Chapter 2:

- Proper fraction: power of x in numerator smaller than in denominator

$$\frac{6x-2}{(x-2)(x+3)} = \frac{A}{(x-2)} + \frac{B}{(x+3)}$$

- metod 1: substitution

$6x-2 = A(x+3) + B(x-2)$

let x = (solution of equations) and sub into eq

let $x=2$ — let $x=-3$

$10 = 5A \rightarrow A=2$ — $-20 = -5B \rightarrow B=4$

$$\frac{2}{(x-2)} + \frac{4}{(x+3)}$$

- method 2: Equating the coefficient

$6x-2 = A(x+3) + B(x-2)$

expand and collect terms

$\underline{6}x \underline{-2} = \underline{(A+B)}x + \underline{(3A-2B)}$

$6 = A+B$ — $-2 = 3A-2B$

simultaneous equation to find A and B

- Always factorise first
- if there is a repeating root you have to split it and then square one of them

$$\frac{3x}{(x+1)^2} = \frac{A}{(x+1)^2} + \frac{B}{(x+1)}$$

when you do substitution for final unknown you can choose x = anything

$x=-1$ and x = anything ex. = 0 or 1

- Improper fraction

case 1: when numerator and denominator powers are equal

do long division

expand brackets first

$$\frac{5x}{x+1} = \underbrace{A}_{\text{constant}} + \frac{B}{x+1}$$

$$\begin{array}{r|l} & A\quad 5 \\ x+1 & 5x \\ & -5x+5 \\ \hline & B\quad -5 \end{array}$$

case 2: when numerator has greater power than denominator

$$\frac{x^3}{(x+2)(x-1)} = Ax+B + \frac{C}{(x+2)} + \frac{D}{(x-1)}$$

$$x-1+\left[\frac{3x-2}{(x+2)(x-1)}\right]$$ proper fraction

$$\begin{array}{r|l} & x-1 \\ x^2+x-2 & x^3 \\ & -\; x^3+x^2-2x \\ \hline & -x^2+2x \\ & -\; -x^2-x+2 \\ \hline & 3x-2 \end{array}$$

Chapter 4:

when power is a fraction or negative:

when constant is 1

$$(1+ax)^n = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + \dots$$

take validity where x is smaller

- validity for x to be valid $|ax|<1$ so $|x| < \frac{1}{a}$ $|x| < \frac{1}{2}$ $\boxed{|x| < \frac{1}{3}}$
- percentage error $= \frac{\text{estimated} - \text{exact}}{\text{exact}} \times 100$ (exact < estimate)

(if no. isn't exact put full calculator display)

when constant isn't 1 $\rightarrow (a+bx)^n$ (divide inside by a)

$$a^n\left(1+\frac{b}{a}x\right)^n = a^n\left[1 + n\left(\frac{b}{a}x\right) + \frac{n(n-1)}{2!}\left(\frac{b}{a}x\right)^2 + \frac{n(n-1)(n-2)}{3!}\left(\frac{b}{a}x\right)^3 + \dots\right]$$

- validity $\left|\frac{b}{a}x\right| < 1 \longrightarrow |x| < \frac{a}{b}$
- percentage error $= \frac{\text{exact} - \text{estimate}}{\text{estimate}} \times 100$ (exact > estimate)

- be careful until what power of x question wants and put $+ \dots$
- be careful ~~until~~ when expanding $\frac{b}{a}$ is also squared or cubed $\left(\frac{b}{a}x\right)^3 = \frac{b^3}{a^3}x^3$
- Expansion with partial fractions

$$\frac{8x+4}{(1-x)(2+x)} = \frac{A}{(1-x)} + \frac{B}{(2+x)} = \frac{4}{(1-x)} - \frac{4}{(2+x)}$$

$$= 4(1-x)^{-1} - 4(2+x)^{-1}$$

expand each part on its own then add them

$$[4 + 4x + 4x^2] - [2 - x + \tfrac{1}{2}x^2] = 2 + 5x + \tfrac{7}{2}x^2$$

validity $\boxed{|x| < 1}$ or $|x| < 2$

Chapter 3 :

- Parametric $\quad x = t+1 \quad y = t^2 \quad , \quad t > 0$

cartesian eq : make t subject in x-formula : $t = x-1$

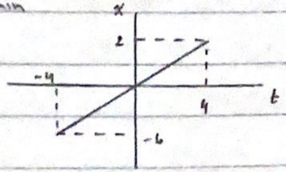
$f(x)$ $\quad$ Substitute t into y-formula : $y = (x-1)^2$

$f(x)$ domain $\rightarrow$ range of $x = p(t)$

$f(x)$ range $\rightarrow$ range of $y = g(t)$

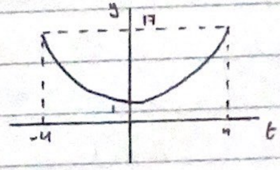
Ex $\quad x = t-2 \quad , \quad y = t^2+1 \quad , \quad -4 \leq t \leq 4$

domain



$-6 \leq x \leq 2$

range



$1 \leq y \leq 17$

$t = x+2 \rightarrow y = (x+2)^2+1$

- Using trig identities ← always think of them

- $(x-a)^2 + (y-b)^2 = r^2$

Ex $\quad x = \sin t + 1 \qquad y = \cos t - 2$

$\sin t = x-1 \qquad \cos t = y+2 \rightarrow \sin^2 t + \cos^2 t = 1$

$(x-1)^2 + (y+2)^2 = 1$

Ex $\quad x = \sin t \qquad y = \sin 3t = \sin(t+2t)$

$\sin(t+2t) = \sin t \cos 2t + \sin 2t \cos t = \sin t(1-2\sin^2 t) + 2\sin t\cos^2 t$

$= \sin t - 2\sin^3 t + 2\sin t(1-\sin^2 t) = \sin t - 2\sin^3 t + 2\sin t - 2\sin^3 t$

$= 3\sin t - 4\sin^3 t = 3x - 4x^3$

$y = 3x - 4x^3$

Chapter 5:

$x = 3t^2$, $y = 2t^3$

$\frac{dx}{dt} = 6t$ $\frac{dy}{dt} = 6t^2$ $\frac{dy}{dx}$ will be in terms of t

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{dy}{dt} \times \frac{1}{\frac{dx}{dt}}$

Ex Show that tangent at C at point P doesn't intersect curve again

tangent at C = $y = 2x + 41$ ← substitute x and y to show only 1 intersection → $b^2 - 4ac = 0$

Curve C $\begin{cases} x = t^2 + t \\ y = t^2 - 10t + 5 \end{cases}$

$t^2 - 10t + 5 = 2(t^2 + t) + 41 \rightarrow t^2 + 12t + 36 = 0$

$b^2 - 4ac = 0 \rightarrow 12^2 - 4(1)(36) = 0$

Implicit $\frac{d}{dx}(y^2) = 2y \cdot \frac{dy}{dx}$ $\frac{d}{dx}(\cos y) = -\sin y \cdot \frac{dy}{dx}$

(differentiation of y). $\frac{dy}{dx}$

Ex $3y^2 - 2y + \boxed{2xy} = x^3$ $2xy$ $u = 2x$ $u' = 2$ $v = y$ $v' = \frac{dy}{dx}$

$6y \cdot \frac{dy}{dx} - 2 \cdot \frac{dy}{dx} + 2y + 2x \cdot \frac{dy}{dx} = 3x^2$ $2y + 2x \cdot \frac{dy}{dx}$

make $\frac{dy}{dx}$ subject $\frac{dy}{dx} = \frac{3x^2 - 2y}{6y - 2 + 2x}$

Rate of change of:

- length → cms^{-1}
- Area → $cm^2 s^{-1}$
- Volume → $cm^3 s^{-1}$

- proportional: direct $y = kx$; inverse $y = \frac{k}{x}$

- increase (+) decrease (-)

Ex gradient is proportional to xy → $\frac{dy}{dx} = kxy$

Chapter 6:

Finding area parametrically $\rightarrow$ originally $\int y . dx$

$f(t) \rightarrow x = 3t$ $\quad$ $y = 2t^2 \rightarrow g(t)$ $\quad$ $\int g(t) . f'(t) . dt$

find $\frac{dx}{dt}$ $\quad$ $\frac{dx}{dt} = 3$

and make dx subject $\quad$ $dx = 3 . dt$ $\quad$ $\int 2t^2 . 3 . dt$

Area $= \int y . dx$ $\quad$ Volume $= \pi \int y^2 . dx$

Integration by substitution

$\boxed{Ex}$ $\quad$ $\int x\sqrt{x+1} \, . dx$ $\quad$ $u = x+1$

① find $\frac{du}{dx}$ and make dx the subject

$\frac{du}{dx} = 1 \rightarrow dx = du$

② make x subject of formula (not always like $u = \sin x$)

$x = u - 1$

③ substitute what you can into equation

$\int (u-1) u^{\frac{1}{2}} . du = \int u^{\frac{3}{2}} - u^{\frac{1}{2}} . du = \frac{2}{5} u^{\frac{5}{2}} - \frac{2}{3} u^{\frac{3}{2}} + c$

④ If question requires substitute back to x

$\frac{2(1+x)^{5/2}}{5} - \frac{2(1+x)^{3/2}}{3} + c$

With integration limits: use correct limits for each one

- limits given by question are for x so only use them for x equation (step 4)
- To find limits for u, substitute x limits into $u = x+1$ eq $\rightarrow$ you might the limits to switch with the top number being smaller than on bottom so to switch just multiply whole equation by (-1)

Integration with partial fractions

- split fraction using chapter 2 and integrate each fraction

$\int \frac{3}{1-4x} = -\frac{3}{4} \ln(1-4x) + c$

$\int a^{kx} = \frac{1}{k} \cdot \frac{1}{\ln a} \cdot a^{x} + c$

Integration by parts

$$\int uv' \cdot dx = uv - \int vu' \cdot dx$$

how to choose u

- $\ln x$ ~~←~~ priority to $\ln x$
- x^n

if there's no $\ln x$ or x^n anything can be u

Ex $\int x^2 \cos x \cdot dx$ $u = x^2$ $u' = 2x$

$v = \sin x$ $v' = \cos x$

$= x^2 \sin x - \int 2x \sin x \longrightarrow \int 2x \sin x \cdot dx$ $u = 2x$ $u' = 2$

$v = -\cos x$ $v' = \sin x$

$= -2\cos x - \int -2\cos x$

$= -2\cos x + \int 2\cos x = (-2\cos x + 2\sin x + c)$

$= x^2 \sin x + 2\cos x - 2\sin x + c$

General solution of differential equation

Ex
$$\frac{dy}{dx} = (1+y)(1-2x)$$

~~terms~~ collect all y terms on dy side and x terms to dx side

raise dx and divide by $(1+y)$

$$\int \frac{1}{1+y} \cdot dy = \int (1-2x) \cdot dx$$

$\ln(1+y) + c = x - x^2 + d \longrightarrow \ln(1+y) = x - x^2 + k$

at this point question would give you when $y = $ "1", $x = $ "0" substitue into equation at this point

$k = \ln 2 \longrightarrow \ln(1+y) = x - x^2 + \ln 2 \longrightarrow y = 2e^{x-x^2} - 1$

Ex $\int \ln x \cdot dx$ $u = \ln x$ $u' = \frac{1}{x}$

$v = x$ $v' = 1$

$= x\ln x - \int 1 \cdot dx = x\ln x - x + c$

Chapter 7:

$(i+j+k)$ $\quad$ column vector $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $\quad$ magnitude $= \sqrt{x^2+y^2+z^2}$

(x, y, z)

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

- to find area use magnitude of vector : area of triangle $= \frac{1}{2}|a|\cdot|b|\cdot\sin\theta$ or $\frac{1}{2}\cdot|b|\cdot|h|$
- equation of line $\rightarrow$ $r = \underset{\sim}{a} + \lambda \underset{\sim}{b}$ (point a, direction vector b)
- if C and D lie on line equation of line : $C + \lambda(\overrightarrow{CD})$ or $D + \mu(\overrightarrow{CD})$
- if lines are parallel they have the same direction vector
- if lines are perpendicular direction vectors multiplied by each other $=0$ $(a\cdot b = 0)$
- skew means not parallel and do not intersect
- to find point of intersection:

  1- write equation as coloumn vector (easier) $\quad l_1 = \begin{pmatrix} 1+\lambda \\ 3-\lambda \\ 5\lambda \end{pmatrix}$ and $l_2 = \begin{pmatrix} -1+\mu \\ -1+\mu \\ -2+2\mu \end{pmatrix}$

  2- set each equation equal to each other $\quad \begin{matrix} 1+\lambda \\ 3-\lambda \\ 5\lambda \end{matrix} = \begin{matrix} -1+\mu \\ -1+\mu \\ -2+2\mu \end{matrix}$

  3- put i in equation on its own and j in equation on its own find λ and μ $\quad$ i: $\lambda - \mu = 2$ $\quad$ j: $\lambda + \mu = 4$

  4- Substitute λ and μ in each k equation for l_1 or l_2, if they're equal then they intersect

  5- To find point substitute λ in l_1 or μ in l_2

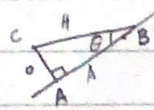
$\cos\theta = \frac{ab}{|a|\cdot|b|}$ $\quad$ a and b are the direction vectors and $|a|$ and $|b|$ are their magnitudes

both vectors have to be going away from point of intersection $\quad$ if not you would find α but you want θ

α θ, how would you know that you have wrong angle? (either question would tell you acute or obtuse if it didn't then mafrood both are accepted) $\hookrightarrow$ $\theta = 180 - \alpha$

- if lines perpendicular $\quad \cos 90 = 0 \rightarrow a\cdot b = 0$
- if lines are parallel $\quad \cos\theta = 1 \rightarrow a\cdot b = |a|\cdot|b|$
- parallel means multiples of direction vector
- when asked to find shortest distance

can be found like this



$\sin\theta = \frac{O}{H} \rightarrow \sin\theta = \frac{|CA|}{|CB|}$

$|CA| = |CB|\sin\theta$

Proofs:

Steps:

1 - Assumption ← opposite of what question wants

2 - Logical steps ← prove your assumption wrong and at the end write → so there is a contradiction

3 - Conclusion ← take from question exactly → the original assumption has been proven false therefore...

Even number: $n = 2k$

Odd number: $n = 2k + 1$

Rational no.: any number can be written in the form $\frac{a}{b}$ where a, b have no common factor

Irrational no.: can't be written as $\frac{a}{b}$

Integer is +ve, -ve or zero (no fractions)

Whole number is +ve or zero (no fractions)

Natural number is +ve (no fraction)

Equations to know:

Area

rectangle = $L \times W$

Parallelogram = $b \times h$

Triangle = $\frac{1}{2}bh$ or $\frac{1}{2}ab\sin\theta$

Trapezium = $\left(\frac{a+b}{2}\right)h$

Circle = πr^2

Surface area

rectangular prism = $2(wl + hl + hw)$

Cube = $6x^2$

Cylinder = $2\pi r(r+h)$

Cone = $\pi r(r+h)$

Sphere = $4\pi r^2$

Hemi-sphere = $3\pi r^2$

Volume

rectangular prism = $l \times w \times h$

Cube = x^3

Cylinder = $\pi r^2 h$

Sphere = $\frac{4}{3}\pi r^3$

Cone = $\frac{1}{3}\pi r^2 h$

Pyramid = $\frac{1}{3} \times h \times$ area of base

$\int k \frac{f'(x)}{f(x)}$ → let $y = \ln(f(x))$ and adjust

$\int k f'(x) \cdot [f(x)]^n$ → let $y = [f(x)]^{n+1}$