Question 1

Consider the letters of the word SUNDAY. Find the number of permutations of four letters that can be chosen if

(i) no restrictions apply

(ii) there must be at least one vowel (A or U)

(iii) both vowels must be chosen and they must be kept together in any permutation.

[6]

Combinations	${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$
Permutations	${}^{n}P_{r} = \frac{n!}{(n-r)!}$

(i) ${}^{6}P_{4} = \frac{6!}{(6-4)!} = \frac{6!}{2!} = \frac{\cancel{2!}\cdot 3\cdot 4\cdot 5\cdot 6}{\cancel{2!}} = \boxed{360}$

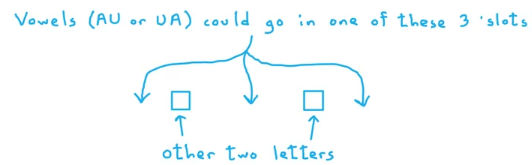
(ii) This means we <u>can't</u> choose S, N, D, Y as the four letters.

$0! = 1$

${}^{4}P_{4} = \frac{4!}{(4-4)!} = \frac{4!}{0!} = 4! = 24$ } number of permutations of S, N, D, Y

$360 - 24 = \boxed{336}$

(iii) There are 2 ways to arrange the vowels (AU or UA), and ${}^{4}P_{2}$ possible permutations of two of the remaining four letters. Finally, there are 3 'slots' into which the vowels can go:



$3 \times 2 \times {}^{4}P_{2} = 3 \times 2 \times \frac{4!}{2!} = 6 \times \frac{\cancel{2!}\cdot 3\cdot 4}{\cancel{2!}} = \boxed{72}$

Question 2

Three letters are chosen at random from the letters in the word AIRFIELD.

(a) Find the number of ways that the selection may contain

(i) no Is

(ii) exactly one I

(iii) two Is.

[4]

(b) Write down the number of arrangements of three letters chosen at random from the word AIRFIELD that have exactly one letter I.

[2]

a) (i) This means choosing three of A, R, F, E, L, D

$^6C_3 = \frac{6!}{3!\,3!} = \frac{\cancel{3!}\cdot 4\cdot 5\cdot \cancel{6}}{\cancel{3!}\,\cancel{3!}} = \boxed{20}$ ($3! = 6$)

(ii) The one I is 'fixed', so this means choosing two of A, R, F, E, L, D

$^6C_2 = \frac{6!}{2!\,4!} = \frac{\cancel{4!}\cdot 5\cdot 6}{2!\,\cancel{4!}} = \frac{30}{2} = \boxed{15}$

(iii) The two I's are 'fixed', so this means choosing one of A, R, F, E, L, D

$^6C_1 = \frac{6!}{5!\,1!} = \frac{\cancel{5!}\cdot 6}{\cancel{5!}\,1!} = \boxed{6}$

Combinations	$^nC_r = \frac{n!}{r!(n-r)!}$
Permutations	$^nP_r = \frac{n!}{(n-r)!}$

Three letters are chosen at random from the letters in the word AIRFIELD.

(a) Find the number of ways that the selection may contain

(i) no Is $\boxed{20}$

(ii) exactly one I $\boxed{15}$

(iii) two Is. $\boxed{6}$

[4]

(b) Write down the number of arrangements of three letters chosen at random from the word AIRFIELD that have exactly one letter I.

[2]

b) Each of the selections in (a)(ii) can be arranged in 3! ways

$15 \times 3! = 15 \times 6 = \boxed{90}$

Combinations	$^nC_r = \frac{n!}{r!(n-r)!}$
Permutations	$^nP_r = \frac{n!}{(n-r)!}$

Question 3

In a maths test students are required to answer four out of seven questions.

(a) Find the number of ways in which the questions can be chosen if there are no restrictions.

[2]

(b) Find the number of ways in which the questions can be chosen if the last question is compulsory.

[3]

(c) Find the number of ways in which the questions can be chosen if the students must do at least 1 of the last two questions.

[3]

a) $^7C_4 = \frac{7!}{4!\,3!} = \frac{\cancel{4!}\cdot 5\cdot \overset{3!=6}{\cancel{6}}\cdot 7}{\cancel{4!}\,\cancel{3!}} = \boxed{35}$

Combinations	$^nC_r = \frac{n!}{r!(n-r)!}$
Permutations	$^nP_r = \frac{n!}{(n-r)!}$

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[3]

(c) Find the number of ways in which the questions can be chosen if the students must do at least 1 of the last two questions.

[3]

b) The last question is 'fixed', so this means choosing three of the remaining 6

$^6C_3 = \frac{6!}{3!\,3!} = \frac{\cancel{3!}\cdot 4\cdot 5\cdot \overset{3!=6}{\cancel{6}}}{\cancel{3!}\,\cancel{3!}} = \boxed{20}$

Combinations	$^nC_r = \frac{n!}{r!(n-r)!}$
Permutations	$^nP_r = \frac{n!}{(n-r)!}$

In a maths test students are required to answer four out of seven questions.

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[3]

(c) Find the number of ways in which the questions can be chosen if the students must do at least 1 of the last two questions.

[3]

Combinations	$^nC_r = \dfrac{n!}{r!(n-r)!}$
Permutations	$^nP_r = \dfrac{n!}{(n-r)!}$

c) There are two options:

Choose one of the last two, and three out of the remaining five

$$^2C_1 \times {}^5C_3 = \frac{2!}{1!\,1!} \times \frac{5!}{3!\,2!} = 2 \times \frac{\cancel{3!}\cdot 4\cdot 5}{\cancel{3!}\,2!} = 2 \times 10 = 20$$

Choose both of the last two, and two out of the remaining five

$$^2C_2 \times {}^5C_2 = \frac{2!}{2!\,0!} \times \frac{5!}{2!\,3!} = 1 \times \frac{\cancel{3!}\cdot 4\cdot 5}{2!\,\cancel{3!}} = 10$$

$0! = 1$

$20 + 10 = \boxed{30}$

Question 4

A farm has a new litter of kittens. Two of the kittens are classed as mostly white, four are mostly black, and five are classed as black and white mixed.

Five of the kittens are selected at random.

(a) Find the number of ways in which the selection might contain:

(i) both of the mostly white kittens

(ii) none of the kittens that are classed as black and white mixed.

[4]

(b) Find the number of ways in which the selection might contain at least two kittens that are classed as mostly black.

[4]

Combinations	${}^nC_r = \dfrac{n!}{r!(n-r)!}$
Permutations	${}^nP_r = \dfrac{n!}{(n-r)!}$

a) (i) The two mostly white kittens are 'fixed', so this means choosing three of the remaining nine:

$${}^9C_3 = \frac{9!}{3!6!} = \frac{7 \cdot 8 \cdot 9}{3!} = \boxed{84}$$

(ii) This just means choosing five of the remaining six kittens:

$${}^6C_5 = \frac{6!}{5!1!} = \boxed{6}$$

A farm has a new litter of kittens. Two of the kittens are classed as mostly white, four are mostly black, and five are classed as black and white mixed.

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[4]

(b) Find the number of ways in which the selection might contain at least two kittens that are classed as mostly black.

[4]

Combinations	${}^nC_r = \dfrac{n!}{r!(n-r)!}$
Permutations	${}^nP_r = \dfrac{n!}{(n-r)!}$

b) This means it doesn't contain zero or one of the mostly black kittens.

${}^{11}C_5 = 462$ } Total number of ways to choose five kittens out of eleven

${}^{7}C_5 = 21$ } Number of ways to choose five kittens from the seven that aren't mostly black

${}^{4}C_1 \times {}^{7}C_4 = 140$ } Number of ways to choose one of the four mostly black kittens, and four of the others

$462 - 21 - 140 = \boxed{301}$

Note: You could also calculate this directly as

$\underbrace{{}^4C_2 \times {}^7C_3}_{\text{2 mostly black}} + \underbrace{{}^4C_3 \times {}^7C_2}_{\text{3 mostly black}} + \underbrace{{}^4C_4 \times {}^7C_1}_{\text{4 mostly black}} = 210 + 84 + 7 = \boxed{301}$

Question 5

A pool table has fifteen different balls including the black ball.

(a) Given that the black ball is the last to be potted and the rest of the balls are potted one at a time in a random order, in how many ways can the fifteen balls be potted?

[2]

The other fourteen balls consist of seven pairs of different coloured balls. One of each pair has a stripe across it and the other has a spot on it.

(b) Given that the black ball is still the last to be potted, in how many ways can the fifteen balls be potted one at a time if

(i) all the balls of one type (striped or spotted) must be potted before any balls of the other type?

(ii) both balls from a coloured pair must be potted (one after the other, in any order) before any balls of another colour are potted?

[5]

Combinations	${}^nC_r = \dfrac{n!}{r!(n-r)!}$
Permutations	${}^nP_r = \dfrac{n!}{(n-r)!}$

a) The final ball is fixed, so this is simply the number of ways that the remaining 14 can be put in order:

$n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$ } number of ways of arranging n distinct objects in a line

$14! = \boxed{87\ 178\ 291\ 200} = 8.71782912 \times 10^{10}$

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(ii) both balls from a coloured pair must be potted (one after the other, in any order) before any balls of another colour are potted?

[5]

Combinations	${}^{n}C_{r} = \dfrac{n!}{r!(n-r)!}$
Permutations	${}^{n}P_{r} = \dfrac{n!}{(n-r)!}$

b) The final ball is fixed, so this is simply the number of ways that the remaining 14 can be put in order:

$n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$ } number of ways of arranging n distinct objects in a line

(i) There are 7! ways of ordering the striped balls and 7! ways of ordering the spotted balls, and either striped or spotted could go first:

$7! \times 7! \times 2 = \boxed{50\,803\,200}$

(ii) There are 7! ways of putting the 7 colours in order, and then each coloured pair could be ordered in one of two ways (striped-spotted or spotted-striped):

$7! \times 2^{7} = \boxed{645120}$

Question 6

Ms Aiba has twelve different maths textbooks on her classroom bookshelf. Five of them are Statistics textbooks and the other seven are Pure Mathematics textbooks. Determine the number of different ways that the books can be arranged on the shelf if

(i) there are no restrictions

(ii) the Statistics textbooks are all first and then the Pure textbooks are all last

(iii) the Statistics textbooks are all together and the Pure textbooks are all together

(iv) only the Statistics textbooks are all together.

[7]

Combinations	${}^nC_r = \dfrac{n!}{r!(n-r)!}$
Permutations	${}^nP_r = \dfrac{n!}{(n-r)!}$

$n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$ } number of ways of arranging n distinct objects in a line

(i) $12! = \boxed{479001600}$

(ii) There are 5! ways to arrange the stats books, and then 7! ways to arrange the pure maths books:

$5! \times 7! = \boxed{604800}$

(iii) Like (ii), but either stats or pure maths could go first:

$2 \times 5! \times 7! = \boxed{1209600}$

(iv) Think of all the stats books as a single object – then those and the 7 pure maths books make 8 objects that can be arranged in 8! ways. But for each of those arrangements the stats books can be arranged in 5! ways:

$8! \times 5! = \boxed{4838400}$

Question 7

Nine shirts for a baseball team are numbered from 1 to 9. Five players are allowed to take one shirt each.

Find the number of ways this can be done if

(i) there are no restrictions

(ii) the first three players all decide to choose an even numbered shirt.

There are four of these (2, 4, 6, 8).

[5]

Combinations	${}^nC_r = \dfrac{n!}{r!(n-r)!}$
Permutations	${}^nP_r = \dfrac{n!}{(n-r)!}$

(i) There are ${}^9C_5$ ways of choosing five shirts, and 5! ways to distribute those among the five players :

$$5! \times {}^9C_5 = \cancel{5!} \times \frac{9!}{\cancel{5!}\,4!} = \frac{9!}{4!} = \boxed{15120}$$

Note : This is just ${}^9P_5$

(ii) There are ${}^4C_3$ ways of choosing three of the four even-numbered shirts, and 3! ways to distribute those among the first three players. Then there are ${}^6C_2$ ways of choosing two of the remaining six shirts, and 2! ways to distribute those among the remaining two players :

$$(3! \times {}^4C_3) \times (2! \times {}^6C_2) = \left(\cancel{3!} \times \frac{4!}{\cancel{3!}\,1!}\right) \times \left(\cancel{2!} \times \frac{6!}{\cancel{2!}\,4!}\right)$$

$$= \underbrace{\frac{4!}{1!}}_{{}^4P_3} \times \underbrace{\frac{6!}{4!}}_{{}^6P_2} = 24 \times 30 = \boxed{720}$$

Question 8a

Riley is going on holiday and is allowed to bring along four of his toys. At home he has nine different plastic dinosaurs, six different toy cars, and five different wooden reptiles.

(a) How many different selections of his toys can he make if he chooses at least one of each type of toy?

[4]

Riley can't decide so he persuades his parents to allow him to bring along five toys instead.

(b) Given that he brings more plastic dinosaurs than any other type of toy, how many different selections can he make now?

[4]

a) 'At least one of each type' means he will choose two of one type, and one each of the other two types:

${}^9C_2 \times {}^6C_1 \times {}^5C_1 = 36 \times 6 \times 5 = 1080$ (2 dinosaurs, 1 car, 1 wooden reptile)

${}^9C_1 \times {}^6C_2 \times {}^5C_1 = 9 \times 15 \times 5 = 675$ (1 dinosaur, 2 cars, 1 wooden reptile)

${}^9C_1 \times {}^6C_1 \times {}^5C_2 = 9 \times 6 \times 10 = 540$ (1 dinosaur, 1 car, 2 wooden reptiles)

$1080 + 675 + 540 = \boxed{2295}$

Combinations	${}^nC_r = \dfrac{n!}{r!(n-r)!}$
Permutations	${}^nP_r = \dfrac{n!}{(n-r)!}$

Question 8b

Riley is going on holiday and is allowed to bring along four of his toys. At home he has nine different plastic dinosaurs, six different toy cars, and five different wooden reptiles.

(a) How many different selections of his toys can he make if he chooses at least one of each type of toy?

[4]

Riley can't decide so he persuades his parents to allow him to bring along five toys instead.

(b) Given that he brings more plastic dinosaurs than any other type of toy, how many different selections can he make now?

[4]

Combinations	$^{n}C_{r} = \dfrac{n!}{r!(n-r)!}$
Permutations	$^{n}P_{r} = \dfrac{n!}{(n-r)!}$

b) 'More plastic dinosaurs than any other type of toy' here means at least 3 plastic dinosaurs:

$^9C_3 \times {}^6C_2 \times {}^5C_0 = 84 \times 15 \times 1 = 1260$ (3 dinosaurs, 2 cars, 0 wooden reptiles)

$^9C_3 \times {}^6C_0 \times {}^5C_2 = 84 \times 1 \times 10 = 840$ (3 dinosaurs, 0 cars, 2 wooden reptiles)

$^9C_3 \times {}^6C_1 \times {}^5C_1 = 84 \times 5 \times 6 = 2520$ (3 dinosaurs, 1 car, 1 wooden reptile)

$^9C_4 \times {}^6C_1 \times {}^5C_0 = 126 \times 6 \times 1 = 756$ (4 dinosaurs, 1 car, 0 wooden reptiles)

$^9C_4 \times {}^6C_0 \times {}^5C_1 = 126 \times 1 \times 5 = 630$ (4 dinosaurs, 0 cars, 1 wooden reptile)

$^9C_5 \times {}^6C_0 \times {}^5C_0 = 126 \times 1 \times 1 = 126$ (5 dinosaurs, 0 cars, 0 wooden reptiles)

$1260 + 840 + 2520 + 756 + 630 + 126 = \boxed{6132}$

Question 9

Two parents, Julie and Malcolm, have 15 household chores to be given to their five children, Biddy, Gus, Mandy, Charlie and Claire.

Find the number of ways in which the 15 household chores can be distributed if Biddy is to do 5 of them, Gus is to do 4 of them, Mandy is to do 3 of them, Charlie is to do 2 of them and Claire is to 1 of them.

[5]

There are ${}^{15}C_5$ ways to choose five tasks for Biddy, then ${}^{10}C_4$ ways to select four of the remaining ten tasks for Gus, and so on:

$${}^{15}C_5 \times {}^{10}C_4 \times {}^{6}C_3 \times {}^{3}C_2 \times {}^{1}C_1 = \boxed{37\ 837\ 800}$$

Note: This is just equal to $\dfrac{15!}{5!\,4!\,3!\,2!\,1!}$

Can you show why?

Combinations	${}^{n}C_r = \dfrac{n!}{r!(n-r)!}$
Permutations	${}^{n}P_r = \dfrac{n!}{(n-r)!}$

Question 10a

A team of 16 is to be split up in order to complete a number of tasks.

(a) Find the number of ways the team can be divided into

(i) two equal groups

(ii) four equal groups.

[4]

A quarter of the team are unable to help with the tasks due to COVID-19 infections.

(b) Find the number of ways that the remaining team members can be divided into

(i) two equal groups

(ii) three equal groups.

[4]

Combinations	$^{n}C_{r} = \dfrac{n!}{r!(n-r)!}$
Permutations	$^{n}P_{r} = \dfrac{n!}{(n-r)!}$

a) (i) There are $^{16}C_8$ ways to select eight of the sixteen people for one group, and then the other eight people automatically go into the other group. But because we don't care 'which group is which' we have to divide by 2! to avoid counting duplicate arrangements:

$$\frac{^{16}C_8}{2!} = \boxed{6435}$$

Note: Because $^{8}C_8 = 1$ (there's only one way to choose eight people out of eight people!), this is the same as:

$$\frac{^{16}C_8 \times {^{8}C_8}}{2!}$$

(ii) Use the same technique (and logic!) for four groups of four:

4 here because there are four in each group

$$\frac{^{16}C_4 \times {^{12}C_4} \times {^{8}C_4} \times {^{4}C_4}}{4!} = \boxed{2627625}$$

4! here because there are four groups

Question 10b

A team of 16 is to be split up in order to complete a number of tasks.

(a) Find the number of ways the team can be divided into

(i) two equal groups

(ii) four equal groups.

[4]

A quarter of the team are unable to help with the tasks due to COVID-19 infections.

(b) Find the number of ways that the remaining team members can be divided into

(i) two equal groups

(ii) three equal groups.

[4]

b) Use the same technique (and logic!) here as in part (a).

(i) Two groups of six:

$$\frac{{}^{12}C_6 \times {}^{6}C_6}{2!} = \boxed{462}$$

(iii) Three groups of four:

$$\frac{{}^{12}C_4 \times {}^{8}C_4 \times {}^{4}C_4}{3!} = \boxed{5775}$$

Combinations	${}^nC_r = \dfrac{n!}{r!(n-r)!}$
Permutations	${}^nP_r = \dfrac{n!}{(n-r)!}$

Question 11

You have a set of four different coloured flashlights that you use to send messages to your friend who lives across from you on the same street.

(For purposes of this question, a 'message' is considered to be a single sequence of one or more coloured lights, displayed one after the other.)

(a) Find the number of different messages you can send using two different coloured flashlights displayed one after the other.

[2]

(b) Find the number of different messages you can send using three different coloured flashlights displayed one after the other.

[2]

(c) Find the total number of different messages you could send using your four flashlights, under the condition that all the colours in a given message must be different from each other.

[3]

Combinations	${}^{n}\mathrm{C}_{r} = \dfrac{n!}{r!(n-r)!}$
Permutations	${}^{n}\mathrm{P}_{r} = \dfrac{n!}{(n-r)!}$

a) There are ${}^4C_2$ ways to choose two out of four colours, and $2!$ ways to order the colours for each of those choices:

$$2! \times {}^4C_2 = \cancel{2!} \times \frac{4!}{\cancel{2!}\,2!} = \frac{4!}{2!} = \frac{\cancel{2!}\cdot 3 \cdot 4}{\cancel{2!}} = \boxed{12}$$

This is just ${}^4P_2$

You have a set of four different coloured flashlights that you use to send messages to your friend who lives across from you on the same street.

(For purposes of this question, a 'message' is considered to be a single sequence of one or more coloured lights, displayed one after the other.)

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[2]

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[2]

(c) Find the total number of different messages you could send using your four flashlights, under the condition that all the colours in a given message must be different from each other.

[3]

Combinations	${}^{n}C_{r} = \dfrac{n!}{r!(n-r)!}$
Permutations	${}^{n}P_{r} = \dfrac{n!}{(n-r)!}$

b) There are ${}^{4}C_{3}$ ways to choose three out of four colours, and 3! ways to order the colours for each of those choices:

$$3! \times {}^{4}C_{3} = \cancel{3!} \times \frac{4!}{\cancel{3!}\,1!} = \frac{4!}{1!} = \frac{\cancel{1}\cdot 2\cdot 3\cdot 4}{\cancel{1}} = \boxed{24}$$

This is just ${}^{4}P_{3}$

You have a set of four different coloured flashlights that you use to send messages to your friend who lives across from you on the same street.

(For purposes of this question, a 'message' is considered to be a single sequence of one or more coloured lights, displayed one after the other.)

(a) Find the number of different messages you can send using two different coloured flashlights displayed one after the other. $\boxed{12}$

[2]

(b) Find the number of different messages you can send using three different coloured flashlights displayed one after the other. $\boxed{24}$

[2]

(c) Find the total number of different messages you could send using your four flashlights, under the condition that all the colours in a given message must be different from each other.

[3]

Combinations	${}^{n}C_{r} = \dfrac{n!}{r!(n-r)!}$
Permutations	${}^{n}P_{r} = \dfrac{n!}{(n-r)!}$

c) There are 4 possible one-colour messages.

${}^{4}P_{1} = \frac{4!}{3!} = \frac{3!\cdot 4}{3!} = 4$, but this should be obvious!

If using all four, they can be ordered in $4! = 24$ different ways.

${}^{4}P_{4} = \frac{4!}{0!} = \frac{4!}{1} = 4! = 24$ (Remember, $0! = 1$)

Add these to the answers from (a) and (b):

$$4 + 12 + 24 + 24 = \boxed{64}$$

Question 12

In this question give your answers in the form $a \times 10^k$, where $1 \le a < 10$ and $k \in \mathbb{Z}$.

John is a builder who has divided his day's work into 15 separate tasks. Two of these tasks John considers 'physically demanding' because one includes moving bricks and the other includes concrete mixing.

(a) Find the number of ways John can complete the tasks given that one 'physically demanding' task is done at the start of the day and the other 'physically demanding' task is done at the end of the day.

[3]

(b) Find the number of ways John can complete the tasks given that the two physically demanding tasks are not done consecutively.

[4]

a) There are 13! ways to order the other thirteen tasks, and then 2! = 2 ways of putting the physically demanding tasks into the 'first' and 'last' slots:

$2 \times 13! = 1.24540416 \times 10^{10}$

$\boxed{1.25 \times 10^{10} \text{ (3 s.f.)}}$

Note: This (= 12 454 041 600) is in fact the exact answer – compare part (b).

Combinations	${}^nC_r = \dfrac{n!}{r!(n-r)!}$
Permutations	${}^nP_r = \dfrac{n!}{(n-r)!}$

In this question give your answers in the form $a \times 10^k$, where $1 \le a < 10$ and $k \in \mathbb{Z}$.

John is a builder who has divided his day's work into 15 separate tasks. Two of these tasks John considers 'physically demanding' because one includes moving bricks and the other includes concrete mixing.

(a) Find the number of ways John can complete the tasks given that one 'physically demanding' task is done at the start of the day and the other 'physically demanding' task is done at the end of the day.

[3]

(b) Find the number of ways John can complete the tasks given that the two physically demanding tasks are not done consecutively.

[4]

b) There are 15! possible ways to order the fifteen tasks.

If the two physically demanding tasks are consecutive, then there are 13! ways to order the other tasks. For each such ordering there are fourteen 'slots' into which the physically demanding tasks could go, and in each slot either 'bricks' or 'concrete' could go first (two options).

Fourteen 'slots' where the two physically demanding tasks could go (×)

× □ × □ × □ × □ × □ × □ × □ × □ × □ × □ × □ × □ × □ ×

Thirteen 'other' tasks (□)

$\underbrace{15! - (2 \times 14 \times 13!)}_{\text{all possible orderings, minus the orderings where the physically demanding tasks are consecutive.}} = 1.133317786 \times 10^{12}$

$\boxed{1.13 \times 10^{12} \text{ (3 s.f.)}}$

Note: This has been rounded to 10 s.f. by the calculator. The exact answer is 1 133 317 785 600

Combinations	${}^nC_r = \dfrac{n!}{r!(n-r)!}$
Permutations	${}^nP_r = \dfrac{n!}{(n-r)!}$