

IB Maths: AI HL

Past Paper 2

Question Paper

These practice questions can be used by students and teachers and is Suitable for IB Maths AI HL Past Papers

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| Course | IB Maths |
| Section | Set A |
| Topic | Past Paper 2 |
| Difficulty | Medium |

Level: IB Maths

Subject: IB Maths AI HL

Board: IB Maths

Topic: Past Paper 2

Question 1

Natasha needs to purchase new sound equipment for her band, the Oscillots. The sound equipment costs £12000, and as she does not have enough money to pay the full amount up front, she is considering two different methods of financing the purchase.

Option 1:

Natasha is eligible to receive a personal loan from her bank for the full amount of £12000. It will be 5 a year loan at a nominal annual interest rate of 3.9% **compounded monthly**. Repayments are made each month.

a)

(i) Find the repayment made each month.

(ii) Find the total amount paid for the sound equipment.

(iii) Find the interest paid on the loan.

[7 marks]

Option 2:

The music shop from which Natasha is buying the sound equipment offers its own financing scheme. This scheme offers her a year loan at a nominal annual interest rate of $r\%$ **compounded quarterly**. The terms of the loan require a 10% payment up front and quarterly repayments of £600.

b)

(i)

Find the amount to be borrowed for this option.

(ii)

Find the annual interest rate, r .

[5 marks]

c)

State which option Natasha should choose. Justify your answer.

[2 marks]

After considering her personal circumstances, Natasha chooses option 2. The music store invests the money from Natasha's payments as soon as each payment is received.

d)

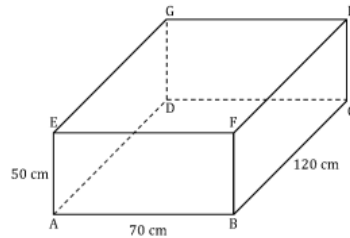
If the music store invests Natasha's payments in a fund yielding 0.9% interest per quarter and inflation is 1.2% per quarter, calculate the real amount of money that the music store has received by the end of the 5 year period.

[4 marks]

Question 2

The Amazing Box Company manufactures rectangular gift boxes.

Their largest box size has length 120 cm, width 70 cm and height 50 cm as shown in the diagram.



Customers often ask for other measurements relating to the gift boxes.

a)

Calculate the length GB.

[2 marks]

b)

Calculate the surface area of the box in m^2 .

[2 marks]

Each month, The Amazing Box Company expects to sell x hundred gift boxes.

It is known that $\frac{dP}{dx} = -2x + 480$, $x \geq 0$ where P is the monthly profit, in euros, from the sale of x hundred boxes.

c)

Find the number of boxes that should be sold each month to maximize the profit.

[3 marks]

It is also known that the company makes a profit of €960 in a month where it sells 15000 boxes.

d)

Find $P(x)$.

[5 marks]

e)

Find the least number of boxes which must be sold each month in order to make a profit.

[3 marks]

Question 3

In an online game there are various 'power-ups' that players can discover while progressing through the game. The power-ups come in two sizes, major and minor, and they all belong to one of two categories, either combat or survival.

It is known that 85% of the power-ups in the game are minor power-ups while the other 15% are major power-ups. Of the major power-ups it is known that 60% are combat power-ups, while of the minor power-ups it is known that 30% are combat power-ups.

a)

Power-ups that are discovered in the game occur at random.

(i)

Find the probability that when a power-up is discovered it will be a survival power-up.

(ii)

Given that a power-up is a combat power-up, find the probability that it is a major power-up.

[4 marks]

In the online game it is possible to purchase 'mystery bundles' containing random assortments of power-ups. According to the game's website, the power-ups in each mystery bundle should be distributed as follows.

| Type | Sword | Lasgun | Armour | Rations | Medical | Other Gear |
|----------------|-------|--------|--------|---------|---------|------------|
| Percentage (%) | 15 | 10 | 15 | 30 | 10 | 20 |

Dylan and his teammates have joined together to purchase a 'mega mystery bundle' containing 60 power-ups. After going through the bundle, they find it contains the following frequencies of each type of power-up:

| Type | Sword | Lasgun | Armour | Rations | Medical | Other Gear |
|--------------------|-------|--------|--------|---------|---------|------------|
| Observed frequency | 4 | 2 | 5 | 21 | 9 | 19 |

Dylan and his friends are upset that they did not receive more weapons and armour. Some of them think that they were simply unlucky this time around, while others insist that the website's claims about the contents of the bundles are misleading.

Dylan is also studying Mathematics as part of his IB course, so he decides to investigate if the sample is consistent with the website's claims by conducting a χ^2 goodness of fit test. The test is carried out at a 5% significance level.

b)

Write down the null hypothesis for this test.

[1 mark]

c)

Copy and complete the following table in your answer booklet.

| Type | Sword | Lasgun | Armour | Rations | Medical | Other Gear |
|--------------------|-------|--------|--------|---------|---------|------------|
| Expected frequency | | | | | | |

[2 marks]

d)

Write down the number of degrees of freedom.

[1 mark]

e)

Find the p -value for the test.

[2 marks]

f)

State the conclusion of the test. Give a reason for your answer.

[2 marks]

Question 4

A city has two supermarkets, X and Y. Each year 22% of the customers using supermarket X move to supermarket Y and 14% of the customers using supermarket Y move to supermarket X. All additional losses and gains of customers by the supermarkets may be ignored.

a)

Write down a transition matrix T representing the movements between the two companies in a particular year

[2 marks]

b)

Find the eigenvalues and corresponding eigenvectors of T .

[4 marks]

c)

Hence write down matrices P and D such that $T = PDP^{-1}$.

[2 marks]

Initially, supermarket X and supermarket Y each have 15000 customers.

d)

Find an expression for the number of customers supermarket X has after n years, where $n \in \mathbb{N}$.

[5 marks]

e)

Hence write down the number of customers that supermarket X can expect to have in the long term.

[1 mark]

Question 5

In a 3D virtual reality computer game, an alien invader's position is given by the coordinates (x, y, z) , relative to a fixed point where x and y are the coordinates in the horizontal plane through O and z is the vertical height above that horizontal plane. All displacements are in cm, and t is the elapsed time from the start of the game in seconds.

The alien moves with constant velocity $\begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix} \text{ cm s}^{-1}$

At $t = 5$, the alien is detected at the position $P(5, -3, 7)$.

a)

Write down the vector equation for the displacement, r of the alien in terms of t .

[2 marks]

The player's avatar is located at point X (11, 12, -5).

b)

If the alien continue to move with the velocity given,

(i) verify that it will pass directly over point .

(ii) Find the vertical distance between the alien and the player's avatar at this point.

[4 marks]

When the alien is 10 cm higher than the player's avatar in the vertical direction, the player angles the avatar's laser beam in the direction of the alien in order to shoot it.

c)

(i)

Find the time at which the alien is 10 cm higher than the player's avatar.

(ii)

Find the direct distance between the alien and the player's avatar at this time.

[5 marks]

d)

Find the angle above the horizontal at which the avatar's laser beam must be positioned in order to hit the alien.

[3 marks]

Question 6

A team of researchers is attempting to model the vertical motion of a skydiver in freefall, before the opening of the skydiver's parachute. Time $t=0$ indicates the moment that the skydiver begins to descend from the airplane.

Initially the researchers propose that the air resistance to the skydiver's motion will be proportional to velocity. The following model is proposed, where the skydiver's vertical displacement, x , since leaving the airplane is determined by the differential equation

$$\frac{d^2x}{dt^2} = 9.81 - 0.00327\left(\frac{dx}{dt}\right)$$

In this equation x represents metres fallen in the *downwards* direction, while t is measured in seconds.

a)

By substituting $v = \frac{dx}{dt}$ into the equation, find an expression for the velocity of the skydiver at time t . Give your answer in the form $v = f(t)$.

[7 marks]

The maximum velocity approached by the skydiver while falling is known as the terminal velocity.

b)

From your solution to part (a), or otherwise, find the terminal velocity of the skydiver that is predicted by the model.

[2 marks]

After considering the predictions resulting from the original model, one of the researchers suggests that it might be more accurate to represent air resistance to the skydiver's motion as being proportional to the square of velocity. Accordingly, the following model is set up:

$$\frac{d^2x}{dt^2} = 9.81 - 0.00327\left(\frac{dx}{dt}\right)^2$$

(c) Write down the differential equation as a system of first order differential equations.

[2 marks]

d)

Use Euler's method, with a step length of 0.5, to find approximations for the displacement and velocity of the skydiver when $t=2$.

[4 marks]

e)

By repeated application of Euler's method, find an approximation of the terminal velocity to five significant figures.

[1 mark]

At terminal velocity the acceleration of an object is equal to zero.

(f) Use the differential equation to find the terminal velocity for the skydiver.

[2 marks]

g)

Use your answers to parts (d), (e) and (f) to comment on the accuracy of the Euler approximation to this model.

[2 marks]

Question 7

Five friends are playing a game in a large field where they have each set up a “house” and created pathways linking some of the houses to each other. The table below shows the lengths, in metres, of the pathways that run between the houses of Annabel, Bernie, Clara, Dionne and Ezra.

| | A | B | C | D | E |
|---|----|----|----|----|----|
| A | | 14 | 11 | | 16 |
| B | 14 | | 10 | 13 | |
| C | 11 | 10 | | | 9 |
| D | | 13 | | | 10 |
| E | 16 | | 9 | 10 | |

a)
Show the direct pathways that link the friends’ houses as a weighted graph.

[2 marks]

b)
Write down the adjacency matrix for this graph.

[2 marks]

c)
Using your answer to part (b), find the number of ways to travel from Annabel’s house to Ezra’s house by travelling along exactly 4 pathways. (Note that a given pathway may be traversed more than once as part of a four-pathway route.)

[2 marks]

d)
Show that the graph in part (a) is a Hamiltonian graph and write down the route of a possible Hamiltonian cycle starting from point A.

[2 marks]

The following table shows the least distances for travelling between the different friends' houses.

| | A | B | C | D | E |
|---|-----|-----|----|-----|-----|
| A | 0 | 14 | 11 | a | 16 |
| B | 14 | 0 | 10 | 13 | b |
| C | 11 | 10 | 0 | 19 | 9 |
| D | a | 13 | 19 | 0 | 10 |
| E | 16 | b | 9 | 10 | 0 |

(e) Find the values of a and b .

[2 marks]

A sixth friend, Fernando, wishes to visit all of his friends' houses starting and finishing at Annabel's house.

f)

Use the nearest neighbor algorithm to find an upper bound for the distance that Fernando will have to walk.

[3 marks]

g)

By deleting vertex A , use the deleted vertex algorithm to find a lower bound for the distance that Fernando will have to walk.

[4 marks]