

# IB Maths: AA SL

## Past Paper 2

### Question Paper

**These practice questions can be used by students and teachers and is Suitable for IB Maths AA SL Past Papers**

Course	IB Maths
Section	Set B
Topic	Past Paper 2
Difficulty	Medium

**Level: IB Maths**

**Subject: IB Maths AA SL**

**Board: IB Maths**

**Topic: Past Paper 2**

### Question 1

A curve  $y = f(x)$  passes through point  $A(4, 2)$  and has a gradient of  $f'(x) = 5x - 2$ .

(a) Find the gradient of the curve at point A.

[2 marks]

(b) Find the equation of the tangent to the curve at point A.

Give your answer in the form  $y = mx + c$ .

[2 marks]

(c) Determine the equation of the curve  $y = f(x)$ .

[3 marks]

### Question 2

Jennifer sells cups of tea at her shop and has noticed that she sells more tea on cooler days. On five different days, she records the maximum daily temperature,  $T$ , measured in degrees Celsius, and the number of cups of teas sold,  $C$ . The results are shown in the following table.

<b>Maximum Daily Temperature, <math>T</math>.</b>	3	5	8	9	12
<b>Cups of tea sold, <math>C</math>.</b>	37	34	33	26	21

The relationship between  $T$  and  $C$  can be modelled by the regression line of  $C$  on  $T$  with equation  $C = aT + b$ .

(a) (i) Find the value of  $a$  and the value of  $b$ .

(ii) Write down the value of Pearson's product-moment correlation coefficient,  $r$ .

[4 marks]

(b) Use your regression equation from part (a)(i) to estimate the number of teas that Jennifer will sell on a day when the maximum temperature is  $11^{\circ}\text{C}$ .

[2 marks]

### Question 3

A scientist is studying the movement of snails and has observed that the distribution of their speeds,  $S$ , follows a normal distribution with a mean of  $48\text{ m/h}$  and a standard deviation of  $1.5\text{ m/h}$ .

(a) Sketch a diagram to represent this information.

[2 marks]

(b) Find the probability that a randomly selected snail has a speed of less than  $46.5\text{ m/h}$ .

[2 marks]

(c) From a sample of 80 snails, calculate the expected number of snails that would have a speed of less than  $46.5\text{ m/h}$ . Give your answer to the nearest integer.

[2 marks]

### Question 4

A hamster runs in its exercise wheel, rotating the wheel at a constant speed. The wheel has a diameter of 14 centimetres and the top of the wheel is positioned at a height of  $k$  centimetres above the floor of the cage.

A point at the top of the wheel is marked before the hamster starts to run, turning the wheel clockwise. The hamster takes 4 seconds to turn the wheel one complete revolution.

After  $t$  seconds, the height of the mark on the wheel above the floor of the cage is given by

$$h(t) = 10 + a \cos\left(\frac{\pi}{2}t\right) \text{ for } 0 \leq t \leq 150$$

(a) After 26 seconds, the mark is 3 cm above the cage floor. Find  $k$ .

[2 marks]

(b) Find the value of  $a$ .

[3 marks]

### Question 5

A particle moves along a straight line with a velocity,  $v \text{ ms}^{-1}$ , given by  $v = 2^t - 2$  where  $t$  is measured in seconds such that  $0 \leq t \leq 4$ .

(a) Find the acceleration of the particle at time  $t = 2$ .

[2 marks]

(b) State the time when the particle comes to rest.

[1 mark]

(c) Find the total distance travelled by the particle.

[3 marks]

### Question 6

In the expansion of  $\left(\frac{1}{2}x + 1\right)^n$ , the coefficient of the  $x^2$  term is  $8n$ , where  $n \in \mathbb{Z}^+$ .

Find  $n$ .

[5 marks]

### Question 7

Three triplets, Aneirin, Bran and Culhwch, are each given a gift of £12 000 by their grandmother on their twenty-first birthday.

Aneirin invests his £12 000 in a bank account that offers an interest rate of 5.84% per annum compounded **annually**.

- (a) (i) Find the value of Aneirin's investment after 9 years, to the nearest pound.
- (ii) Determine the number of years it would take for the value of Aneirin's investment to double.

[5 marks]

Bran invests his £12 000 in a bank account that offers an interest rate of  $r\%$  per annum, compounded **monthly**, where  $r$  is set to two decimal places.

- (b) Find the minimum value of  $r$  needed if Bran is to have at least as much money in his account after 9 years as Aneirin has in his.

[3 marks]

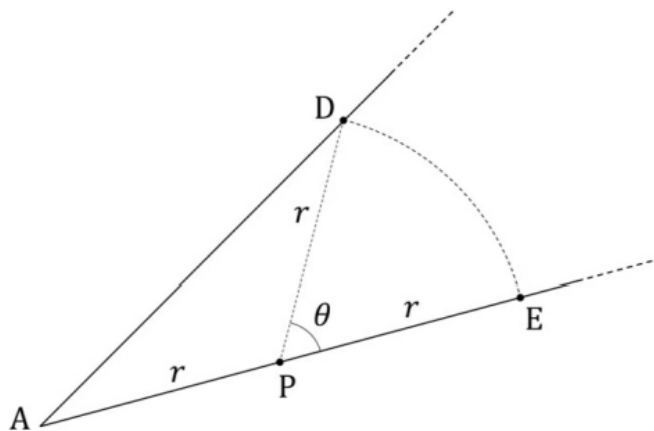
Culhwch doesn't trust banks, and so he puts all of his £12 000 in a metal box buried in his garden where he does not earn any interest. His savings plan is simply to add more money to the box each year, such that the money added each year will be a fixed multiple of the money added the previous year.

- (c) (i) Given that each year Culhwch adds one third the amount he added in the previous year, show that his savings will never reach £20 000.
- (ii) Determine the minimum fixed multiplier he would need to use in order for his savings to reach £20 000 by the end of 9 years, given that the multiplier used will be set to 3 significant figures.

[8 marks]

### Question 8

James has an awkwardly-shaped corner of his garden that is hard to mow. The corner is bordered by two straight fences that meet at point  $A$ , with the garden lying between them. James' solution to his mowing problem is to buy a goat and tie it to one of the fences at point  $P$ , so that the goat can eat all the grass it can reach. The rope is of length  $r$  metres. Points  $D$  and  $E$  (each on a different fence) are such that  $PD = PE = PA = r$  and  $\widehat{DPE} = \theta$  radians. This is shown in the following diagram.



The length of arc  $DE$  in the diagram is 16 m.

- (a) Write down an expression for  $r$  in terms of  $\theta$ .

[1 mark]

(b) Show that the area of James' garden that the goat can reach is  $\frac{128}{\theta^2}(\theta + \sin \theta) \text{ m}^2$ .

You may treat the goat as a point particle located at the end of the rope.

[4 marks]

The area of James' garden that the goat can reach is equal to  $240 \text{ m}^2$ .

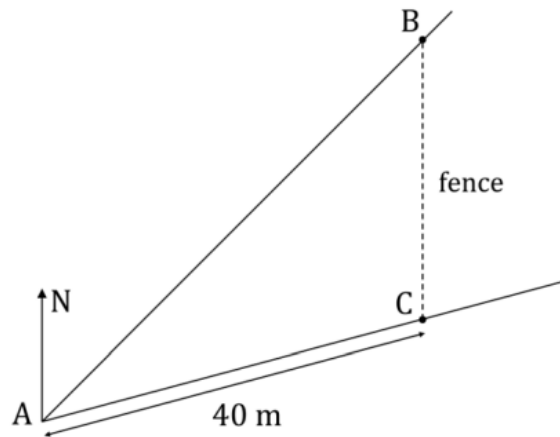
(c) Find the value of  $\theta$ .

[2 marks]

(d) Hence find the size of  $\widehat{DAE}$ .

[2 marks]

After the goat chews through the rope and eats all the flowers in a flowerbed at the far end of the garden, James decides to build a new fence between points B and C to enclose the goat's portion of the garden. This is shown in the following diagram.



Point B is due north of point C and  $AC = 40$  m. The bearing of B from A is  $45^\circ$ .

- (e) (i) Find the size of  $\widehat{ABC}$ .
- (ii) Find the length of new fence required.

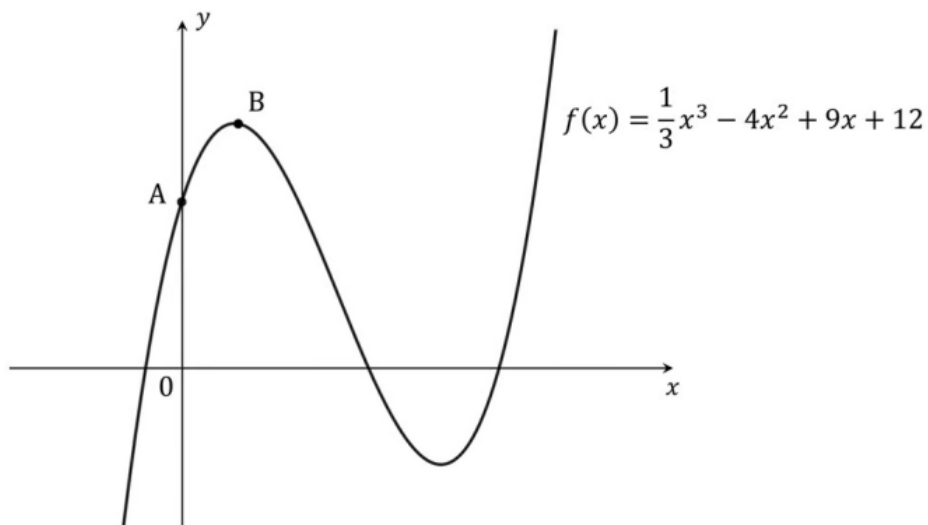
[4 marks]



### Question 9

The diagram below shows a part of the graph of the function

$$f(x) = \frac{1}{3}x^3 - 4x^2 + 9x + 12$$



- (a) Point A is the point of intersection between the graph and the y-axis. Write down the coordinates of point A.

[1 mark]

- (b) Find  $f'(x)$ .

[2 marks]

- (c) Using the graph, explain why the equation  $f'(x) = 0$  must have exactly two distinct real solutions.

[3 marks]

Point B is the point on the graph with  $x$ -coordinate  $\frac{8-\sqrt{26}}{2}$ .

(d) Find the gradient of the tangent line to the graph at point B.

[2 marks]

Points C and D are the points on the graph at which the tangent lines are perpendicular to the tangent line at point B.

(e) By first determining the gradient of the tangents at points C and D, find the  $x$ -coordinates of points C and D.

[4 marks]

(f) Given that point C lies between points A and B on the graph, find the equation of the tangent line to the graph at point C.

[4 marks]