Chapter 1 :

- only simplify fraction vertically and diagonally
- ALWAYS FACTORISE
- when there are 3 terms in the question work on 2 first then bring back 3rd term
- when x^2 has a coefficient, after factorising make sure the coefficient would still be there
- if you want to have same denominator here

$\frac{3x}{(x+4)^2} - \frac{1}{(x+4)}$ you cannot do this $\times \left(\frac{1}{(x+4)}\right)^2 \rightarrow$ multiply the whole fraction by $(x+4)$

- Improper Algebric fractions
  - the exponent in the numerator is larger than in the denominator
    $\frac{x^2+5x+8}{x-2}$ $\qquad x^2 > x^1$
  - the exponent in the numerator is equal to the one in the denominator
    $\frac{x^3+5x-9}{x^3-4x^2+7x-3}$ $\qquad x^3 = x^3$

To turn into mixed fraction:

1- long division $\frac{x^3+2x^2+3x-4}{x+1} \equiv Q(x) + \frac{\text{remainder}}{\text{divisor}}$

$$x+1 \overline{\smash{)}\, x^3+2x^2+3x-4} \quad \text{quotient } x^2+x+2 \quad \text{remainder } -6 \rightarrow x^2+x+2 + \frac{-6}{x+1}$$

2- $f(x) = Q(x) \times \text{divisor} + \text{remainder}$

$$\underbrace{8x^4+2x^2+5}_{f(x)} \equiv \underbrace{(Ax+B)}_{Q}\underbrace{(2x^2+2)}_{Di} + \underbrace{[Cx+D]}_{\text{Remainder}}$$

can switch but use whats easier where Di is known

$$Di \overline{\smash{)}\, f(x)} \quad \text{quotient } Q(x) \quad \text{remainder } R$$

- with long division only divide with first term ~~with~~ in divisor and first time in $f(x)$ and then multiply the term you get with all of the divisor

Chapter 2:

- if solving equation has 2 absolute value equations take once as (+ve +ve) and then (-ve -ve)
- solving absolute value take once as (+) and once (-) get 2 answers
- if $|x-2| = -7$ then it has no solution
- Function → one domain (x) has one image from range (y)  one-to-one
  → multiple domains (x) have one image from range (y)  many-to-one
- Not a function → One element in domain (x) has multiple images in range (y)  one-to-many
- * vertical line test (|): passes one point → F
  passes two points → NF
  if it was a function do Horizontal line test (—): passes one point → one-to-one
  passes two points → many-to-one
- any equation with asymptotes is not a function
- ∘ → > or <   • → ≥ or ≤
- if you have quadratic equation and want the inverse put it in the form of completing the square
  $a\left(\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2\right) + c$
- when asked for range and domain draw small sketch
- inverse function is reflection of original function across $y=x$
- domain of main = range of inverse
  range of main = domain of inverse
- $|f(x)|$ → reflect any parts where $f(x)<0$ over x-axis (no -ve y and reflect) 
- $f(|x|)$ → erase $x \leq 0$ then reflect $x \geq 0$ over y-axis (erase -ve x then reflect +ve x) 

Translation
- $f(x+a)$ (x axis) +a → left -a → right   (y-axis) $f(x)+a$ +a → up -a → down
- Stretch $f(ax)$ multiply x by $\frac{1}{a}$ $af(x)$ multiply y by a
- Reflection $-f(x)$ over x-axis ⇅ $f(-x)$ over y-axis ⇆

Ex: $f(x) = 5(3x-13)+2$ with x translate (±) then stretch ($\frac{1}{a}$)
$(0,0) \to \left(\frac{13}{3}, 2\right)$ with y stretch (a) then translate (±)

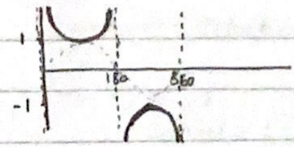
- be careful $|0-1| = 1$
- $f(x) = f^{-1}(x) = x$] make one of the functions $= x$] as $f^{-1}(x)$ and $f(x)$ intersect at $y=x$
- if $g(a) = f^{-1}(a)$ → $fg(a) = ff^{-1}(a)$ → $fg(a) = a$
- when asked $f^{-1}(x)$ give domain even if not asked
- write $f^{-1}(x)$ not $y=$

Chapter 3:

- $\sin\theta = \sin(180 - \theta)$
- don't forget to add 360 and -360 to angles you find or if 2θ its okay if its not in range still
- $\boxed{\operatorname{cosec} x = \frac{1}{\sin x}}$

  range: $y \leq -1$ or $y \geq 1$

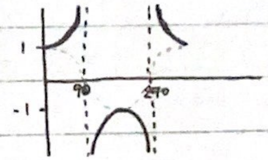
  domain: $0 < x < 180$ and $180 < x < 360$



- $\boxed{\sec x = \frac{1}{\cos x}}$

  range: $y \leq -1$ or $y \geq 1$

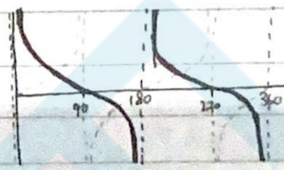
  domain: $0 \leq x < 90$ and $90 < x < 270$ and $270 < x \leq 360$



- $\boxed{\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}}$

  range: $y \in \mathbb{R}$

  domain: $0 < x < 180$ and $180 < x < 360$



- $\sin^2\theta + \cos^2\theta = 1$

  $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1}{\cot\theta}$

  $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$

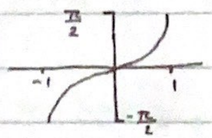
  $\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} \rightarrow \tan^2\theta + 1 = \sec^2\theta$

  $\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta} \rightarrow \cot^2\theta + 1 = \operatorname{cosec}^2\theta$

$\arcsin x = \sin^{-1} x$

range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

domain: $-1 \leq x \leq 1$



$\arctan x = \tan^{-1} x$

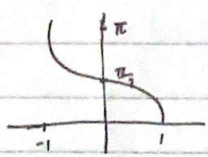
range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$

domain: $x \in \mathbb{R}$

$\arccos x = \cos^{-1} x$

range: $0 \leq y \leq \pi$

domain: $-1 \leq x \leq 1$



Chapter 4:

$\sin(A+B) \equiv \sin A\cos B + \sin B\cos A$

$\sin(A-B) \equiv \sin A\cos B - \sin B\cos A$

$\cos(A+B) \equiv \cos A\cos B - \sin A\sin B$

$\cos(A-B) \equiv \cos A\cos B + \sin A\sin B$

$\tan(A+B) \equiv \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B) \equiv \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

Proof for $\tan(A+B)$:

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A\cos B + \sin B\cos A}{\cos A\cos B - \sin A\sin B}$$

divide numerator and denominator by $\cos A\cos B$

$$\frac{\frac{\sin A\cos B}{\cos A\cos B} + \frac{\sin B\cos A}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} - \frac{\sin A\sin B}{\cos A\cos B}} = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

Proof for $\tan(A-B)$:

$$\tan(A-B) = \frac{\sin(A-B)}{\cos(A-B)} = \frac{\sin A\cos B - \sin B\cos A}{\cos A\cos B + \sin A\sin B}$$

divide numerator and denominator by $\cos A\cos B$

$$\frac{\frac{\sin A\cos B}{\cos A\cos B} - \frac{\sin B\cos A}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} + \frac{\sin A\sin B}{\cos A\cos B}} = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

$\sin 2A = \sin(A+A) = \sin A\cos A + \sin A\cos A = 2\sin A\cos A$

$\tan 2A = \tan(A+A) = \dfrac{\tan A + \tan A}{1 + \tan A\tan A} = \dfrac{2\tan A}{1+\tan^2 A}$

$\cos 2A = \cos(A+A) = \cos A\cos A - \sin A\sin A = \cos^2 A - \sin^2 A$

$\cos^2 A - (1-\cos^2 A) = 2\cos^2 A - 1$

$(1-\sin^2 A) - \sin^2 A = 1 - 2\sin^2 A$

$\sin 2A \equiv 2\sin A\cos A$

$\tan 2A \equiv \dfrac{2\tan A}{1+\tan^2 A}$

$\cos 2A \equiv \cos^2 A - \sin^2 A$

$\cos 2A \equiv 2\cos^2 A - 1$

$\cos 2A \equiv 1 - 2\sin^2 A$

- never divide by any $\sin\theta$ / $\cos\theta$ anything, that way you would lose an answer
- focus if they want min or max if the $\sin\theta$ is numerator or denominator
  - ↳ if they want max and $\sin\theta$ in denominator then $\sin\theta$ min

$a\sin\theta + b\cos\theta \rightarrow R\sin(\theta+\alpha)$

$a\cos\theta + b\sin\theta \rightarrow R\cos(\theta-\alpha)$

$a\sin\theta - b\cos\theta \rightarrow R\sin(\theta-\alpha)$

$a\cos\theta - b\sin\theta \rightarrow R\cos(\theta+\alpha)$

same thing depends on what question wants

$3\sin\theta + 4\cos\theta$:

- Expand $R\sin(\theta+\alpha) = R[\sin\theta\cos\alpha + \sin\alpha\cos\theta] = R\sin\theta\cos\alpha + R\sin\alpha\cos\theta$

- $R\sin\theta\cos\alpha + R\sin\alpha\cos\theta = 3\sin\theta + 4\cos\theta$

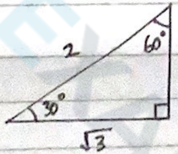
- $R\sin\theta\cos\alpha = 3\sin\theta \rightarrow R\cos\alpha = 3$

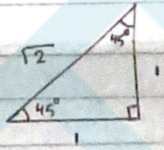
  $R\sin\alpha\cos\theta = 4\cos\theta \rightarrow R\sin\alpha = 4$

  be careful if smth is -ve

$\frac{R\sin\alpha}{R\cos\alpha} = \frac{4}{3} = \tan\alpha = \frac{4}{3}$ $\quad \alpha = \tan^{-1}\left(\frac{4}{3}\right)$ should always be +ve and inside range

$$\boxed{R = \sqrt{a^2 + b^2}}$$

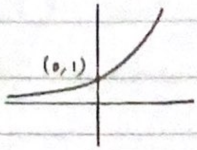




Chapter 5:

$y = a^x$

$y = 0$ asymptote



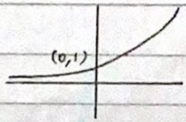
$a > 1$

increasing function

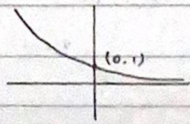
$0 < a < 1$

decreasing function

- a can't be -ve because its neither increasing or decreasing
- a can't equal 1 bc $y = 1^x$ then all $y = 1$ so becomes straight line
- the smaller the base (a) the closer the cure is to x-axis

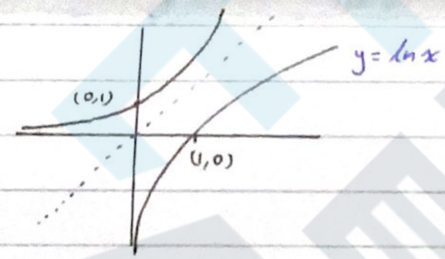


$y = e^x$



$y = e^{-x}$

$y = \ln x$ ← inverse of $y = e^x$



$\ln e^x = e^{\ln x} = x$

ln has same rules as log

$f(x) = Ae^{Bx} + C$ asymptote

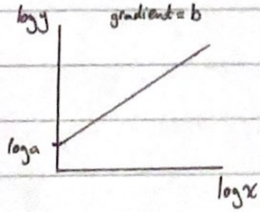
$y = ax^b$ (curve)

$\log y = \log ax^b$

$\log y = \log a + b \log x$

$\boxed{\log y} = b\boxed{\log x} + \log a$

$y = mx + c$



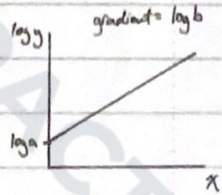
$y = ab^x$ (exponential)

$\log y = \log ab^x$

$\log y = x \log b + \log a$

$\boxed{\log y} = (\log b)\boxed{(x)} + \log a$

$y = mx + c$



Chapter 6:

- $f(x) = \sin kx$ $\quad f'(x) = k\cos kx$
- $f(x) = \cos kx$ $\quad f'(x) = -k\sin kx$

$f(x) = e^{kx}$ $\quad f'(x) = ke^{kx}$

$f(x) = \ln x$ $\quad f'(x) = \frac{1}{x}$

$f(x) = a^x$ $\quad f'(x) = \ln(a) \cdot a^x$

$f(x) = a^{kx}$ $\quad f'(x) = k \cdot \ln(a) \cdot a^{kx}$

don't forget
$f(x) = \ln 1$
$f'(x) = 0$

differentiation using first principle for $\cos x$ and $\sin x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

using $\sin(A \pm B)$
$\cos(A \pm B)$

given that $\frac{\sin h}{h} = 1$

$\frac{(\cos h - 1)}{h} = 0$

$f(x) = \sin x$

$$f'(x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$\lim_{h \to 0} \frac{\sin x \cos h + \sin h \cos x - \sin x}{h}$$

$$\lim_{h \to 0} \sin x \frac{(\cos h - 1)}{h} + \frac{\sin h}{h} \cos x$$

$$\sin x(0) + (1)\cos x = \cos x$$

$f(x) = \sin x$
$f'(x) = \cos x$

- Proof for $y = a^x \rightarrow \frac{dy}{dx} = \ln(a) \cdot a^x$

$$y = a^x \rightarrow y = e^{\ln a^x} \rightarrow y = e^{x \ln a}$$

$$\frac{dy}{dx} = \ln a \cdot e^{x \ln a} \rightarrow \frac{dy}{dx} = \ln a \cdot e^{\ln a^x}$$

$$\frac{dy}{dx} = \ln a \cdot a^x$$

- Proof for $y = a^{kx} \rightarrow \frac{dy}{dx} = k\ln a \cdot a^{kx}$

$$y = a^{kx} \rightarrow y = e^{\ln a^{kx}} \rightarrow y = e^{x k \ln a}$$

$$\frac{dy}{dx} = k\ln a \cdot e^{xk\ln a} \rightarrow \frac{dy}{dx} = k\ln a\, e^{\ln a^{kx}}$$

$$\frac{dy}{dx} = k\ln a \cdot a^{kx}$$

- $f(x) = e^{2x^2 - 1}$ $\quad f'(x) = 4x e^{2x^2-1}$] derivative of power $\cdot$ $e^{\text{original power}}$
- $f(x) = \ln(x^2)$ $\quad f'(x) = \frac{2x}{x^2} = \frac{2}{x}$] $\frac{\text{derive inside } \ln}{\text{inside } \ln}$

- Function rotation

$f(x)^n$ $\quad f'(x) = nf(x)^{n-1} f'(x)$

$f(x) = (\sin x)^3$ $\quad f'(x) = 3\sin^2 x \cos x$

- Chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\boxed{\begin{aligned} y &= f(g(x)) \\ \frac{dy}{dx} &= f'(g(x))\, g'(x) \end{aligned}}$$

$y = (1+2x)^5$

let $u = 1+2x$

$\cancel{dy}\ \frac{du}{dx} = 2$

$y = u^5$

$\frac{dy}{du} = 5u^4$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= 5u^4 \times 2 = 10(1+2x)^4$$

- $f(x) = \sin(ax+b)$ $\qquad f'(x) = a\cos(ax+b)$
- $f(x) = \cos(ax+b)$ $\qquad f'(x) = \cancel{a\sin}\ -a\sin(ax+b)$

- $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$

- Product rule:

doesn't matter which is which

$$y = \overset{u}{f(x)}\, \overset{v}{g(x)}$$

$$\frac{dy}{dx} = f'(x)g(x) + f(x)g'(x)$$

$$\frac{dy}{dx} = vu' + v'u$$

- Quotient rule

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{vu' - v'u}{v^2}$$

- point of inflection $\rightarrow$ $\frac{d^2y}{dx^2} = 0$

$f(x)$	$f'(x)$
$\sin kx$	$k\cos kx$
$\cos kx$	$-k\sin kx$
$\tan kx$	$k\sec^2 kx$
$\cot kx$	$-k\,\mathrm{cosec}^2 kx$
$\sec kx$	$k\tan kx\sec kx$
$\mathrm{cosec}\, kx$	$-k\cot kx\,\mathrm{cosec}\, kx$
$\arcsin kx$	$\frac{k}{\sqrt{1-(kx)^2}}$ ← derivative of inside
$\arccos kx$	$-\frac{k}{\sqrt{1-(kx)^2}}$
$\arctan kx$	$\frac{k}{1+(kx)^2}$
$\mathrm{arccot}\, kx$	$-\frac{k}{1+(kx)^2}$
$\mathrm{arcsec}\, kx$	$\frac{k}{kx\sqrt{(kx)^2-1}}$
$\mathrm{arccosec}\, kx$	$-\frac{k}{kx\sqrt{(kx)^2-1}}$

Proof for each one:

$u = \sin x \quad u' = \cos x$

$v = \cos x \quad v' = -\sin x$

$$y = \tan x = \frac{\sin x}{\cos x}$$

$$\frac{dy}{dx} = \frac{vu' - v'u}{v^2} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

$$y = \tan x \qquad \frac{dy}{dx} = \sec^2 x$$

$u = \cos x \quad u' = -\sin x$

$v = \sin x \quad v' = \cos x$

$$y = \cot x = \frac{\cos x}{\sin x}$$

$$\frac{dy}{dx} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = -\frac{1}{\sin^2 x} = -\mathrm{cosec}\, x$$

$$y = \cot x \qquad \frac{dy}{dx} = -\mathrm{cosec}\, x$$

$$y = \sec x = \frac{1}{\cos x} = (\cos x)^{-1}$$

$$\frac{dy}{dx} = -1(\cos x)^{-2}(-\sin x) = \frac{\sin x}{\cos^2 x} = \cancel{\tan x \sec x}\ \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x} = \tan x \cdot \sec x$$

$$y = \sec x \qquad \frac{dy}{dx} = \tan x \sec x$$

$$y = \mathrm{cosec}\, x = \frac{1}{\sin x} = (\sin x)^{-1}$$

$$\frac{dy}{dx} = -1(\sin x)^{-2}(\cos x) = -\frac{\cos x}{\sin^2 x} = -\frac{\cos x}{\sin x}\cdot\frac{1}{\sin x} = -\cot x\,\mathrm{cosec}\, x$$

$$y = \mathrm{cosec}\, x \qquad \frac{dy}{dx} = -\cot x\,\mathrm{cosec}\, x$$

$y = \arcsin x \rightarrow \sin y = x$

$\frac{dx}{dy} = \cos y \rightarrow \frac{dy}{dx} = \frac{1}{\cos y}$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

$\sin^2 y + \cos^2 y = 1$

$\cos y = \sqrt{1-\sin^2 y}$

$\cos y = \sqrt{1-x^2}$

$y = \arcsin x \qquad \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

$y = \arccos x \rightarrow \cos y = x$

$\frac{dx}{dy} = -\sin y \rightarrow \frac{dy}{dx} = -\frac{1}{\sin y}$

$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$

$\sin^2 y + \cos^2 y = 1$

$\sin y = \sqrt{1-\cos^2 y}$

$\sin y = \sqrt{1-x^2}$

$y = \arccos x \qquad \frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$

$y = \arctan x \rightarrow \tan y = x$

$\frac{dx}{dy} = \sec^2 y \rightarrow \frac{dy}{dx} = \frac{1}{\sec^2 y}$

$\frac{dy}{dx} = \frac{1}{1+x^2}$

$\sec^2 y = 1 + \tan^2 x$

$\sec^2 y = 1 + x^2$

$y = \arctan x \qquad \frac{dy}{dx} = \frac{1}{1+x^2}$

$y = \text{arccot}\, x \rightarrow \cot y = x$

$\frac{dx}{dy} = -\text{cosec}^2 x \rightarrow \frac{dy}{dx} = -\frac{1}{\text{cosec}^2 x}$

$\frac{dy}{dx} = -\frac{1}{1+x^2}$

$\text{cosec}^2 y = 1 + \cot^2 y$

$\text{cosec}^2 y = 1 + x^2$

$y = \text{arccot}\, x \qquad \frac{dy}{dx} = -\frac{1}{1+x^2}$

$y = \text{arcsec}\, x \rightarrow \sec y = x$

$\frac{dx}{dy} = \sec y \tan y \rightarrow \frac{dy}{dx} = \frac{1}{\sec y \tan y}$

$\frac{dy}{dx} = \frac{1}{x\sqrt{x^2-1}}$

$\sec^2 y = \tan^2 y + 1$

$\tan y = \sqrt{\sec^2 y - 1}$

$\tan y = \sqrt{x^2 - 1}$

$y = \text{arcsec}\, x \qquad \frac{dy}{dx} = \frac{1}{x\sqrt{x^2-1}}$

$y = \text{arccosec}\, x \rightarrow \text{cosec}\, y = x$

$\frac{dx}{dy} = -\text{cosec}\, y \cot y \rightarrow \frac{dy}{dx} = -\frac{1}{\text{cosec}\, y \cot y}$

$\frac{dy}{dx} = -\frac{1}{x\sqrt{x^2-1}}$

$\text{cosec}^2 y = \cot^2 y + 1$

$\cot y = \sqrt{\text{cosec}^2 y - 1}$

$\cot y = \sqrt{x^2 - 1}$

$y = \text{arccosec}\, x \qquad \frac{dy}{dx} = -\frac{1}{x\sqrt{x^2-1}}$

Chapter 7:

$\int x^n \cdot dx = \frac{x^{n+1}}{n+1} + c$

$\int e^x \cdot dx = e^x + c$

$\int \frac{1}{x} \cdot dx = \int x^{-1} \cdot dx = \ln|x| + c$

$\int \sin x \cdot dx = -\cos x + c$

$\int \cos x \cdot dx = \sin x + c$

$\int \sec^2 x \cdot dx = \tan x + c$

$\int \operatorname{cosec}^2 x \cdot dx = -\cot x + c$

$\int \sec x \tan x \cdot dx = \sec x + c$

$\int \operatorname{cosec} x \cot x \cdot dx = -\operatorname{cosec} x + c$

when you have limits with integration always work in radians

only for linear $\int \cos(2x-1) \cdot dx$

$\frac{1}{\text{derivative}} = \frac{1}{a}$ which would mean coefficient of x bc this is only for linear $\longrightarrow \frac{1}{2}\sin(2x-1) + c$

$\int \frac{1}{2x+1} \cdot dx \longrightarrow \frac{1}{2}\ln|2x+1| + c$

this isn't linear so $\int (3x-1)^3 \cdot dx \xrightarrow{\text{act like}} \int u^3 \cdot dx = \frac{u^4}{4}$

$\frac{1}{3} \times \frac{(3x-1)^4}{4} + c = \frac{(3x-1)^4}{12} + c$

use trigonometric identities

$\sin 2x \equiv 2\cos x \sin x \longrightarrow \cos x \sin x = \frac{1}{2}\sin 2x$

$\cos 2x \equiv 2\cos^2 x - 1 \longrightarrow \cos^2 x = \frac{\cos 2x + 1}{2}$

$\cos 2x \equiv 1 - 2\sin^2 x \longrightarrow \sin^2 x = \frac{1 - \cos 2x}{2}$

$\sec^2 x \equiv \tan^2 x + 1 \longrightarrow \tan^2 x = \sec^2 x - 1$

$\operatorname{cosec}^2 x \equiv \cot^2 x + 1 \longrightarrow \cot^2 x = \operatorname{cosec}^2 x - 1$

with integration there can't be multiplication

you can take multiplication with numbers outside $\int \sin 2x \cos 2x = \int \frac{1}{2} \sin 4x = \frac{1}{2}\int \sin 4x$

$= \frac{1}{2}\left[-\frac{1}{4}\cos 4x\right]$

Reverse chain rule

case ①

$\int k \frac{f'(x)}{f(x)} \cdot dx$ let $y = \ln|f(x)|$ find $\frac{dy}{dx}$ to check

and then adjust any constants

Ex $\int \frac{e^{2x}}{e^{2x}+1} \cdot dx$

$y = \ln|e^{2x}+1|$

$\frac{dy}{dx} = \frac{2e^{2x}}{e^{2x}+1}$

↳ adjust $y = \frac{1}{2}\ln|e^{2x}+1| + c$

case ②

$\int k\, f'(x) \cdot [f(x)]^n \cdot dx$ let $y = [f(x)]^{n+1}$ find $\frac{dy}{dx}$ to check

and then adjust any constant

Ex $\int \frac{\sin 2x}{(3+\cos 2x)^3} \cdot dx = \int \sin 2x\,(3+\cos 2x)^{-3}$

$y = (3+\cos 2x)^{-2}$

$\frac{dy}{dx} = -2(3+\cos 2x)^{-3}(-2\sin 2x) = 4\sin 2x\,(3+\cos 2x)^{-3}$

$\rightarrow y = \frac{1}{4}\sin 2x\,(3+\cos 2x)^{-3} + c$

exception :

in case ② : e

- you don't do +1 to the power you take e^{x} as $f(x)$ and find $\frac{dy}{dx}$ for e^{x} alone then act as normal $\rightarrow \int \sec^2 x\, e^{4\tan x} \rightarrow y = e^{4\tan x}$

$\frac{dy}{dx} = 4\sec^2 x\, e^{4\tan x}$

↳ $y = \frac{1}{4} e^{4\tan x}$

Chapter 8:

Locating roots $f(x_1) = +ve > 0$ $f(x_2) = -ve < 0$

sign change implies root

Locating turning point $f'(x_1) = +ve > 0$ $f'(x_2) = -ve < 0$

switch depend on xs

Ex. $f'(4) = -ve$ $f'(10) = +ve$

dec to inc

$f'(1) = +ve$ $f'(7) = -ve$

inc to dec

sign change implies slope changes from dec to inc over interval, which implies turning point

when asked to show that for example $x = 1.441$ is a root (or tp)

add dp to x → 1.4410 find upper and lower bound

1.4405 1.4415

one should be +ve and the other -ve so: (you write answers and < 0 or > 0)

sign change implies x lies in the range $1.4405 < x < 1.4415$ so

$x = 1.441$ correct to 3 decimal places

Iterative method

$f(x) = 0$

"question tells you how to rearrange formula"

Ex: $x^2 - x + 1 = 0$

ex. ~~so x^2 is subject~~ $x = 1 - \frac{1}{x}$

$x^2 = x - 1$

$x = 1 - \frac{1}{x}$

$$\left[x_{n+1} = 1 - \frac{1}{x_n} \right]$$

question will give you first x like x_0 then will either tell you what x's to find or until the anwer is constant to 3 dp

* any answer you get from any rearrangement of the equation is an answer to the original equation

$$\int (3x+1)^4 = u^4 \quad - \quad \frac{u^5}{5}$$

$$\frac{1}{3} \times \frac{(3x+1)^5}{5}$$

$$\cos^2 x + \sin^2 x + 2\cos x \sin x = (\cos x + \sin x)^2$$

$$1 - \sin^2 x = (1 + \sin x)(1 - \sin x)$$