EXAM PAPERS PRACTICE

IB Maths: AA HL

Other Functions & Graphs

Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AA HL Topic Questions

Course	IB Maths
Section	2. Functions
Topic	2.4 Other Functions & Graphs
Difficulty	Medium

Level: IB Maths

Subject: IB Maths AA HL

Board: IB Maths

Topic: Other Functions & Graphs

Question 1

Let $f(x) = \frac{3x-2}{2x+1}$, for $x \neq -\frac{1}{2}$, and $g(x) = -x - 2$, for $x \in \mathbb{R}$.

The graphs of f and g intersect at points A and B.

(a) Find the coordinates of A and B.

[5 marks]

(b) Find the length of the line segment AB.

[3 marks]

Question 2

Consider the functions $f(x) = -x^5 + 2020$ and $g(x) = \frac{1}{\sqrt{(1-x)^3}} - 2$.

(a) Find the coordinates of the y-intercepts for the graph of

(i) f

(ii) g.

[2 marks]

(b) Find the coordinates of the x-intercepts for the graph of

(i) f

(ii) g.

[3 marks]

(c) For the graph of g, find the equation of

(i) the vertical asymptote

(ii) the horizontal asymptote.

[2 marks]

Question 3

Consider the function f defined by $f(x) = \frac{x+2}{2x-3}$, for $x \neq \frac{3}{2}$, and the line $x - 7y + 2 = 0$.

The graph of f and the line intersect at points A and B.

(a) Find the coordinates of A and B.

[5 marks]

(b) Find the midpoint of the line segment AB.

[2 marks]

Question 4

Let $f(x) = \ln(x + 2)$, $x > -2$.

(a) Find the coordinates of:

(i) the x –intercept

(ii) the y –intercept.

[2 marks]

(b) State the equation of the vertical asymptote to the graph of f.

[2 marks]

The graph of $y = f(x)$ intersects with its inverse, twice.

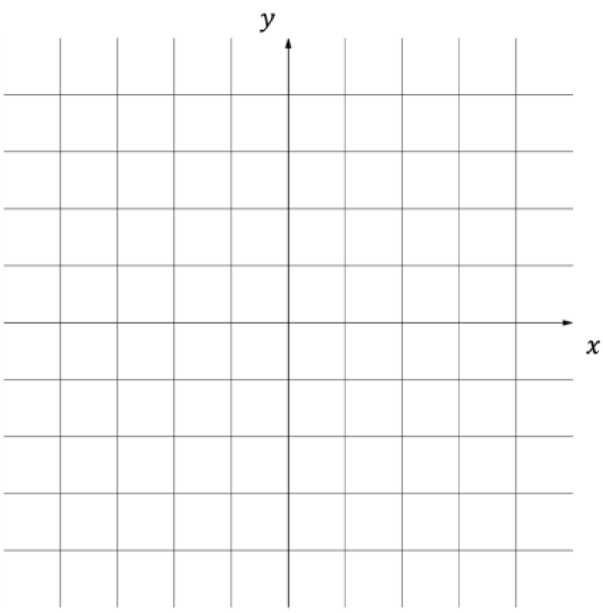
(c) Find the two coordinates where $f(x) = f^{-1}(x)$.

[2 marks]

Question 5

Let $f(x) = 0.5e^{2x} + 1,\ \text{for} -1 \leq x \leq 2.$

(a) On the following grid, sketch the graph of $y = f(x)$.



[3 marks]

(b) The inverse of f can be written in the form of $f^{-1}(x) = A \ln b(x - c)$.
Find the values of A, b and of c.

[4 marks]

Question 6

Carbon-14 is a radioactive isotope of the element carbon.
Carbon-14 decays exponentially – as it decays it loses mass.
Carbon-14 is used in carbon dating to estimate the age of objects.
The time it takes the mass of carbon-14 to halve (called its half-life) is approximately 5700 years.

A model for the mass of carbon-14, m g, in an object of age t years is

$$m = m_0 e^{-kt}$$

where m_o and k are constants.

(a) For an object initially containing 100g of carbon-14, write down the value of m_0.

[1 mark]

(b) Briefly explain why, if $m_0 = 100$, m will equal 50g when $t = 5700$ years.

[2 marks]

(c) Using the values from part (b), show that the value of k is 1.22×10^{-4} to three significant figures.

[2 marks]

(d) A different object currently contains 60g of carbon-14.
In 2000 years' time how much carbon-14 will remain in the object?

[2 marks]

Question 7

A small company makes a profit of £2500 in its first year of business and £3700 in the second year. The company decides they will use the model

$$P = P_0 y^k$$

to predict future years' profits.
£P is the profit in the y^{th} year of business.
P_0 and k are constants.

(a) Write down two equations connecting P_0 and k.

[2 marks]

(b) Find the values of P_0 and k.

[2 marks]

(c) Find the predicted profit for years 3 and 4.

[2 marks]

(d) Show that

$$P = P_0 y^k$$

can be written in the form

$$\log P = \log P_0 + k \log y.$$

[2 marks]

Question 8

In an effort to prevent extinction scientists released some rare birds into a newly constructed nature reserve.
The population of birds, within the reserve, is modelled by

$$B = 16e^{0.85t}$$

B is the number of birds after t years of being released into the reserve.

(a) Write down the number of birds the scientists released into the nature reserve.

[1 mark]

(b) According to this model, how many birds will be in the reserve after 3 years?

[2 marks]

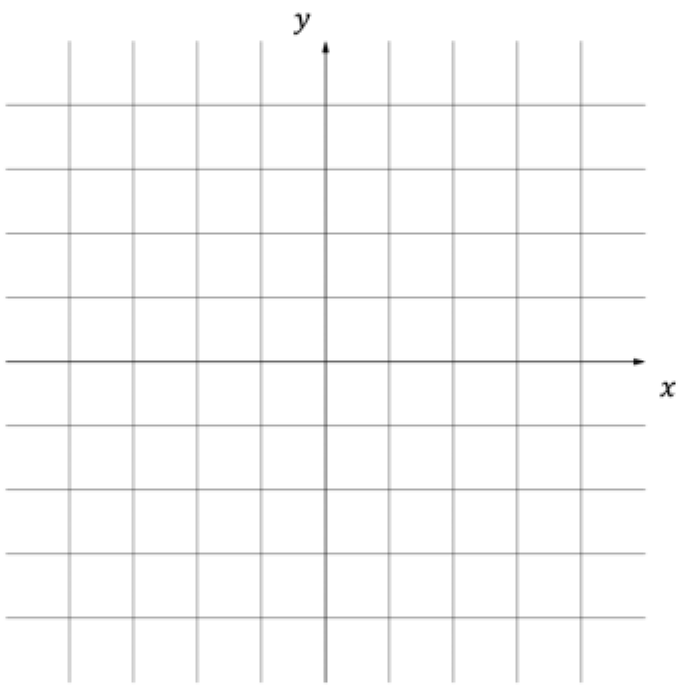
(c) How long will it take for the population of birds within the reserve to reach 500?

[2 marks]

Question 9

Rebecca recently had the COVID-19 vaccine. The volume, V, of the vaccine in her blood over time can be modelled by an equation of the form $V_1(t) = 1.7te^{-1.25t}$, where V is the concentration (in mg) of the vaccine in the bloodstream and t is time measured in days after 9am on Monday.

(a) On the following grid, sketch the graph of $y = V_1(t)$.



[3 marks]

(b) Find, to the nearest minute, the time when the vaccine volume, V_1 reaches a maximum value.

[2 marks]

(c) Rebecca experienced side-effects from the vaccine between the times when the volume reached its maximum value until it had dropped to half of its maximum value. Find, to the nearest minute, the length of time that Rebecca experienced side-effects from taking the vaccine.

[3 marks]

(d) The vaccine is medically determined to be no longer in Rebecca's bloodstream when it drops down to 1% of its maximum value. Find the time that the vaccine is no longer in Rebecca's bloodstream.

[2 marks]

(e) Rebecca's friend, Zara, also had the vaccine on the same day. The volume in Zara's bloodstream can be modelled by an equation of the form of $V_2(t) = 1.766te^{-1.3t}$. Calculate, to the nearest minute, how much faster V_2 took to reach a maximum volume compared to V_1.

[1 mark]

Question 10

Let $f(x) = e^x + 1$ and $g(x) = 4x + a$, where $x \in \mathbb{R}$ and a is a constant.

(a) Find $(g \circ f)(x)$.

[2 marks]

(b) Given that $(g \circ f)(0) = 2$, find the value of a.

[2 marks]

(c) Solve the equation $(g \circ f)(x) = 0$.

[3 marks]

Question 11

Let $f(x) = ab^x$, where $x, a, b \in \mathbb{R}$ and $x \geq 0,\ a, b > 1$.

The graph of f contains the points $(0, 3)$ and $(2, 75)$.

(a) Find the values of a and b.

[3 marks]

(b) Find an expression for $f^{-1}(x)$.

[3 marks]

(c) Find the value of $f^{-1}(375)$.

[2 marks]

Question 12

Consider $f(x) = \ln\left(\sqrt{x^2 - 16}\right)$.

(a) Find the largest possible domain D_f for f to be a function.

[2 marks]

Let $f(x) = \ln\left(\sqrt{x^2 - 16}\right)$, for $x \in D_f$.

(b) Explain why

(i) f is an even function

(ii) the inverse function f^{-1} does not exist.

[3 marks]

Question 13

Let $f(x) = \frac{2(x+1)}{x-1}$, for $x \neq 1$, and $g(x) = x + 1$, for $x \in \mathbb{R}$.

The graphs of f and g intersect at points A and B.

(a) Find the coordinates of A and B.

[5 marks]

(b) Find the equation of the straight line that passes through A and B, giving your answer in the form $ax + by + d = 0$.

[3 marks]