EXAM PAPERS PRACTICE

GCSE OCR Math J560
Perpendicular Lines

Answers

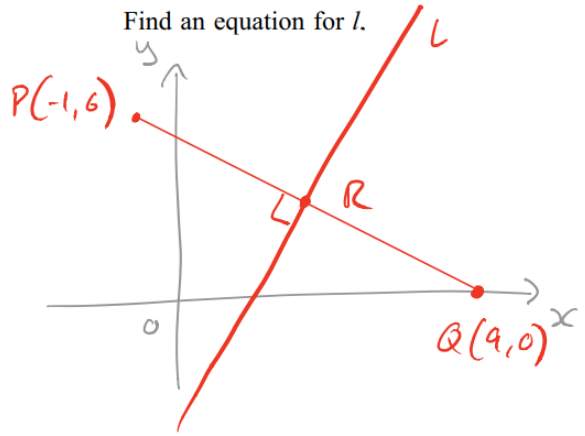
"We will help you to achieve A Star "

Answer 1

The points P and Q have coordinates $(-1, 6)$ and $(9, 0)$ respectively.

The line l is perpendicular to PQ and passes through the mid-point of PQ.

Find an equation for l.



$R\left(\frac{-1+9}{2}, \frac{6+0}{2}\right)$ so $R(4, 3)$

$\text{GRAD}_{PQ} = \frac{0-6}{9-(-1)} = \frac{-6}{10} = \frac{-3}{5}$

$m = \frac{5}{3}$ (SINCE $-\frac{3}{5} \times \frac{5}{3} = -1$)

EQN OF L: $y = \frac{5}{3}x + c$

$R(4,3)$: $3 = \frac{5}{3} \times 4 + c$

$\frac{9}{3} - \frac{20}{3} = \frac{20}{3} - \frac{20}{3} + c$

$-\frac{11}{3} = c$

$L: y = \frac{5}{3}x - \frac{11}{3}$

Mulitiply both sides by 3

$3y = 5x - 11$

rearrange

$5x - 3y - 11 = 0$

EQUATION OF A STRAIGHT LINE

$y = mx + c$

GRADIENT, y-INTERCEPT

MIDPOINT

(AVERAGE POINT) OF (x_1, y_1) AND (x_2, y_2) IS $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

GRADIENT

$m = \frac{\text{RISE}}{\text{RUN}}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

FOR TWO POINTS (x_1, y_1) AND (x_2, y_2)

PERPENDICULAR GRADIENTS MULTIPY TO GIVE -1

Answer 2

The line l_1 has equation $y = -2x + 3$

The line l_2 is perpendicular to l_1 and passes through the point (5, 6).

(a) Find an equation for l_2.

(a) $m_1 = -2$ so $m_2 = \frac{1}{2}$ (since $-2 \times \frac{1}{2} = -1$)

PERPENDICULAR GRADIENTS MULTIPY TO GIVE -1

EQN OF L_2: $y = \frac{1}{2}x + c$

$(5, 6)$: $6 = \frac{1}{2} \times 5 + c$ so $c = \frac{7}{2}$

$-\frac{5}{2}$ $-\frac{5}{2}$

L_2: $y = \frac{1}{2}x + \frac{7}{2}$

EQUATION OF A STRAIGHT LINE

$y = mx + c$

GRADIENT, y-INTERCEPT

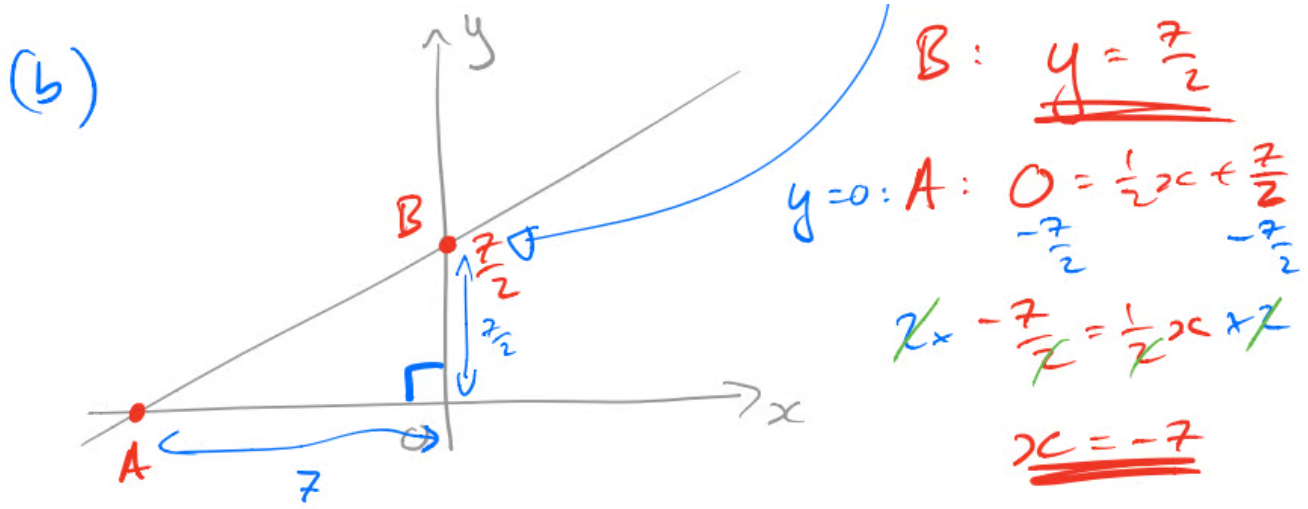
Multiply both sides by 2

$2y = x + 7$

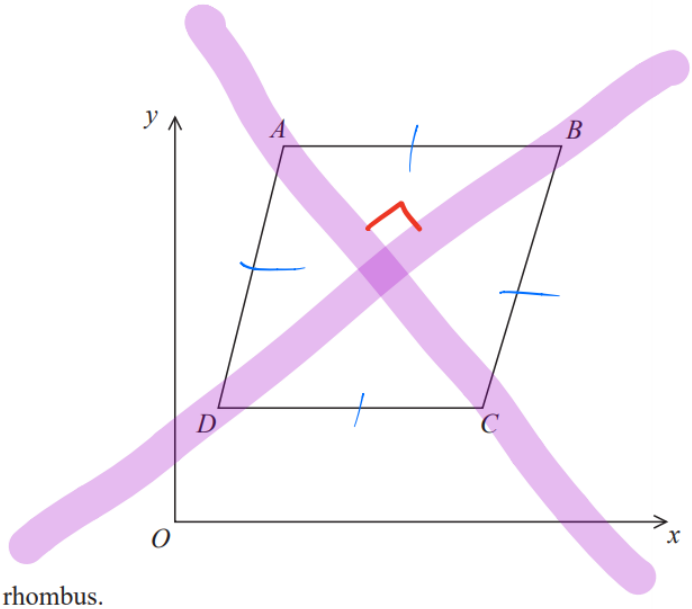
rearrange

$x - 2y + 7 = 0$

Answer 3


(b)
y
x
B
A
7
$\frac{7}{2}$
$\frac{7}{2}$
B: $y = \frac{7}{2}$
$y=0$: A: $0 = \frac{1}{2}x + \frac{7}{2}$
$-\frac{7}{2}$ $-\frac{7}{2}$
$2 \times -\frac{7}{2} = \frac{1}{2}x \times 2$
$x = -7$

Answer 4



ABCD is a rhombus.
The coordinates of *A* are (5, 11)
The equation of the diagonal *DB* is $y = \frac{1}{2}x + 6$

Find an equation of the diagonal *AC*.

DIAGONALS OF A RHOMBUS MEET AT RIGHTANGLES

SO

$m \times \frac{1}{2} \times 2 = -1 \times 2$

$m = -2$

AC: $y = -2x + c$

USE A(5, 11) $11 = -2 \times 5 + c$

$11 = -10 + c$

$+10 \quad +10$

$c = 21$

EQUATION OF A STRAIGHT LINE

$y = mx + c$

m: GRADIENT; c: y-INTERCEPT

PERPENDICULAR GRADIENTS MULTIPY TO GIVE -1

Answer 5

The line l_1 has equation $3x + 5y - 2 = 0$

(a) Find the gradient of l_1.

(a) l_1: $3x + 5y - 2 = 0$

$-3x \quad +2 \quad -3x + 2$

$\frac{5y}{5} = \frac{-3x+2}{5}$

$y = -\frac{3}{5}x + \frac{2}{5}$

So $m_1 = -\frac{3}{5}$

EQUATION OF A STRAIGHT LINE

$y = mx + c$

m: GRADIENT, c: y-INTERCEPT

Answer 6

(b) $m_1 = -\frac{3}{5}$ so $m_2 = \frac{5}{3}$ (since $-\frac{3}{5} \times \frac{5}{3} = -1$)

PERPENDICULAR GRADIENTS MULTIPY TO GIVE -1

EQN OF L_2: $y = \frac{5}{3}x + c$

$(3, 1)$: $1 = \frac{5}{3} \times 3 + c$

$-5 \quad -5$

$-4 = c$

L_2: $y = \frac{5}{3}x - 4$

Answer 7

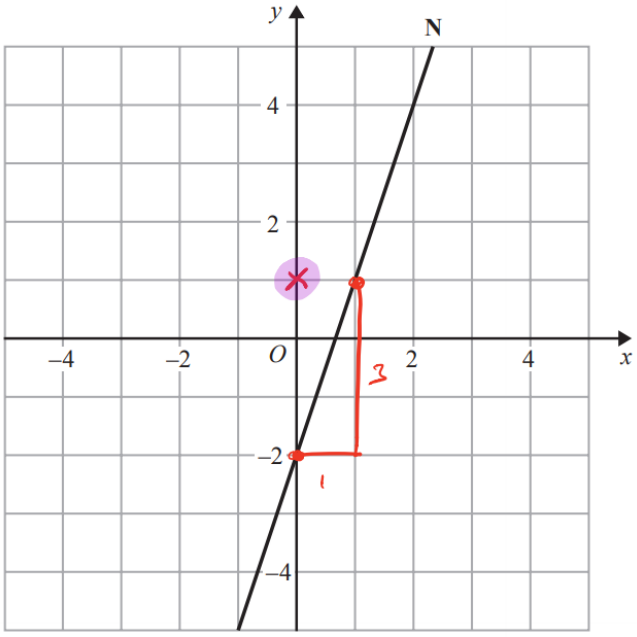
The line **N** is drawn below.

EQUATION OF A STRAIGHT LINE

$y = mx + c$

m: GRADIENT, c: y-INTERCEPT

PERPENDICULAR GRADIENTS MULTIPY TO GIVE -1



GRAD = RISE / RUN

$= \frac{3}{1}$

$= 3$

Find an equation of the line perpendicular to line **N** that passes through the point (0, 1).

GRADIENT OF N = 3

PERPENDICULAR $\Rightarrow \frac{3 \times m}{3} = \frac{-1}{3}$

$m = -\frac{1}{3}$

GOES THRU (0, 1) SO $c = 1$

EQUATION: $y = -\frac{1}{3}x + 1$

Answer 8

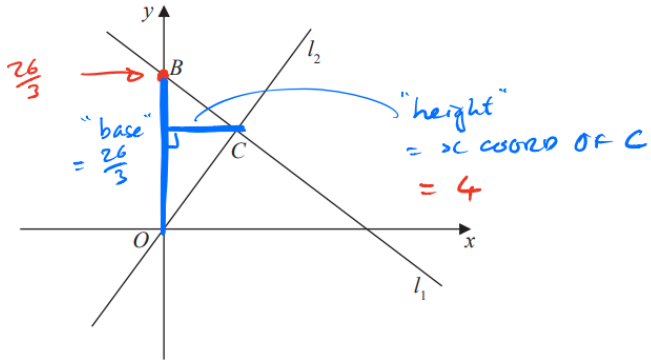


Figure 2

The line l_1, shown in Figure 2 has equation $2x + 3y = 26$

The line l_2 passes through the origin O and is perpendicular to l_1

(a) Find an equation for the line l_2

(a) l_1: $2x + 3y = 26$

$-2x \quad -2x$

$3y = -2x + 26$

$\frac{3y}{3} = \frac{-2x}{3} + \frac{26}{3}$

$y = -\frac{2}{3}x + \frac{26}{3}$

So $m_1 = -\frac{2}{3}$ so $m_2 = \frac{3}{2}$

(since $-\frac{2}{3} \times \frac{3}{2} = -1$)

l_2: $y = \frac{3}{2}x$

$c = 0$ as l_2 passes through $(0, 0)$

Answer 9

The line L has equation $y = 5 - 2x$. → $y = -2x + 5$

(a) Show that the point $P\,(3, -1)$ lies on L.

(a) $x = 3$: $y = -2 \times 3 + 5$

$y = -1$

So $P(3, -1)$ LIES ON L.

Answer 10

(b) $\text{GRAD}_L = -2$

So $m = \frac{1}{2}$ (since $-2 \times \frac{1}{2} = -1$)

EQN: $y = \frac{1}{2}x + c$

P(3, -1): $-1 = \frac{1}{2} \times 3 + c$

$-3/2 \quad -3/2$

$-\frac{5}{2} = c$

EQN: $y = \frac{1}{2}x - \frac{5}{2}$

Multiply both sides by 2

$2y = x - 5$

rearrange

$x - 2y - 5 = 0$

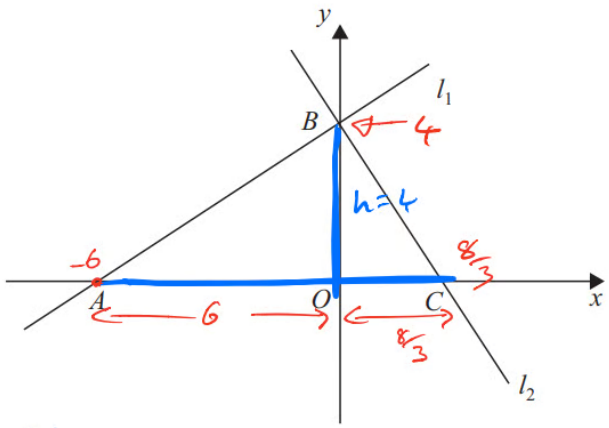
EQUATION OF A STRAIGHT LINE

$y = mx + c$

GRADIENT, y-INTERCEPT

PERPENDICULAR GRADIENTS MULTIPY TO GIVE -1

Answer 11



$b = 6 + \frac{8}{3} = \frac{18}{3} + \frac{8}{3} = \frac{26}{3}$

Figure 1

The line l_1 has equation $2x - 3y + 12 = 0$

(a) Find the gradient of l_1.

$\frac{2x}{3} + \frac{12}{3} = \frac{3y}{3}$ (+3y, +3y)

$y = \frac{2}{3}x + 4$

$m_1 = \frac{2}{3}$

EQUATION OF A STRAIGHT LINE

$y = mx + c$

m: GRADIENT; c: y-INTERCEPT

Answer 12

(b) Find an equation of l_2.

(b) $m_1 = \frac{2}{3}$ so $m_2 = -\frac{3}{2}$ (since $-\frac{3}{2} \times \frac{2}{3} = -1$)

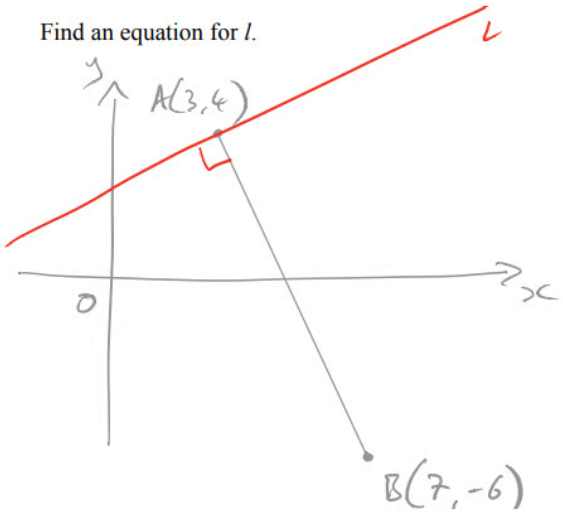
PERPENDICULAR GRADIENTS MULTIPY TO GIVE -1

EQN of L_2: $y = -\frac{3}{2}x + 4$

Answer 13

The points A and B have coordinates (3, 4) and (7, -6) respectively. The straight line l passes through A and is perpendicular to AB.

Find an equation for l.



$\text{GRAD}_{AB} = \frac{-6-4}{7-3} = \frac{-10}{4} = -\frac{5}{2}$

So $m = \frac{2}{5}$ (since $-\frac{5}{2} \times \frac{2}{5} = -1$)

Eqn of l: $y = \frac{2}{5}x + c$

$A(3,4)$: $4 = \frac{2}{5} \times 3 + c$

$-\frac{6}{5} \quad -\frac{6}{5}$

$\frac{14}{5} = c$

So l: $y = \frac{2}{5}x + \frac{14}{5}$ Mulitiply both sides by 5

$5y = 2x + 14$ rearrange

$2x - 5y + 14 = 0$

Equation of a Straight Line

$y = mx + c$

m: Gradient; c: y-intercept

Perpendicular Gradients Multipy To Give -1

Gradient

$m = \frac{\text{Rise}}{\text{Run}}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

For two points (x_1, y_1) and (x_2, y_2)

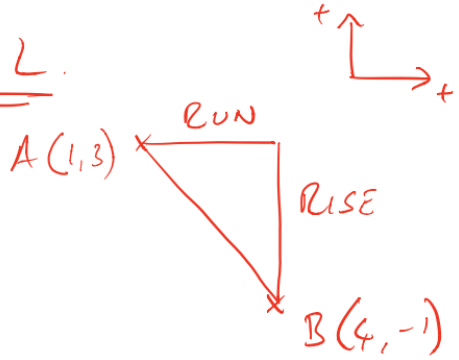
Answer 14

A is the point with coordinates (1, 3)

B is the point with coordinates (4, –1)
The straight line L goes through both A and B.

Is the line with equation $2y = 3x - 4$ perpendicular to line L?
You must show how you got your answer.

LINE L.



$$m = \frac{-1-3}{4-1}$$

$$= \frac{-4}{3}$$

$$2y = 3x - 4$$

$$y = \frac{3}{2}x - 2$$

$$m = \frac{3}{2}$$

MULTIPLY TOGETHER

$$-\frac{4}{3} \times \frac{3}{2} = -2 \neq -1$$ SO NOT PERPENDICULAR

EQUATION OF A STRAIGHT LINE

$y = mx + c$

GRADIENT, Y-INTERCEPT

PERPENDICULAR GRADIENTS MULTIPY TO GIVE -1

GRADIENT

$$m = \frac{\text{RISE}}{\text{RUN}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

FOR TWO POINTS (x_1, y_1) AND (x_2, y_2)

Answer15

The point P has coordinates $(3, 4)$
The point Q has coordinates (a, b)

A line perpendicular to PQ is given by the equation $3x + 2y = 7$

Find an expression for b in terms of a.

$$\text{GRAD}_{PQ} = \frac{b-4}{a-3}$$

$$3x + 2y = 7$$
$$-3x \qquad -3x$$
$$2y = -3x + 7$$
$$\frac{2y}{2} = \frac{-3x}{2} + \frac{7}{2}$$
$$y = -\frac{3}{2}x + \frac{7}{2}$$

$m = -\frac{3}{2}$

$$\text{GRAD}_{PQ} \times -\frac{3}{2} = -1$$

$$\text{GRAD}_{PQ} = \frac{2}{3}$$

So
$$3(a-3)\frac{(b-4)}{a-3} = \frac{2}{3} \times 3(a-3)$$
$$3(b-4) = 2(a-3)$$
$$3b - 12 = 2a - 6$$
$$+12 \qquad +12$$
$$3b = 2a + 6$$
$$\frac{3b}{3} = \frac{2a+6}{3}$$
$$b = \frac{2a+6}{3}$$

EQUATION OF A STRAIGHT LINE

$y = mx + c$

m: GRADIENT, c: y-INTERCEPT

PERPENDICULAR GRADIENTS MULTIPY TO GIVE -1

GRADIENT

$$m = \frac{\text{RISE}}{\text{RUN}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

FOR TWO POINTS (x_1, y_1) AND (x_2, y_2)