

Surds

1. $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
2. $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$

Indices

1. $a^m \times a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$
3. $(a^m)^n = a^{mn}$
4. $a^0 = 1$
5. $a^{-n} = \frac{1}{a^n}$
6. $a^{\frac{1}{n}} = \sqrt[n]{a}$
7. $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

Rules of rationalizing

- Fractions in the form $\frac{1}{\sqrt{a}}$, multiply the top and bottom by \sqrt{a} .
- Fractions in the form $\frac{1}{\sqrt{a} + \sqrt{b}}$, multiply the top and bottom by $\sqrt{a} - \sqrt{b}$.
- Fractions in the form $\frac{1}{\sqrt{a} - \sqrt{b}}$, multiply the top and bottom by $\sqrt{a} + \sqrt{b}$.

Logarithms

1. $\log a + \log b = \log ab$
2. $\log a - \log b = \log \frac{a}{b}$
3. $a \log_x y = \log_x y^a$
4. $\log_a a = 1$
5. $\log_x \frac{\log b^x}{\log b} = \frac{\log b^x}{\log b}$
6. $\log_a 1 = 0$
7. $\log_a b = \frac{1}{\log b} a$

Quadratic Equation

Solving quadratic equation

Quadratic equation can be solved by:

1. factorization
2. completing the square:

$$x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$

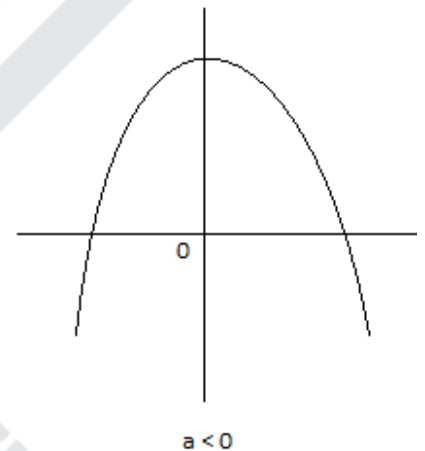
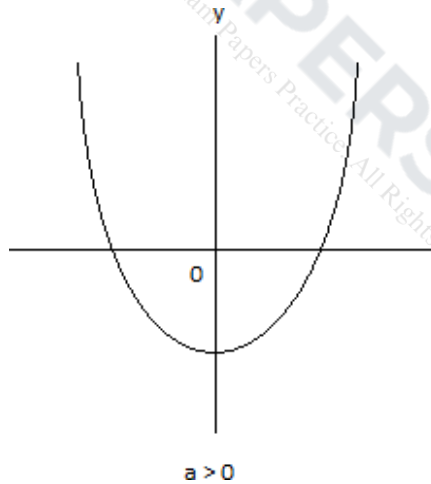
3. using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

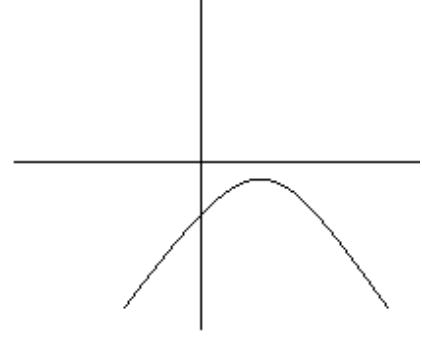
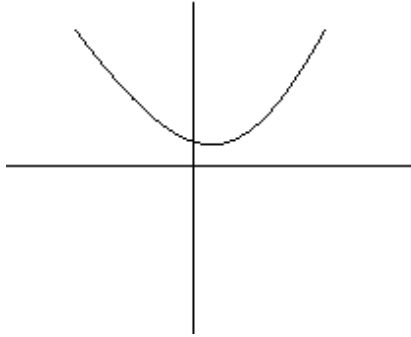
Nature of roots

- $ax^2 + bx + c = 0$

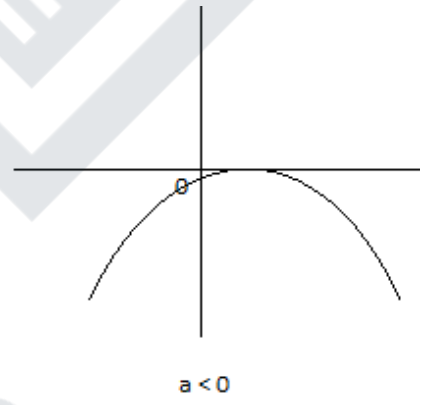
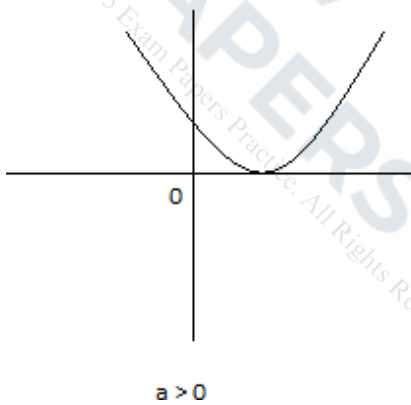
1. If $b^2 - 4ac > 0$, roots are real & different / real and distinct and the curve $y = ax^2 + bx + c$ will cut the x axis at two real and distinct points



2. If $b^2 - 4ac < 0$, roots are not real/ imaginary / complex and the curve $y = ax^2 + bx + c$ will lie entirely above the x axis if $a > 0$ and entirely below the x axis if $a < 0$.



3. If $b^2 - 4ac = 0$, roots are real and equal / repeated / coincident and the curve $y = ax^2 + bx + c$ touches the x-axis.



4. If $b^2 - 4ac \geq 0$, roots are real.

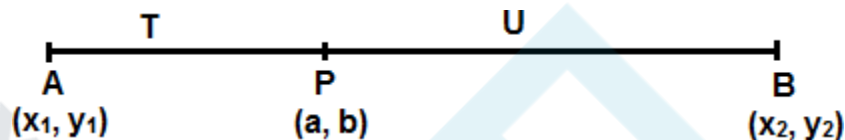
Solving Quadratic Inequality

When α and β ($\alpha < \beta$) are two roots of $ax^2 + bx + c = 0$ ($a > 0$) and

1. If $ax^2 + bx + c > 0$, range of values of x : $x < \alpha$, $x > \beta$
2. If $ax^2 + bx + c \geq 0$, range of values of x : $x \leq \alpha$, $x \geq \beta$
3. If $ax^2 + bx + c < 0$, range of values of x : $\alpha < x < \beta$
4. If $ax^2 + bx + c \leq 0$, range of values of x : $\alpha \leq x \leq \beta$

Co – ordinate Geometry

1. The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
2. The gradient of the line joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\frac{y_2 - y_1}{x_2 - x_1}$
3. The coordinates of the mid-point of the line joining $A(x_1, y_1)$ and $B(x_2, y_2)$ are $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$.
4. Finding coordinates when a point divides a line internally.



$$a = \frac{(U \times x_1) + (T \times x_2)}{T + U}$$

$$b = \frac{(U \times y_1) + (T \times y_2)}{T + U}$$

5. The equation of the straight line having a gradient m and passing through the point (x_1, y_1) is given by: $y - y_1 = m(x - x_1)$.
6. Two lines are parallel if their gradients are equal.
7. Two lines are perpendicular to each other if the product of their gradients is -1 .

Equation of circle

Centre (a, b) and radius $= r$

$$(x - a)^2 + (y - b)^2 = r^2$$

Arithmetic Progression (A.P)

1. n th term $= a + (n - 1)d$
2. $S_n = \frac{n}{2}\{2a + (n - 1)d\}$

Geometric Progression (G.P)

1. n th term $= ar^{n-1}$
2. $S_n = \frac{a(r^n - 1)}{r - 1}$, $r > 1$
3. $S_n = \frac{a(1 - r^n)}{1 - r}$, $r < 1$

$-1 < r < 1$ or $|r| < 1$.

The series is convergent. It has sum to infinity.

1. $S_\infty = \frac{a}{1 - r}$

Otherwise the series is divergent. It has does not have sum to infinity.

Differentiation

1. For a curve $y = f(x)$ represents the gradient of the tangent to the curve at any point x .
2. If $y = ax^n$, then $\frac{dy}{dx} = anx^{n-1}$, where a and n are constants.
3. $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$
4. If y is a function of u , and u is a function of x , then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ (chain rule).
5. If y , u and v are functions of x and $y = uv$, then $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$ (product rule).
6. If y , u and v are functions of x and $y = \frac{u}{v}$, then $\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$ (quotient rule).

The following are true only when x is in radians:

7. $\frac{d}{dx}(\sin x) = \cos x$
8. $\frac{d}{dx}(\cos x) = -\sin x$

Other formulae

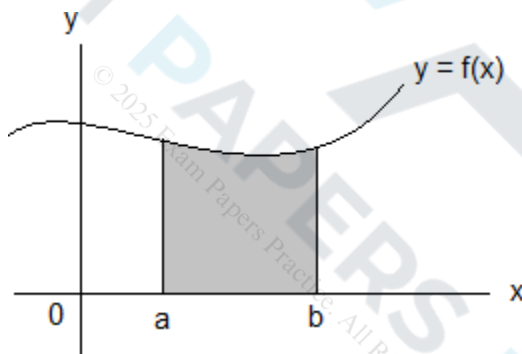
9. $\frac{d}{dx}(\sin^n x) = n \sin^{n-1} x (\cos x)$
10. $\frac{d}{dx}(\cos^n x) = n \cos^{n-1} x (-\sin x)$

Application of Differentiation

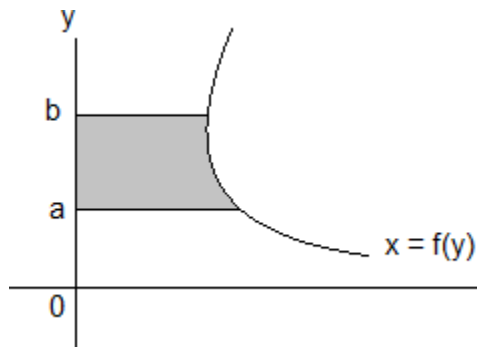
11. For an increasing function $f(x)$ in the interval (a, b) $f'(x) > 0$ in the interval $a \leq x \leq b$.
12. For a decreasing function $f(x)$ in the interval (a, b) $f'(x) < 0$ in the interval $a \leq x \leq b$.
13. Stationary points or turning points of a function $y = f(x)$ occur when $\frac{dy}{dx} = 0$.
14. The second derivative ($\frac{d^2y}{dx^2}$) determines the nature of the stationary points:
 - (a) If $\frac{d^2y}{dx^2}$ is negative, the stationary point is a maximum point.
 - (b) If $\frac{d^2y}{dx^2}$ is positive, the stationary point is a minimum point.
 - (c) If $\frac{d^2y}{dx^2}$ is zero, the point could be either a maximum or a minimum point or a point of inflexion.
 - (d) If $\frac{d^2y}{dx^2}$ is zero $\frac{d^3y}{dx^3}$ is not equal to zero, then the stationary point is point of inflexion.
15. To sketch a curve, note
 - (i) the points where $x = 0$ or $y = 0$
 - (ii) the nature and position of the stationary points
 - (iii) the direction of the curve as x and y approach infinity.
 - (iv) the interval on which the gradient is positive or negative.

Integration

1. $\int ax^n dx = \frac{ax^{n+1}}{n+1} + c \quad n \neq -1$
2. $\int (ax + b)^n dx = \frac{(ax+b)^{n+1}}{(n+1)a} + c$
3. $\int \cos x dx = \sin x + c$
4. $\int \sin x dx = -\cos x + c$
5. $\int \cos bx dx = \frac{1}{b} \sin bx + c$
6. $\int \sin bx dx = -\frac{1}{b} \cos bx + c$
7. $\int \cos(ax + b) dx = \frac{1}{a} \sin(ax + b) + c$
8. $\int \sin(ax + b) dx = -\frac{1}{a} \cos(ax + b) + c$
9. The area bounded by the curve $y = f(x)$, the x -axis and the lines $x = a$ and $x = b$ is given by $\int_a^b y dx$.

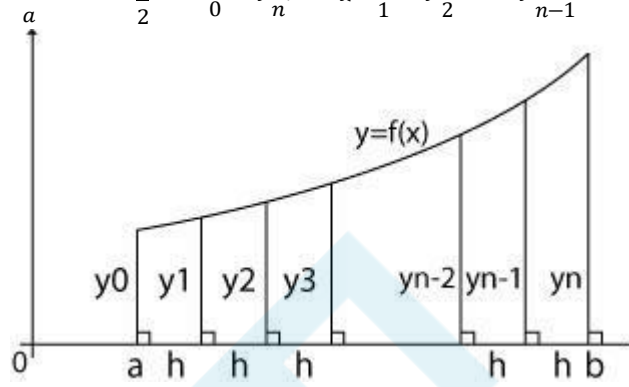


10. The area bounded by the curve $x = f(y)$, the y -axis and the lines $y = a$ and $y = b$ is given by $\int_a^b x dy$.



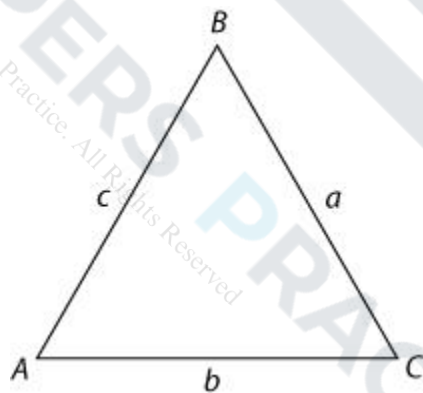
11. Area between $g(x)$ and $f(x) = \int_a^b |g(x) - f(x)| dx$

12. When the area bounded by $y = f(x)$, the x -axis and the lines $x = a$ and $x = b$ is rotated through 360° about the x -axis, the volume of solid of revolution is given by $\pi \int_a^b y^2 dx$.
13. When the area bounded by $y = f(x)$, the y -axis and the lines $y = a$ and $y = b$ is rotated through 360° about the y -axis, the volume of solid of revolution is given by $\pi \int_a^b x^2 dy$.
14. **The trapezium rule:** $\int_a^b y dx = \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$



Triangle

Sine rule



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$\cos A = \frac{c^2 + b^2 - a^2}{2bc}$$

Area of triangle

$$\text{area} = \frac{1}{2}ab \sin C$$

Circular Measure

1. π radian = 180°
2. For a sector of a circle enclosed by two radii that subtend an angle of θ radians at the centre, the arc length s is given by

$$s = r\theta$$
 and the area of the sector A is given by

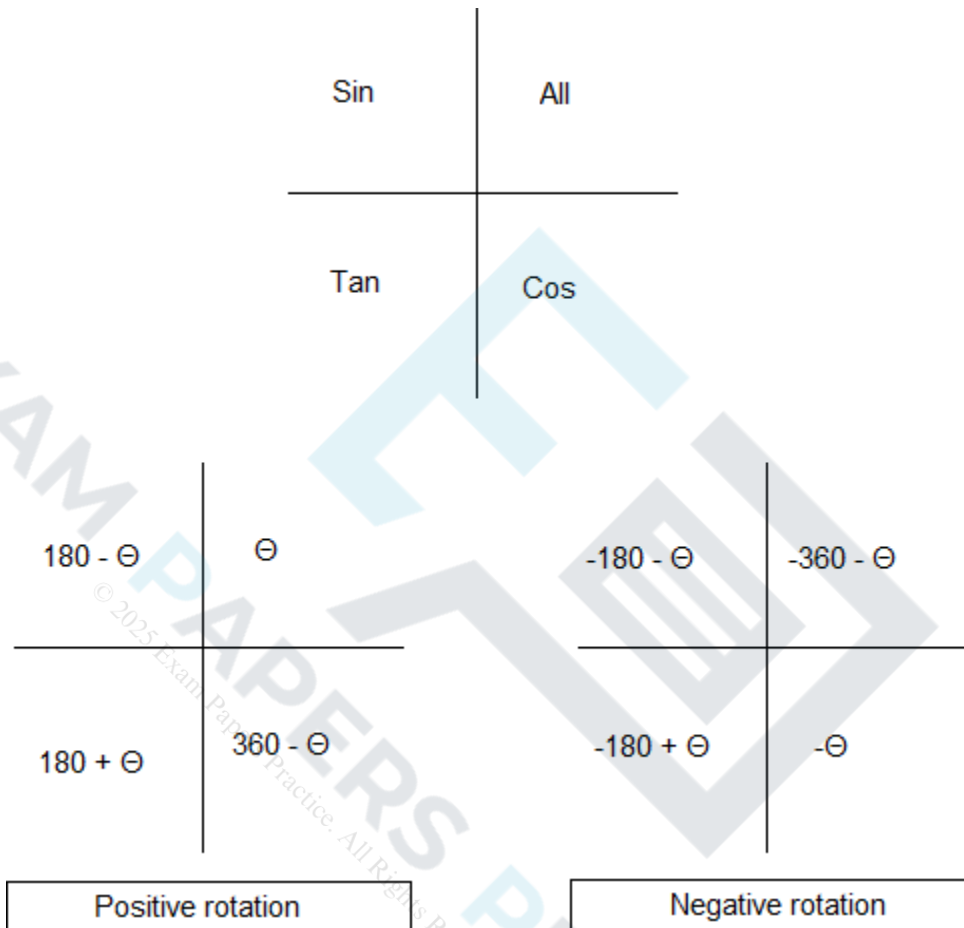
$$A = \frac{1}{2} r^2 \theta$$
 where r is the radius of the circle.

Binomial Expansion

1. $n! = n(n-1)(n-2)(n-3) \dots$
2. $\frac{n!}{(n-2)!} = \frac{n(n-1)(n-2)}{(n-2)} = n(n-1)$
3. $n_{c_1} = n$
4. $n_{c_2} = \frac{n(n-1)}{2!}$
5. $n_{c_3} = \frac{n(n-1)(n-2)(n-3)}{3!}$
6. $(a+x)^n = a^n + n_{c_1} a^{n-1}x + n_{c_2} a^{n-2}x^2 + n_{c_3} a^{n-3}x^3 + \dots$
7. $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$
8. $(r+1)^{\text{th}} \text{ term} = ({}^n C_r x^{n-r} y^r)$

Trigonometry

Rotation



1. $\sin \theta = \frac{\text{opp}}{\text{hyp}}$
2. $\cos \theta = \frac{\text{adj}}{\text{hyp}}$
3. $\tan \theta = \frac{\text{opp}}{\text{adj}}$
4. $\sin^2 + \cos^2 x = 1$
5. $\tan A = \frac{\sin A}{\cos A}$

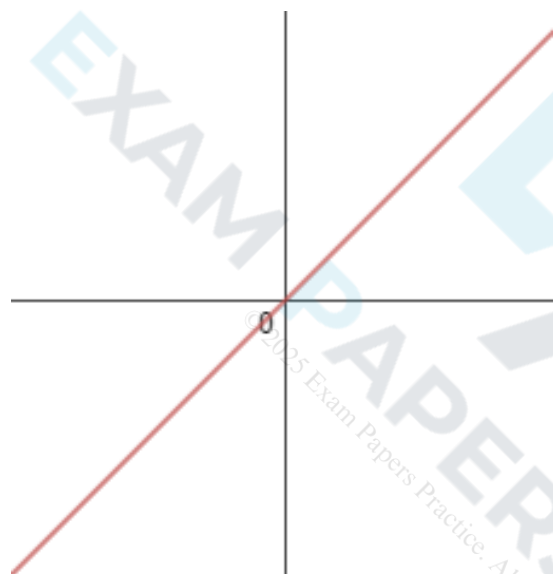
Ratios

The trigonometric ratios of 30° , 45° and 60° have exact forms, given below:

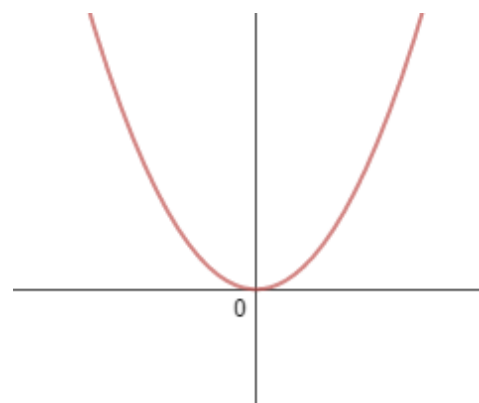
$\sin 30^\circ = \frac{1}{2}$	$\cos 30^\circ = \frac{\sqrt{3}}{2}$	$\tan 30^\circ = \frac{\sqrt{3}}{3}$
$\sin 45^\circ = \frac{\sqrt{2}}{2}$	$\cos 45^\circ = \frac{\sqrt{2}}{2}$	$\tan 45^\circ = 1$
$\sin 60^\circ = \frac{\sqrt{3}}{2}$	$\cos 60^\circ = \frac{1}{2}$	$\tan 60^\circ = \sqrt{3}$

Graphs

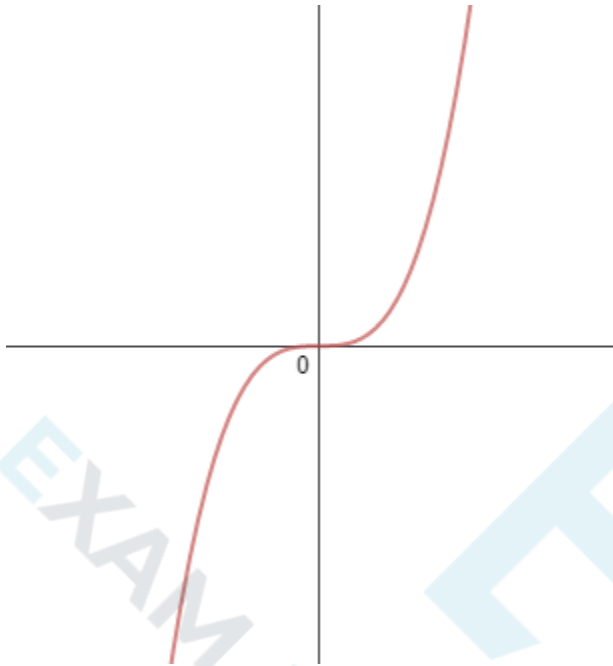
1. $y = x$



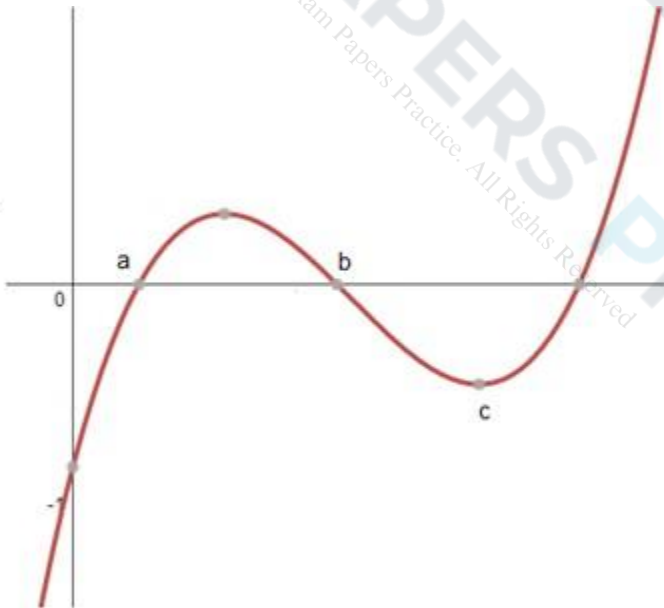
2. $y = x^2$



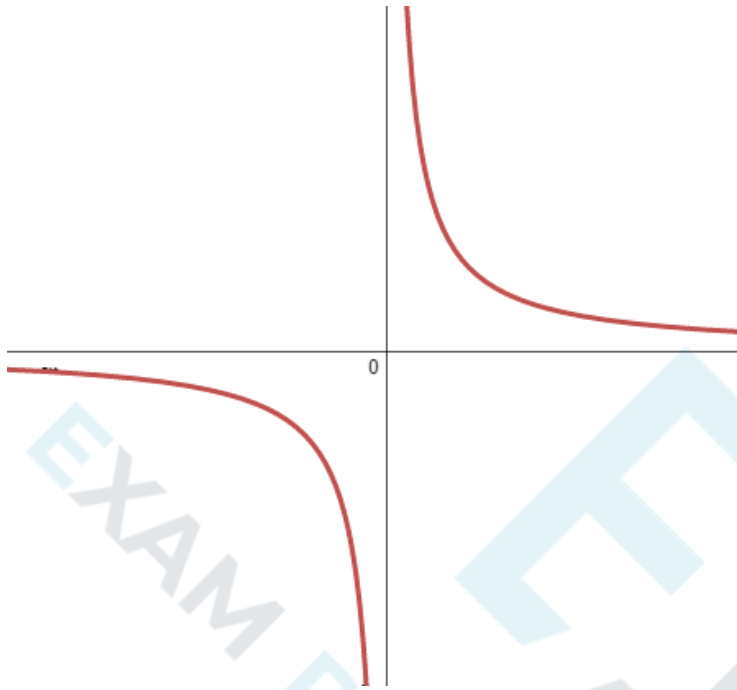
3. $y = x^3$



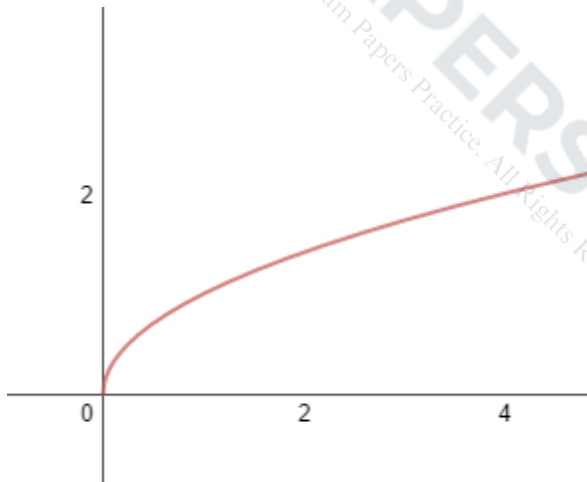
4. $y = (x - a)(x - b)(x - c)$



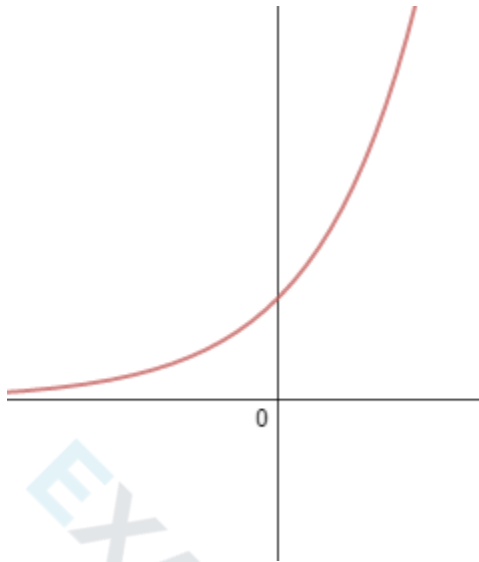
4. $y = \frac{1}{x}$



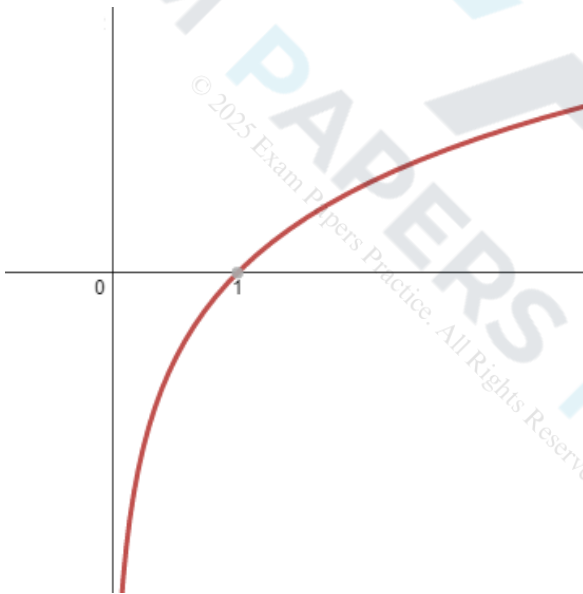
5. $y = \sqrt{x}$



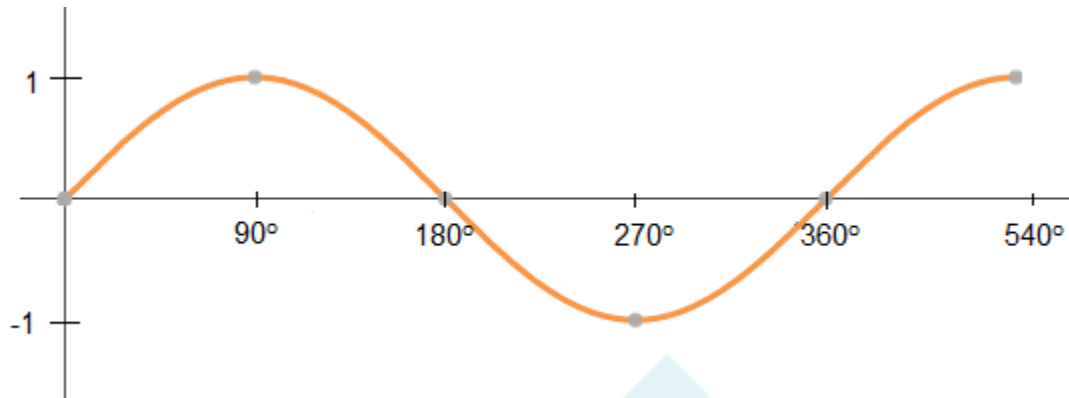
6. $y = e^x$



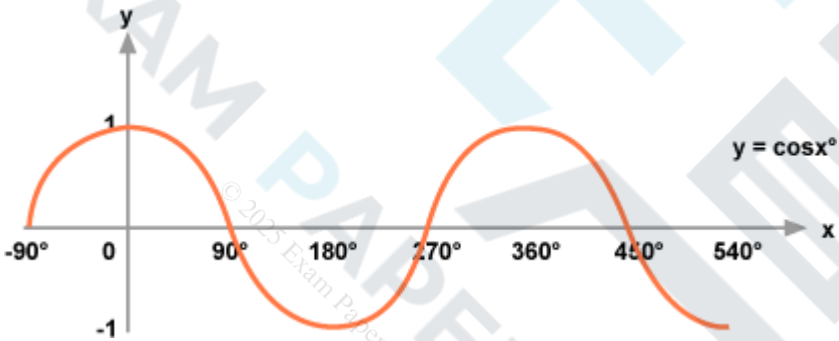
7. $y = \ln x$



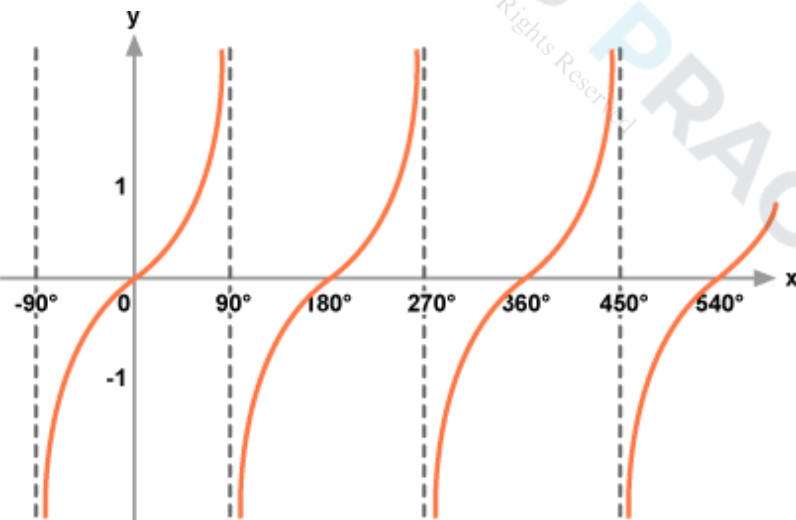
8. $y = \sin x$



9. $y = \cos x$



10. $y = \tan x$



Transformation

$f(x + a)$ is a translation of $-a$ in the x -direction.

$f(x) + a$ is a translation of $+a$ in the y -direction.

$f(ax)$ is a stretch of $\frac{1}{a}$ in the x -direction (multiply x -coordinates by $\frac{1}{a}$).

$af(x)$ is a stretch of a in the y -direction (multiply y -coordinates by a).

