IB Maths: AI HL

Past Paper 2

Question Paper

These practice questions can be used by students and teachers and is Suitable for IB Maths AI HL Past Papers

Course	IB Maths
Section	Set B
Topic	Past Paper 2
Difficulty	Medium

Level: IB Maths

Subject: IB Maths AI HL

Board: IB Maths

Topic: Past Paper 2

Question 1

Anna decides she wants to buy a farm and the bank agree to give her a loan provided she makes a 13.9% deposit of $40 000.

(a) Calculate the value of the farm.

[1 mark]

She currently has $15 000 saved up and decides to invest it in some high risk high growth shares forecasted to grow at 65% annually.

(b) Calculate the forecasted number of years it will take for her to be able to afford the deposit.

[2 marks]

1.5 years later, the shares outperform their forecasted growth rate and Anna is able to afford the deposit on the farm.

(c) Calculate the percentage error between the forecasted annual growth rate and the actual annual growth rate of the shares.

[3 marks]

Anna now takes out the loan from the bank.

(d) Write down the amount of the loan.

[1 mark]

The loan is for 25 years, compounded monthly, with equal monthly payments of $1200.

(e) For this loan, find

(i) the amount of interest paid by Anna,

(ii) the annual interest rate of the loan.

[5 marks]

After 15 years of paying off this loan, Anna decides to pay the **remainder** in one final payment.

(f) Find the amount of Anna's final payment.

[3 marks]

(g) Find how much money Anna saved by making one final payment after 20 years.

[3 marks]

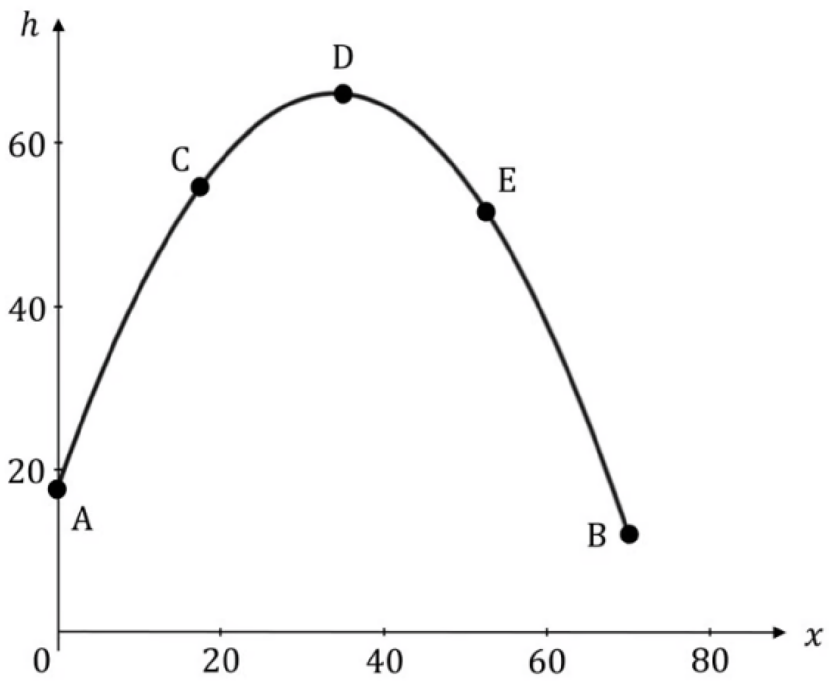
Question 2

The cross-sectional profile of a hill is modelled by the function

$$h(x) = \frac{1}{6}\left(17x + 107 - \frac{x^2}{4}\right), \qquad 0 \leq x \leq 70,$$

where h is the altitude above mean sea level, in metres, and x is the horizontal distance, in metres, from a fixed point O.

The cross-sectional profile of the hill can be seen in the diagram below.



Point A has coordinates $(0, 17.8)$ correct to 3 significant figures, and point B has exact coordinates $(70, 12)$.

(a) Calculate the altitude at $x = 3$.

[2 marks]

A point P is at an altitude of 40 m.

(b) Find the possible values of its horizontal distance from O.

[3 marks]

(c) Find $h'(x)$.

[2 marks]

(d) Hence calculate the maximum altitude of the hill.

[2 marks]

When $x = 17.5$, the altitude of the hill is 54.7 m, when $x = 35$, the altitude of the hill is 66.0 m, and when $x = 52.5$ the altitude of the hill is 51.7 m. These points are shown on the diagram as C, D and E respectively, and the altitudes in each case are given correct to 3 significant figures.

(e) Use the trapezoidal rule with four intervals to estimate the cross-sectional area of the hill

[3 marks]

(f) (i) Write down the integral which can be used to find the cross-sectional area of the hill.

(ii) Hence find the cross-sectional area of the hill to the nearest square metre.

[4 marks]

Question 3

It is believed that the time in minutes, T, that a customer spends on hold during a call when calling a customer service line can be modelled by a normal distribution, with $T \sim \mathrm{N}(17, 2.8^2)$.

(a) Using the model find, correct to four decimal places, the probability that during a call a customer chosen at random spends

(i) less than 15 minutes on hold

(ii) more than 23 minutes on hold.

[3 marks]

500 customer service calls are monitored and the length of time that a customer is put on hold for during each call is measured.

(b) By again using the model find, correct to one decimal place, the expected number of the 500 calls in which a customer is put on hold for between

(i) 15 and 20 minutes

(ii) 20 and 23 minutes.

[3 marks]

For the 500 monitored calls, the measured lengths of time that a customer was put on hold for during each call are summarised in the following table.

Length of time on hold, *T*	**Number of calls**
Less than 15 minutes	98
Between 15 and 20 minutes	309
Between 20 and 23 minutes	79
More than 23 minutes	14

It is decided to perform a χ^2 goodness of fit test at the 10% level of significance to decide whether the length of time that a customer is put on hold for during a call can indeed be modelled by a normal distribution, with $T \sim \mathrm{N}(17, 2.8^2)$.

(c) State the null and alternative hypotheses.

[2 marks]

(d) Find the p-value for the test.

[3 marks]

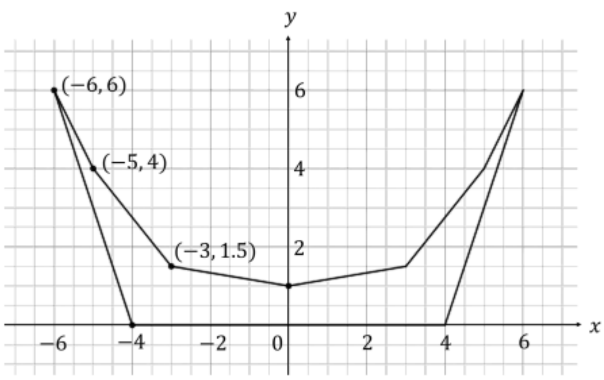
(e) State the conclusion of the test. Give a reason for your answer.

[2 marks]

Question 4

Paul finds an unusually shaped bowl when excavating his garden. It appears to be made out of bronze, and Paul decides to model the shape in order to work out its volume.

By uploading a photograph of the object onto some graphing software, Paul identifies that the cross-section of the bowl goes through the points $(-4,0)$, $(-6,6)(-5,4)$,$(-3,1.5)$ and $(0,1)$. The cross-section is symmetrical about the y-axis as shown in the diagram. All of the units are in centimetres.



He models the section from $(-4,0)$ to $(-6,6)$ as a straight line.

a)
Find the equation of the line passing through these two points.

[2 marks]

Paul models the section of the bowl that passes through the points $(-6,6)$,$(-5,4)$,$(-3,1.5)$ and $(0,1)$with a quadratic curve.

b)
(i)
Find the equation of the least squares quadratic curve for these four points.

(ii)
By considering the gradient of this curve when $x = -0.5$, explain why it may not be a good model.

[3 marks]

Paul thinks that a quadratic with a minimum at $(0,1)$ and passing through the point $(-6,6)$ is a better option.

c)
Find the equation of the new model.

[4 marks]

Believing this to be a better model for the bowl, Paul finds the volume of revolution about the y-axis to estimate the volume of the bowl.

d)
Re-arrange the answers to parts (a) and (c) to make x a function of y.

[3 marks]

e)
(i)
Write down an expression for Paul's estimate of the volume as the difference of two integrals.

(ii)
Hence find the value of Paul's estimate.

[5 marks]

Question 5

In the town of Petersham, all the residents belong to either one or the other of the town's two curling clubs – the Slippery Sliders (S) or the Wild Sweepers (W). Competition between the clubs is fierce, and members tend to be quite loyal. Still, each year 2% of the members of S switch to W and 3% of the members of W switch to S. Any other losses or gains of members by the two curling clubs may be ignored.

a)
Write down a transition matrix $\boldsymbol{T}$ representing the movement of members between the two clubs in a particular year.

[2 marks]

b)
Diagonalise the matrix T by writing it in the form $\boldsymbol{T} = \boldsymbol{PDP}^{-1}$ for appropriate matrices $\boldsymbol{P}$ and $\boldsymbol{D}$.

[6 marks]

Initially there are 574 members of and 621 members of W.

c)
Show that the numbers of members of S and W after n years will be $(717 - 143(0.95^n))$ and $(478 + 143(0.95^n))$, respectively.

[6 marks]

d)
Hence write down the number of members that each of the curling clubs can expect to have in the long term.

[2 marks]

Question 6

A boat is moving such that its position vector when viewed from above at time t seconds can be modelled by

$$r = \begin{pmatrix} 10 - a\sin\left(\frac{\pi t}{600}\right) \\ b\left(1 - \cos\left(\frac{\pi t}{600}\right)\right) \end{pmatrix}$$

with respect to a rectangular coordinate system from a point O, where the non-zero constants a and b can be determined. All distances are given in metres.

The boat leaves its mooring point at time $t = 0$ seconds and 5 minutes later is at the point with coordinates $(-20, 40)$.

a)
Find
(i)
the values of a and b,

(ii)
the displacement of the boat from its mooring point.

[4 marks]

b)
Find the velocity vector of the boat at time t seconds.

[2 marks]

After setting off, the boat reaches a point P where it is moving parallel to the x-axis.

c)
Find OP.

[6 marks]

d)
Find the time that the boat returns to its mooring point and the acceleration of the boat at this moment.

[3 marks]

Question 7

On a particular island, a particular species of bird was initially recorded as having a population of 80 at the start of a programme of observations. Over time, the scientists conducting the programme determined that the growth rate of the bird population could be modelled by the following differential equation

$$\frac{dx}{dt} = \frac{7}{5}x$$

where x is the size of the bird population, and t is the length of time in years since the start of the programme.

a)
Find the population of the bird species two years after the start of the programme.

[5 marks]

When the population of the bird species reaches 2000, a new reptile species is introduced to the island in order to control the bird population. Initially 280 reptiles are introduced to the island. Based on their research the scientists believe that the interaction between the two species after the introduction of the reptiles can be modelled by the system of coupled differential equations

$$\frac{dx}{dt} = (3 - 0.012y)x$$

$$\frac{dx}{dt} = (0.0007x - 1)y$$

Where x and y represent the size of the bird and reptile populations respectively.

(b) Using the Euler method with a step size of 0.5, find an estimate for

(i) the bird population 2 years after the reptiles were introduced

(ii) the reptile population 2 years after the reptiles were introduced.

[6 marks]

c)
Explain how the approximation in part (b) could be improved.

[1 mark]

d)
Show that the origin is an equilibrium point for the system, and determine the coordinates of the other equilibrium point.

[3 marks]