a different but consistent set of values, i.e., a <i>T</i> difference in the heating process that corresponds to a thermal energy provided for that difference.	2
1.	e	i	percentage uncertainty in <i>c</i> is 8 % \checkmark $\Delta c = 917.6 \times 0.08 = 73 \llcorner \text{J kg}^{-1} \text{K}^{-1} \gg \checkmark$	Allow ECF from d) for MP1 , if candidates did not use the rate of cooling, accepting either 8% or 4%. Allow ECF from (d) for MP2 . Allow ECF from MP1 .	2
1.	e	ii	$c = (9.2 \pm 0.7) \times 10^2 \text{ J kg}^{-1} \text{K}^{-1} \checkmark$	Accept 1 or 2 significant figures for <i>c</i> with a 1 s.f. uncertainty OR 3 s.f. for <i>c</i> with a 2 s.f. uncertainty, i.e., consistent expressions. Allow ECF from d) and e) i). Accept an alternative unit, e.g. in g or in °C.	1

Question			Answers	Notes	Total
2.	a		<p>there is background radiation present</p> <p>OR</p> <p>there is a systematic error in the rate counter ✓</p>	<p><i>Award [1] for an explanation that suggests that other radioactive elements of higher half-life could be present in the sample.</i></p>	1
2.	b		<p>realization that 2 «Bq» is the background activity ✓</p> <p>calculation, reading or work on graph that shows understanding of half-life ✓</p> <p>half-life = 70 (±5) «s» ✓</p>	<p><i>Allow [2] max for a correct calculation of a half-life if MP1 is not scored, range accepted 65 to 85.</i></p> <p><i>Award MP2 for the correct use of any number of half-lives.</i></p> <p><i>For MP3, accept answers in the range 65 «s» to 75 «s»</i></p>	3

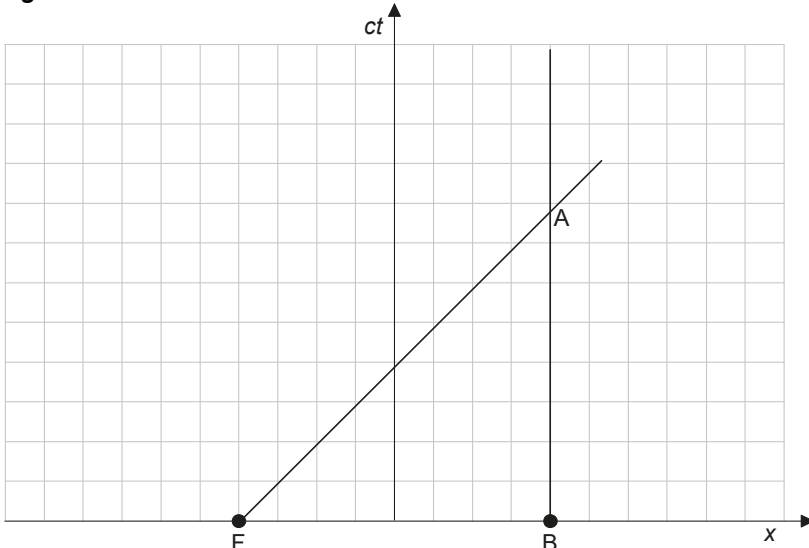
Section B

Option A — Relativity

Question			Answers	Notes	Total
3.	a	i	a moving muon creates a magnetic field «at the position of the other muon» ✓		1
3.	a	ii	the net force must be repulsive in all frames ✓ hence the electric force is greater than the magnetic force ✓	<i>MP2 requires MP1.</i>	2
3.	b		muons decay into electrons / muons have short lifetime ✓ without relativity few muons would reach the ground ✓ experiments show the arrival of more muons than expected ✓ therefore the lifetime of the muons has been increased «in the ground frame» ✓ «so time dilation has occurred»	<i>Accept lifetime / half-life</i>	3 max

4.		<p>ALTERNATIVE 1</p> <p>the clock at a distance d from the origin «is set ahead by an amount equal to $\frac{d}{c}$» ✓</p> <p>a light signal leaves the origin «when the origin clock reads 0» ✓</p> <p>when the signal arrives at the clock the clock is started ✓</p> <p>ALTERNATIVE 2</p> <p>both clocks are set to read zero ✓</p> <p>a light signal is emitted from the midpoint of the two clocks ✓</p> <p>when signals arrive at the clocks the clocks are started ✓</p> <p>ALTERNATIVE 3</p> <p>both clocks are set to read zero together ✓</p> <p>one is moved to d ✓</p> <p>at a very slowly /negligible velocity✓</p>		3
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Question			Answers	Notes	Total
5.	a	i	$\ll \frac{12.0}{0.800} = \gg 15.0 \text{ « yrs » } \checkmark$		1
5.	a	ii	$\gamma = \frac{1}{\sqrt{1-0.8^2}} \text{ OR } \gamma = \frac{5}{3} \checkmark$ $t = \frac{15}{\gamma} = 9.00 \text{ « yrs » } \checkmark$	Allow ECF from MP1	2
5.	b	i	the length of an object in the object's rest frame \checkmark		1
5.	b	ii	$\frac{-0.800c - 0.600c}{1 + \frac{0.800c \times 0.600c}{c^2}} \checkmark$ $\ll = 0.946c \gg$	Correct substitution Allow positive values in the numerator	1
5.	b	iii	$\gamma = \frac{1}{\sqrt{1-0.946^2}} \text{ OR } \gamma = 3.08 \checkmark$ $L = \ll \frac{992}{\gamma} = \gg 322 \text{ « m » } \checkmark$	Allow ECF from MP1	2

5.	c	i	45 degree line to the right from F ✓ intersecting vertical line from B ✓	<p>eg:</p> 	2
5.	c	ii	$\Delta t' = 3.08 \times \left(\frac{992}{c} + \frac{0.946 \times 992}{c} \right) \checkmark$ $= 1.98 \times 10^{-5} \text{ «s» } \checkmark$		2

Question			Answers	Notes	Total
6.	a		$E = \sqrt{0.938^2 + 2.5^2} \checkmark$ Subtracts rest energy and states that PD is 1.73 «GV» \checkmark	Accept $2.77 \times 10^{-10} \text{J}$ for MP1	2
6.	b		$\gamma = 2.85 \checkmark$ $v = 0.936c \checkmark$	Accept ECF from MP1 Allow $\gamma = 2.84$ from different rounding	2

Question			Answers	Notes	Total
7.	a		the massive galaxy bends/curves/warps spacetime in its vicinity ✓ light from the bright galaxy bends around massive galaxy ✓ extensions of lights rays arriving at Earth give multiple images of galaxy ✓	Accept labelled diagram for MP2 Accept labelled diagram for MP3	3
7.	b		area around black hole where escape speed is the speed of light OR area from within which nothing can be communicated to the outside OR area from which not even light can escape ✓		1
7.	c		$\Delta t = \frac{1.0}{\sqrt{1 - \frac{R}{1.25R}}} \checkmark$ $\Delta t = 2.24 \approx 2.2 \text{ «s» } \checkmark$		2

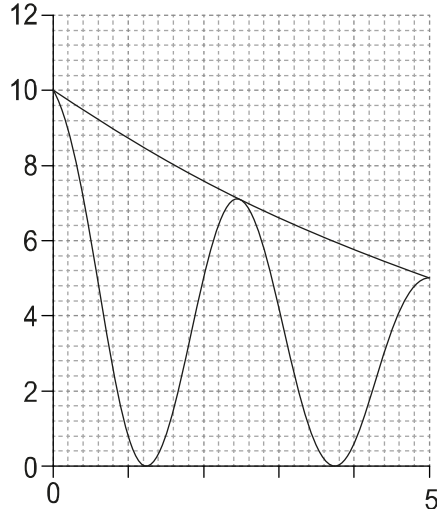
Option B — Engineering physics

Question			Answers	Notes	Total
8.	a		$\alpha = \frac{12.5}{2.0}$ OR 6.25 «rad s ⁻² » ✓	Allow the use of any point from the graph.	1
8.	b		ALTERNATIVE 1 $I = \frac{1}{2} \times 0.2 \times 0.4^2 = \ll 0.016 \gg$ ✓ Torque = «I x α =» 0.1 «Nm» ✓ ALTERNATIVE 2 $\Delta L = \frac{1}{2} \times 0.20 \times 0.40^2 \times 12.5 = 0.20$ «Js» ✓ $\Gamma = \frac{\Delta L}{\Delta t} = \frac{\frac{1}{2} \times 0.20 \times 0.40^2 \times 12.5}{2.0} = 0.10$ «Nm» ✓	Use of 6 gives an answer of 0.096 Nm. Allow ECF from MP1	2
8.	c	i	initial momentum of block = 0.016 x 12.5 OR 0.20 ✓ moment of inertia of the object is 0.10 x 0.40 ² ✓ conservation of angular momentum $0.20 = \left(\frac{1}{2} \times 0.20 \times 0.40^2 + 0.10 \times 0.40^2 \right) \omega'$ so $\omega' = 6.25$ «rad s ⁻¹ » ✓	Allow ECF for MP3 Allow ECF from 8) b)	3
8.	c	ii	initial KE is $\frac{1}{4} \times 0.20 \times 0.40^2 \times 12.5^2$ OR = 1.25 «J» ✓ final KE is $\frac{1}{4} \times 0.20 \times 0.40^2 \times 6.25^2 + \frac{1}{2} \times 0.10 \times 0.40^2 \times 6.25^2 = 0.625$ «J» ✓ fraction lost is 50 % ✓	Allow ECF from c) i) Allow ECF for MP3	3

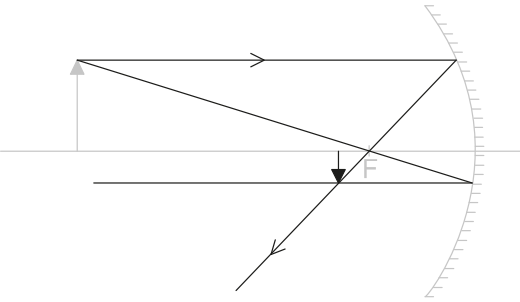
Question			Answers	Notes	Total
9.	a	i	$V = \frac{6.00 \times 10^{-4} \times 8.31 \times 712}{1.00 \times 10^5} \llcorner = 3.55 \times 10^{-5} \text{ m}^3 \llcorner \checkmark$ <p>quotes $p_1 V_1^{5/3} = p_2 V_2^{5/3}$ or $p V^{5/3} = \text{constant}$ with at least one substitution correct \checkmark</p> $p = \llcorner \frac{1 \times 10^5 \times (3.55 \times 10^{-5})^{5/3}}{(8 \times 10^{-5})^{5/3}} \llcorner = 25800 \text{ Pa } \checkmark$	Award [3] for a BCA	3
9.	a	ii	$T = \frac{25800 \times 8 \times 10^{-5}}{6 \times 10^{-4} \times 8.31} \text{ OR } T = 414 \text{ «K» } \checkmark$ <p>Q = 0 OR W = - ΔU \checkmark</p> $W = \llcorner \frac{3}{2} n R \Delta T = -\frac{3}{2} \times 6 \times 10^{-4} \times 8.31 \times (414 - 712) \Rightarrow 2.23 \text{ «J» } \checkmark$	<p>MP2 seen or implied in MP3.</p> <p>Ignore sign</p> <p>Award [3] for a BCA</p>	3

9.	b	i	<p>ALTERNATIVE 1</p> $Q = p\Delta V + \frac{3}{2}p\Delta V = \frac{5}{2}p\Delta V \checkmark$ $Q = \frac{5}{2} \times 25800 \times (3.55 - 8) \times 10^{-5} \checkmark$ $Q = -2.87 \text{ «J» } \checkmark$ <p>ALTERNATIVE 2</p> $T = \frac{25800 \times 3.55 \times 10^{-5}}{6 \times 10^{-4} \times 8.31} \text{ OR } 184 \text{ «K» } \checkmark$ $Q = \text{«}W + \Delta U \text{»} = 25800 \times (3.55 - 8) \times 10^{-5} + \frac{3}{2} \times 6 \times 10^{-4} \times 8.31 \times (184 - 414) \checkmark$ $Q = -2.87 \text{ «J» } \checkmark$	<p>MP1 seen or implied in MP2.</p> <p>Ignore sign</p> <p>Allow ECF from 9 a) i) and 9 a) ii)</p> <p>Award [3] for a BCA</p>	3
9.	b	ii	$\text{efficiency} = \text{«} \frac{3.9 - 2.87}{3.9} \text{»} = 0.26 \text{ OR } 26\% \checkmark$	Allow ECF from (b)(i)	1
9.	b	iii	<p>so that minimum energy is wasted/ejected</p> <p>OR</p> <p>useful work as close as possible to energy input</p> <p>OR</p> <p>beneficial for the environment / to the availability of resources</p> <p>\checkmark</p>	Allow to obtain maximum power possible	1

Question			Answers	Notes	Total
10.	a		$v_2 = \sqrt{2g\Delta h + v_1^2} = \sqrt{2 \times 9.8 \times 0.20 + 1.2^2} \checkmark$ $2.3 \text{ m s}^{-1} \checkmark$		2
10.	b		the continuity equation applies / $Av = \text{constant} \checkmark$ v increases \checkmark so area must decrease \checkmark «so diameter decreases»		3

Question			Answers	Notes	Total
11.	a	i	« $Q = 2\pi \times \frac{10}{5} = \rangle 4\pi \approx 13 \checkmark$		1
11.	a	ii	first maximum at $t=0$, second maximum at $t=2.5$ and third maximum at $t=5$ s \checkmark maxima must touch the other curve and minima the time axis \checkmark	 <p>Allow [1] for half the cycle drawn over the 5 seconds</p>	2
11.	b		because structures will develop large amplitudes / close to the natural frequency \checkmark these can lead to collapse of the structure \checkmark		2

Option C — Imaging

Question			Answers	Notes	Total
12.	a		first ray ✓ second ray and position of image ✓		2
12.	b		image moves towards F/goes to F OR light rays now come from far away / «near» infinity ✓ magnification « tends to $\frac{f}{u}$ which » tends to 0 ✓	<i>Award [1 max] for a response of magnification/image gets smaller.</i>	2

Question			Answers	Notes	Total
13.	a		<p>too difficult to make «a mirror of such large diameter»</p> <p>OR</p> <p>many small panels can form a curved surface</p> <p>OR</p> <p>surface of reflector does not have to be smooth due to the long radio wavelength ✓</p>		1
13.	b		<p>far away radio sources are focused at different positions «above mirror» ✓</p> <p>so antenna is moved to that position «to collect signal» ✓</p>		2
13.	c		<p>avoids absorption «of certain wavelengths» by the atmosphere ✓</p> <p>the refractive index of the atmosphere varies making images blurry ✓</p> <p>convection currents/atmosphere turbulence makes images blurry ✓</p> <p>there is no light pollution «for satellite-based telescopes» ✓</p> <p>«a telescope in space» can operate day and night ✓</p>		2 max

Question			Answers	Notes	Total
14.	a	i	$\left\langle \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{10} - \frac{1}{10.6} \right\rangle v = 177 \text{ «mm» } \checkmark$		1
14.	a	ii	$m_o = -\frac{177}{10.6} \text{ OR } m_e = 1 + \frac{250}{50} \checkmark$ $M \ll m_o \times m_e \gg = 100$	<p>Allow ECF from a) i) Award [2] for a BCA</p> <p>Allow ECF from MP1 if candidates multiply two magnifications.</p>	2
14.	b		$\left\langle \frac{1}{u} = \frac{1}{f} - \frac{1}{v} = \frac{1}{50} - \frac{1}{-250} \right\rangle u = 41.7 \text{ «mm» } \checkmark$ $d = \ll v_o + u_e = 41.7 + 177 \Rightarrow 218 \text{ «mm» } \checkmark$	<p>Allow ECF from 14 a) Allow ECF from MP1 Accept 219 «mm»</p> <p>MP1 can be present in 14 a) ii)</p>	2

Question			Answers	Notes	Total
15.	a	i	$\theta_c = \sin^{-1}\left(\frac{1.45}{1.47}\right) \approx 80.5^\circ \checkmark$		1
15.	a	ii	$-5.0 = 10 \times \log\left(\frac{I}{I_0}\right) \Rightarrow I = I_0 \times 10^{-\frac{5}{10}} \checkmark$ $I = 0.0038 \text{ « W »} \approx 3.8 \text{ « mW »} \checkmark$	Award [2] for a BCA	2
15.	b		monochromatic consists of only one wavelength / frequency \checkmark «material dispersion means that» signals of different wavelengths travel at different speeds \checkmark so they arrive at different times \checkmark producing a degradation/distortion/garbling the signal / monochromatic produces a clearer signal \checkmark		3 max

Question			Answers	Notes	Total
16.	a	i	an alternating potential difference is applied to a crystal/quartz ✓ which is forced to vibrate «producing ultrasound» ✓		2
16.	a	ii	an A scan is one dimensional «series of signals reflected from various boundaries» ✓ a B scan is a superposition of many A scans / a 2D image ✓ identifies A-scan with amplitude and B-scan with brightness ✓		2 max
16.	b	i	fraction transmitted at skin–tissue boundary is $1 - R_{st}$ ✓ fraction reflected from tissue–liver boundary is $R_{tl} \times (1 - R_{st})$ ✓ fraction received at transducer is $= R_t \times (1 - R_{st})^2 = 5.62 \times 10^{-6}$ ✓	Award [3] for a bald correct answer Award [2 max] for an answer of 5.66×10^{-6} as it indicates the final transmission at skin has been omitted	3
16.	b	ii	wavelength $= \frac{5.0 \times 10^{-2}}{250} = 2.0 \times 10^{-4}$ «m» ✓ frequency $= \frac{c}{\lambda} = \frac{1500}{2.0 \times 10^{-4}} = 7.5 \times 10^6$ «Hz» ✓	Award [2] for a BCA	2
16.	c		no exposure to ionizing/harmful radiation ✓ not intrusive procedure ✓		1 max

Option D — Astrophysics

Question			Answers	Notes	Total
17.	a	i	the angle at which the star subtends 1 AU OR half the angle between star and Earth measured six months apart ✓	Accept a sketch that identifies the correct angle, i.e., half the angle subtended from star to Earth at both sides of Sun.	1
17.	a	ii	$d = \frac{1}{0.131} \times 3.26$ OR $d = 24.9 \text{ «ly»}$ ✓		1
17.	b	i	$\frac{b_V}{b_\beta} = \ll \frac{L_V}{L_\beta} \left(\frac{d_\beta}{d_V} \right)^2 = \gg \frac{54}{40000} \left(\frac{780}{25} \right)^2 =$ OR calculates one brightness in terms of L_\odot ✓ $\frac{b_V}{b_\beta} = 1.3$ OR calculates a second brightness in terms of L_\odot ✓ so Vega appears brighter ✓	Award [0] for a bald correct answer.	3

17.	b	ii	$\frac{L_{\beta}}{L_v} = \frac{40000}{54} = \ll \left(\frac{R_{\beta}}{R_v}\right)^2 \left(\frac{11000}{9600}\right)^4 = \gg \checkmark$ $\frac{R_{\beta}}{R_v} = \sqrt[4]{\frac{40000}{54} \left(\frac{9600}{11000}\right)^4} \checkmark$ $= 21 \checkmark$		3
17.	c	i	$\frac{M_v}{M_{\odot}} = 54^{\frac{1}{3.5}} \text{ OR } \frac{M_v}{M_{\odot}} = 3.1 \checkmark$		1
17.	c	ii	luminosity of β Ori is too high AND temperature is comparable to Vega / mass «if main sequence would be» 20.6 M_{\odot} \checkmark so it cannot be a main sequence star \checkmark	MP2 requires MP1. <i>If AND not present, i.e., if only one of the three features is mentioned, with conclusion present, award [1] max.</i>	2
17.	c	iii	Vega is a low mass star / remnant will be lower than Chandrasekhar limit / it will go through red giant phase \checkmark will evolve to become a white dwarf \checkmark a very small/dense/hot/dead/low luminosity star \checkmark		2 max

Question			Answers	Notes	Total
18.	a		black-body radiation ✓ filling the universe ✓ «highly» isotropic ✓ originated when radiation decoupled from matter/the early universe ✓	<i>Allow radiation from the Big Bang for MP4</i>	2 max
18.	b		$\lambda = \ll \frac{2.9 \times 10^{-3}}{2.8} = \gg = 1.0 \times 10^{-3} \ll m \gg$ ✓		1
18.	c	i	a measure of the size of the universe ✓ a parameter that describes the expansion of the universe ✓ a scale factor at a given time ✓ compares physical distances at another time ✓	<i>Allow in terms of a scale factor or in terms of ratios.</i>	2 max
18.	c	ii	correctly relates wavelength to scale factor «so wavelength was 1100 times smaller at emission» ✓ correctly relates wavelength to temperature «so T must have been 1100 times higher» ✓		2

Question			Answers	Notes	Total
19.	a	i	«need of overcoming higher electrostatic forces so» higher temperatures/densities are required ✓ high mass stars have greater pressure «at their cores and so higher temperatures/densities» ✓	<i>Allow [1] mark for an answer that refers to a higher neutron flux in high mass stars.</i>	2
19.	a	ii	because iron is near the peak of the binding energy curve / most stable nucleus OR because energy would have to be supplied to produce anything heavier than iron /energy cannot be released by further fusions ✓	<i>Accept element instead of nucleus</i>	1
19.	b		leads to elements with higher mass number / isotopes / reference to s or r processes ✓ mention of beta decay OR «further decay» leads to elements with higher atomic number ✓		2

Question			Answers	Notes	Total
20.	a	i	<p>ALTERNATIVE 1</p> <p>total energy of m is $E = \frac{1}{2}mv^2 - \frac{G \times \frac{4}{3} \times \pi r^3 \rho}{r}$ ✓</p> <p>sets $E = 0$ and replaces $v = H_0 r$ ✓</p> <p>«to get result»</p> <p>ALTERNATIVE 2</p> <p>uses escape velocity $v = \sqrt{\frac{2GM}{r}}$ and $M = \frac{4}{3} \pi r^3 \rho$ ✓</p> <p>replaces $v = H_0 r$ ✓</p> <p>«to get result»</p>		2
20.	a	ii	$\rho = \left\langle \frac{3 \times \left(\frac{68 \times 10^3}{10^6 \times 3.08 \times 10^{16}} \right)^2}{8\pi \times 6.67 \times 10^{-11}} \right\rangle = 8.7 \times 10^{-27} \approx 9 \times 10^{-27} \text{ «kgm}^{-3}\text{» } \checkmark$		1
20.	b		<p>critical density is used to predict whether the universe is flat or curved / the future of the universe ✓</p> <p>density needs to include «mass and» dark energy ✓</p> <p>relates dark energy to a faster/accelerating rate of expansion ✓</p>	<p><i>Allow any discussion of alternative models of the Universe for MP1</i></p>	2 max