Question 1

Let $f(x) = \frac{3x-2}{2x+1}$, for $x \neq -\frac{1}{2}$, and $g(x) = -x - 2$, for $x \in \mathbb{R}$.

The graphs of f and g intersect at points A and B.

(a) Find the coordinates of A and B. [5]

(b) Find the length of the line segment AB. [3]

a) $f(x) = g(x)$

$\frac{3x-2}{2x+1} = -x-2$

$3x - 2 = (-x-2)(2x+1) = -2x^2 - 5x - 2$

$2x^2 + 8x = 2x(x+4) = 0 \quad \therefore x = 0, -4$

Sub $x = 0, -4$ into $f(x)$ or $g(x)$.

$g(-4) = -(-4) - 2 = 2 \quad \therefore (-4, 2)$

$g(0) = -(0) - 2 = -2 \quad \therefore (0, -2)$

$\boxed{A(-4, 2) \text{ and } B(0, -2)}$

Let $f(x) = \frac{3x-2}{2x+1}$, for $x \neq -\frac{1}{2}$, and $g(x) = -x - 2$, for $x \in \mathbb{R}$.

The graphs of f and g intersect at points A and B.

(a) Find the coordinates of A and B. $\boxed{A(-4, 2) \text{ and } B(0, -2)}$ [5]

(b) Find the length of the line segment AB. [3]

b) Distance between two points $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\boxed{|AB| = \sqrt{(0+4)^2 + (-2-2)^2} = \sqrt{32} = 4\sqrt{2} \text{ units}}$

Question 2

Consider the functions $f(x) = -x^5 + 2020$ and $g(x) = \frac{1}{\sqrt{(1-x)^3}} - 2$.

(a) Find the coordinates of the y-intercepts for the graph of

(i) f

(ii) g. [2]

(b) Find the coordinates of the x-intercepts for the graph of

(i) f

(ii) g. [2]

(c) For the graph of g, find the equation of

(i) the vertical asymptote

(ii) the horizontal asymptote. [3]

a) i) y-intercepts occur when $x = 0$.

Sub $x = 0$ into $f(x)$.

$f(0) = -(0)^5 + 2020$

$f(0) = 2020$

$\boxed{\text{Hence the } y\text{-intercept for } f \text{ is } (0, 2020).}$

ii) Sub $x = 0$ into $g(x)$.

$g(0) = \frac{1}{\sqrt{(1-(0))^3}} - 2$

$g(0) = -1$

$\boxed{\text{Hence the } y\text{-intercept for } g \text{ is } (0, -1).}$

Consider the functions $f(x) = -x^5 + 2020$ and $g(x) = \frac{1}{\sqrt{(1-x)^3}} - 2$.

(a) Find the coordinates of the y-intercepts for the graph of

(i) f

(ii) g. [2]

(b) Find the coordinates of the x-intercepts for the graph of

(i) f

(ii) g. [2]

(c) For the graph of g, find the equation of

(i) the vertical asymptote

(ii) the horizontal asymptote. [3]

b) i) x-intercepts occur when the function equals zero.

Set $f(x) = 0$ and solve for x on your GDC.

$-x^5 + 2020 = 0$

$x \approx 4.58$

$\boxed{\text{Hence the } x\text{-intercept for } f \text{ is } (4.58, 0).}$

ii) Set $g(x) = 0$ and solve for x on your GDC.

$\frac{1}{\sqrt{(1-x)^3}} - 2 = 0$

$x \approx 0.370$

$\boxed{\text{Hence the } x\text{-intercept for } g \text{ is } (0.37, 0).}$

Consider the functions $f(x) = -x^5 + 2020$ and $g(x) = \frac{1}{\sqrt{(1-x)^3}} - 2$.

(a) Find the coordinates of the y-intercepts for the graph of

(i) f

(ii) g. [2]

(b) Find the coordinates of the x-intercepts for the graph of

(i) f

(ii) g. [2]

(c) For the graph of g, find the equation of

(i) the vertical asymptote

(ii) the horizontal asymptote. [3]

c) i) The vertical asymptote is when the denominator of $g(x)$ equals zero.

$$[\text{denominator of } g] = 0$$

$$\sqrt{(1-x)^3} = 0$$

$$x = 1$$

$\boxed{\text{Hence the equation of the vertical asymptote is } x = 1}$

ii) As x tends towards negative infinity $(-\infty)$, $\frac{1}{\sqrt{(1-x)^3}}$ tends towards zero.

$$g(x) = \frac{1}{\sqrt{(1-x)^3}} - 2$$

$$\lim_{x \to -\infty} g(x) = 0 - 2 = -2$$

$\boxed{\text{Hence the equation of the horizontal asymptote is } y = -2.}$

Question 3

Consider the function f defined by $f(x) = \frac{x+2}{2x-3}$, for $x \neq \frac{3}{2}$, and the line $x - 7y + 2 = 0$.

The graph of f and the line intersect at points A and B.

(a) Find the coordinates of A and B. [5]

(b) Find the midpoint of the line segment AB. [2]

a) Let $y = f(x)$

$$y = \frac{x+2}{2x-3}$$

Sub $y = f(x)$ into the line equation

$$x - 7\left(\frac{x+2}{2x-3}\right) + 2 = 0$$

$$x - \frac{7x - 14}{2x-3} + 2 = 0$$

$$x(2x-3) - 7x - 14 + 2(2x-3) = 0$$

$$2x^2 - 3x - 7x - 14 + 4x - 6 = 2x^2 - 6x - 20 = 0$$

$$2(x^2 - 3x - 10) = 2(x-5)(x+2) = 0 \qquad \therefore x = -2, 5$$

Sub $x = -2, 5$ into f.

$$f(-2) = \frac{(-2)+2}{2(-2)-3} = 0 \qquad \boxed{\therefore A(-2, 0)}$$

$$f(5) = \frac{(5)+2}{2(5)-3} = 1 \qquad \boxed{\therefore B(5, 1)}$$

Consider the function f defined by $f(x) = \frac{x+2}{2x-3}$, for $x \neq \frac{3}{2}$, and the line $x - 7y + 2 = 0$.

The graph of f and the line intersect at points A and B.

(a) Find the coordinates of A and B. [5]

$\boxed{\therefore A(-2, 0)}$ $\boxed{\therefore B(5, 1)}$

(b) Find the midpoint of the line segment AB. [2]

b) Midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$$\text{Midpoint} = \left(\frac{-2+5}{2}, \frac{0+1}{2}\right)$$

$$\boxed{\text{Midpoint} = \left(\frac{3}{2}, \frac{1}{2}\right)}$$

Question 4

Let $f(x) = \ln(x+2)$, $x > -2$.

(a) Find the coordinates of:

(i) the x −intercept

(ii) the y −intercept.

[2]

(b) State the equation of the vertical asymptote to the graph of f.

[2]

The graph of $y = f(x)$ intersects with its inverse, twice.

(c) Find the two coordinates where $f(x) = f^{-1}(x)$.

[2]

a) x-intercepts occur when $f(x) = 0$.

$0 = \ln(x+2)$

$1 = x + 2$

$x = -1$

$\boxed{x\text{-intercept at } (-1, 0).}$

y-intercepts occur when $x = 0$.

$f(0) = \ln(0+2)$

$f(0) = 0.6931\ldots$

$= 0.693 \quad (3sf)$

$\boxed{y\text{-intercept at } (0, 0.693).}$

Let $f(x) = \ln(x+2)$, $x > -2$.

(a) Find the coordinates of:

(i) the x −intercept

(ii) the y −intercept.

[2]

(b) State the equation of the vertical asymptote to the graph of f.

[2]

The graph of $y = f(x)$ intersects with its inverse, twice.

(c) Find the two coordinates where $f(x) = f^{-1}(x)$.

[2]

b) $f(x)$ is undefined when $x + 2 < 0$.

$x + 2 < 0$

$\boxed{\text{Vertical asymptote: } x = -2}$

EXAM PAPERS PRACTICE

Let $f(x) = \ln(x + 2)$, $x > -2$.

(a) Find the coordinates of:

(i) the x −intercept

(ii) the y −intercept.

[2]

(b) State the equation of the vertical asymptote to the graph of f.

[2]

The graph of $y = f(x)$ intersects with its inverse, twice.

(c) Find the two coordinates where $f(x) = f^{-1}(x)$.

[2]

c) Find $f^{-1}(x)$.

$y = f(x)$

$y = \ln(x + 2)$

swap x and y

$x = \ln(y + 2)$

$y = e^{x} - 2$

$f^{-1}(x) = e^{x} - 2$

$f(x) = f^{-1}(x)$

$\ln(x + 2) = e^{x} - 2$

Graph $f(x)$ and $f^{-1}(x)$ on your GDC and find their intersection.

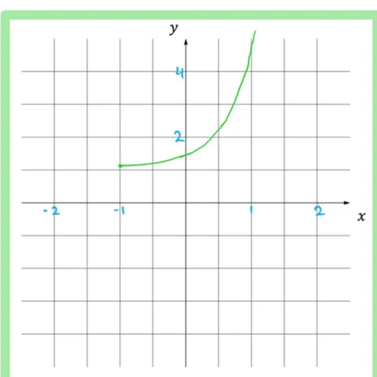
Intersection :

$(-1.84, -1.84)$ and $(1.14, 1.14)$

Question 5

Let $f(x) = 0.5e^{2x} + 1$, for $-1 \leq x \leq 2$.

(a) On the following grid, sketch the graph of $y = f(x)$.



[3]

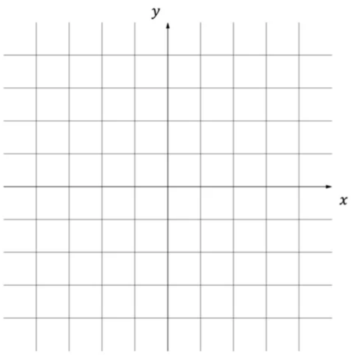
a) Graph $f(x)$ on your GDC.

(b) The inverse of f can be written in the form of $f^{-1}(x) = A \ln b(x - c)$.
Find the values of A, b and of c.

[4]

Let $f(x) = 0.5e^{2x} + 1$, for $-1 \le x \le 2$.

(a) On the following grid, sketch the graph of $y = f(x)$.



[3]

(b) The inverse of f can be written in the form of $f^{-1}(x) = A\ln b(x - c)$.
Find the values of A, b and of c.

[4]

b) Find $f^{-1}(x)$

$y = f(x)$

$y = 0.5e^{2x} + 1$

$x = 0.5e^{2y} + 1$

$\frac{x-1}{0.5} = e^{2y}$

$2y = \ln 2(x-1)$

$y = \frac{1}{2}\ln 2(x-1)$

$\boxed{\therefore\ A = \frac{1}{2} \quad b = 2 \quad c = 1}$

Question 6

Carbon-14 is a radioactive isotope of the element carbon.
Carbon-14 decays exponentially – as it decays it loses mass.
Carbon-14 is used in carbon dating to estimate the age of objects.
The time it takes the mass of carbon-14 to halve (called its half-life) is approximately 5700 years.

A model for the mass of carbon-14, m g, in an object of age t years is

$$m = m_0 e^{-kt}$$

where m_o and k are constants.

(a) For an object initially containing 100g of carbon-14, write down the value of m_0

[1]

(b) Briefly explain why, if $m_0 = 100$, m will equal 50g when $t = 5700$ years.

[2]

(c) Using the values from part (b), show that the value of k is 1.22×10^{-4} to three significant figures.

[2]

(d) A different object currently contains 60g of carbon-14.
In 2000 years' time how much carbon-14 will remain in the object?

[2]

a) $t=0 \quad m=100$

$100 = m_0 \underbrace{e^{-k(0)}}\ = m_0$

$\boxed{m_0 = 100}$

Carbon-14 is a radioactive isotope of the element carbon.
Carbon-14 decays exponentially – as it decays it loses mass.
Carbon-14 is used in carbon dating to estimate the age of objects.
The time it takes the mass of carbon-14 to halve (called its half-life) is approximately 5700 years.

A model for the mass of carbon-14, m g, in an object of age t years is

$$m = m_0 e^{-kt}$$

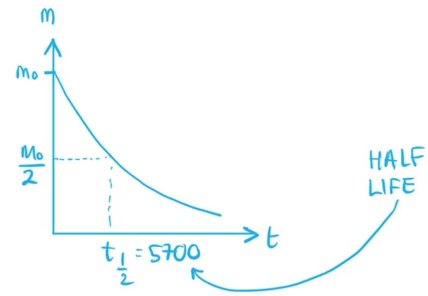
where m_o and k are constants.

(a) For an object initially containing 100g of carbon-14, write down the value of m_0.

$m_0 = 100$ [1]

(b) Briefly explain why, if $m_0 = 100$, m will equal 50g when $t = 5700$ years. [2]

b)



Since half life is 5700y, this is the time it takes for the initial mass m_0 (100g) to half to 50g.

(c) Using the values from part (b), show that the value of k is 1.22×10^{-4} to three significant figures. [2]

(d) A different object currently contains 60g of carbon-14.
In 2000 years' time how much carbon-14 will remain in the object? [2]

Carbon-14 is a radioactive isotope of the element carbon.
Carbon-14 decays exponentially – as it decays it loses mass.
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A model for the mass of carbon-14, m g, in an object of age t years is

$$m = m_0 e^{-kt}$$

where m_o and k are constants.

(a) For an object initially containing 100g of carbon-14, write down the value of m_0. [1]

(b) Briefly explain why, if $m_0 = 100$, m will equal 50g when $t = 5700$ years. [2]

(c) Using the values from part (b), show that the value of k is 1.22×10^{-4} to three significant figures. [2]

c) Sub values from (b) into model

$$50 = 100e^{-k5700}$$

$$\frac{1}{2} = e^{-k5700}$$

$$\ln\frac{1}{2} = \underbrace{\ln e^{-5700k}}$$

$$\ln\frac{1}{2} = -5700k$$

$$k = \frac{-\ln\frac{1}{2}}{5700} = \boxed{1.22 \times 10^{-4}} \ (3sf)$$

(d) A different object currently contains 60g of carbon-14.
In 2000 years' time how much carbon-14 will remain in the object? [2]

Carbon-14 is a radioactive isotope of the element carbon.
Carbon-14 decays exponentially – as it decays it loses mass.
Carbon-14 is used in carbon dating to estimate the age of objects.
The time it takes the mass of carbon-14 to halve (called its half-life) is approximately 5700 years.

A model for the mass of carbon-14, m g, in an object of age t years is

$$m = m_0 e^{-kt}$$

where m_o and k are constants.

(a) For an object initially containing 100g of carbon-14, write down the value of m_0.

[1]

(b) Briefly explain why, if $m_0 = 100$, m will equal 50g when $t = 5700$ years.

[2]

(c) Using the values from part (b), show that the value of k is 1.22×10^{-4} to three significant figures.

[2]

(d) A different object currently contains 60g of carbon-14.
In 2000 years' time how much carbon-14 will remain in the object?

[2]

d) Sub $m_0 = 60$ and $t = 2000$ into model

$m = 60e^{-1.22 \times 10^{-4}(2000)}$

$\boxed{m = 47.0\text{g}}$ (3sf)

Question 7

A small company makes a profit of £2500 in its first year of business and £3700 in the second year. The company decides they will use the model

$$P = P_0 y^k$$

to predict future years' profits.
£P is the profit in the y^{th} year of business.
P_0 and k are constants.

(a) Write down two equations connecting P_0 and k.

[2]

(b) Find the values of P_0 and k.

[2]

(c) Find the predicted profit for years 3 and 4.

[2]

(d) Show that

$$P = P_0 y^k$$

can be written in the form

$$\log P = \log P_0 + k \log y.$$

[2]

a) $P = 2500$ $y = 1$

$P = 3700$ $y = 2$

$\boxed{\begin{array}{l} 2500 = P_0(1)^k \\ 3700 = P_0(2)^k \end{array}}$

A small company makes a profit of £2500 in its first year of business and £3700 in the second year. The company decides they will use the model

$$P = P_0 y^k$$

to predict future years' profits.
£P is the profit in the y^{th} year of business.
P_0 and k are constants.

(a) Write down two equations connecting P_0 and k.

$2500 = P_0(1)^k$ ①

$3700 = P_0(2)^k$ ② [2]

(b) Find the values of P_0 and k. [2]

(c) Find the predicted profit for years 3 and 4. [2]

(d) Show that

$$P = P_0 y^k$$

can be written in the form

$$\log P = \log P_0 + k \log y.$$

[2]

b) dividing ① by ②

$$\frac{2500}{3700} = \left(\frac{1}{2}\right)^k$$

$$k = \log_{\frac{1}{2}}\left(\frac{2500}{3700}\right) = \boxed{0.566 = k} \ (3sf)$$

sub $k = 0.566$ into ①

$$2500 = P_0(1)^{0.566} = P_0$$

$$\boxed{P_0 = £2500}$$

A small company makes a profit of £2500 in its first year of business and £3700 in the second year. The company decides they will use the model

$$P = P_0 y^k$$

to predict future years' profits.
£P is the profit in the y^{th} year of business.
P_0 and k are constants.

(a) Write down two equations connecting P_0 and k. [2]

(b) Find the values of P_0 and k.

$P_0 = 2500$ $k = 0.566$ [2]

(c) Find the predicted profit for years 3 and 4. [2]

(d) Show that

$$P = P_0 y^k$$

can be written in the form

$$\log P = \log P_0 + k \log y.$$

[2]

c) Using P_0 and k from (b)

$$P = 2500y^{0.566}$$

at $y = 3$ $P = 2500(3)^{0.566} = £4653.70$

to the nearest penny

$y = 4$ $P = 2500(4)^{0.566} = £5476$

exactly!

A small company makes a profit of £2500 in its first year of business and £3700 in the second year. The company decides they will use the model

$$P = P_0 y^k$$

to predict future years' profits.
£P is the profit in the y^{th} year of business.
P_0 and k are constants.

(a) Write down two equations connecting P_0 and k.

[2]

(b) Find the values of P_0 and k.

[2]

(c) Find the predicted profit for years 3 and 4.

[2]

(d) Show that

$$P = P_0 y^k$$

can be written in the form

$$\log P = \log P_0 + k \log y.$$

[2]

d) $\log P = \log P_0 y^k$

rewrite as two terms

$= \log P_0 + \log y^k$

rewrite power as coefficient

$\boxed{\log P = \log P_0 + k \log y}$

Question 8

In an effort to prevent extinction scientists released some rare birds into a newly constructed nature reserve.
The population of birds, within the reserve, is modelled by

$$B = 16e^{0.85t}$$

B is the number of birds after t years of being released into the reserve.

(a) Write down the number of birds the scientists released into the nature reserve.

[1]

(b) According to this model, how many birds will be in the reserve after 3 years?

[2]

(c) How long will it take for the population of birds within the reserve to reach 500?

[2]

a) $t=0$

$B = 16e^{0.85(0)} = \boxed{16}$

In an effort to prevent extinction scientists released some rare birds into a newly constructed nature reserve.
The population of birds, within the reserve, is modelled by

$$B = 16e^{0.85t}$$

B is the number of birds after t years of being released into the reserve.

(a) Write down the number of birds the scientists released into the nature reserve.

[1]

(b) According to this model, how many birds will be in the reserve after 3 years?

[2]

(c) How long will it take for the population of birds within the reserve to reach 500?

[2]

b) $t=3$

$B=16e^{0.85(3)} = \boxed{205}$ (3sf)

In an effort to prevent extinction scientists released some rare birds into a newly constructed nature reserve.
The population of birds, within the reserve, is modelled by

$$B = 16e^{0.85t}$$

B is the number of birds after t years of being released into the reserve.

(a) Write down the number of birds the scientists released into the nature reserve.

[1]

(b) According to this model, how many birds will be in the reserve after 3 years?

[2]

(c) How long will it take for the population of birds within the reserve to reach 500?

[2]

c) Sub B=500 into equation

$500 = 16e^{0.85t}$

$\frac{125}{4} = e^{0.85t}$

$\ln \frac{125}{4} = \ln e^{0.85t}$

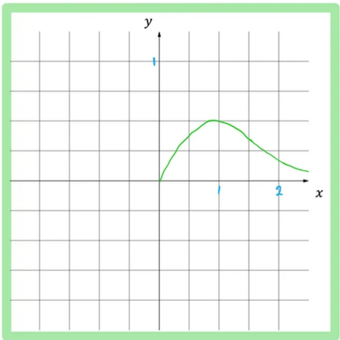
$= 0.85t \ln e^{1}$

$t = \frac{1}{0.85} \ln \frac{125}{4} = \boxed{4.05y}$ (3sf)

Question 9

Rebecca recently had the COVID-19 vaccine. The volume, V, of the vaccine in her blood over time can be modelled by an equation of the form $V_1(t) = 1.7te^{-1.25t}$, where V is the concentration (in mg) of the vaccine in the bloodstream and t is time measured in days after 9am on Monday.

(a) On the following grid, sketch the graph of $y = V_1(t)$.



[3]

(b) Find, to the nearest minute, the time when the vaccine volume, V_1 reaches a maximum value.

[2]

(c) Rebecca experienced side-effects from the vaccine between the times when the volume reached its maximum value until it had dropped to half of its maximum value. Find, to the nearest minute, the length of time that Rebecca experienced side-effects from taking the vaccine.

[3]

(d) The vaccine is medically determined to be no longer in Rebecca's bloodstream when it drops down to 1% of its maximum value. Find the time that the vaccine is no longer in Rebecca's bloodstream.

[2]

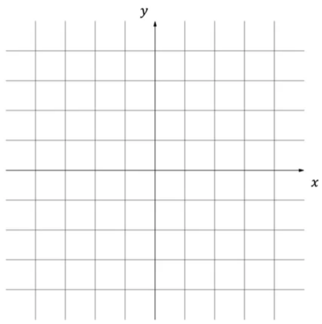
(e) Rebecca's friend, Zara, also had the vaccine on the same day. The volume in Zara's bloodstream can be modelled by an equation of the form of $V_2(t) = 1.766te^{-1.3t}$.
Calculate, to the nearest minute, how much faster V_2 took to reach a maximum volume compared to V_1.

[1]

a) Graph $V_1(t)$ on your GDC.

Rebecca recently had the COVID-19 vaccine. The volume, V, of the vaccine in her blood over time can be modelled by an equation of the form $V_1(t) = 1.7te^{-1.25t}$, where V is the concentration (in mg) of the vaccine in the bloodstream and t is time measured in days after 9am on Monday.

(a) On the following grid, sketch the graph of $y = V_1(t)$.



[3]

(b) Find, to the nearest minute, the time when the vaccine volume, V_1 reaches a maximum value.

[2]

(c) Rebecca experienced side-effects from the vaccine between the times when the volume reached its maximum value until it had dropped to half of its maximum value. Find, to the nearest minute, the length of time that Rebecca experienced side-effects from taking the vaccine.

[3]

(d) The vaccine is medically determined to be no longer in Rebecca's bloodstream when it drops down to 1% of its maximum value. Find the time that the vaccine is no longer in Rebecca's bloodstream.

[2]

(e) Rebecca's friend, Zara, also had the vaccine on the same day. The volume in Zara's bloodstream can be modelled by an equation of the form of $V_2(t) = 1.766te^{-1.3t}$.
Calculate, to the nearest minute, how much faster V_2 took to reach a maximum volume compared to V_1.

[1]

b) Find the maximum of $V_1(t)$ on your GDC.

maximum : $(0.8, 0.5)$

0.8×24 hours $= 19.2$ hours

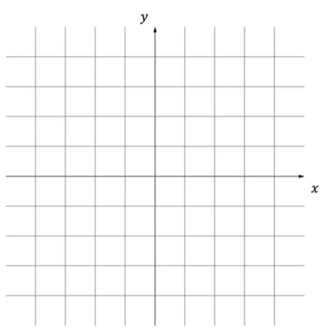
19.2 hours $=$ 19 hrs and 12 mins

9 am + 19 hrs and 12 mins = 4:12 am

$\boxed{\text{4:12 am on Tuesday}}$

Rebecca recently had the COVID-19 vaccine. The volume, V, of the vaccine in her blood over time can be modelled by an equation of the form $V_1(t) = 1.7te^{-1.25t}$, where V is the concentration (in mg) of the vaccine in the bloodstream and t is time measured in days after 9am on Monday.

(a) On the following grid, sketch the graph of $y = V_1(t)$.



[3]

(b) Find, to the nearest minute, the time when the vaccine volume, V_1 reaches a maximum value.

[2]

(c) Rebecca experienced side-effects from the vaccine between the times when the volume reached its maximum value until it had dropped to half of its maximum value. Find, to the nearest minute, the length of time that Rebecca experienced side-effects from taking the vaccine.

[3]

(d) The vaccine is medically determined to be no longer in Rebecca's bloodstream when it drops down to 1% of its maximum value. Find the time that the vaccine is no longer in Rebecca's bloodstream.

[2]

(e) Rebecca's friend, Zara, also had the vaccine on the same day. The volume in Zara's bloodstream can be modelled by an equation of the form of $V_2(t) = 1.766te^{-1.3t}$.
Calculate, to the nearest minute, how much faster V_2 took to reach a maximum volume compared to V_1.

[1]

c) Maximum : $(0.8, 0.5)$

Find t when $V(t) = 0.25$

$0.25 = 1.7te^{-1.25t}$

$t = 2.14348$

$\therefore$ days $: 2.14348 - 0.8$
$= 1.34348$

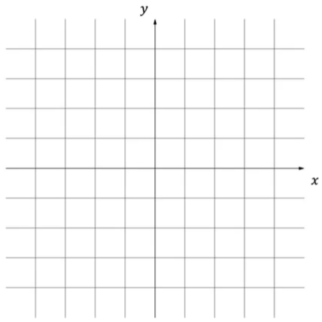
$0.34348 \times 24 = 8.24352$

$0.24352 \times 60 = 14.6 \approx 15$

$\boxed{\text{1 day 8hrs and 15mins}}$

Rebecca recently had the COVID-19 vaccine. The volume, V, of the vaccine in her blood over time can be modelled by an equation of the form $V_1(t) = 1.7te^{-1.25t}$, where V is the concentration (in mg) of the vaccine in the bloodstream and t is time measured in days after 9am on Monday.

(a) On the following grid, sketch the graph of $y = V_1(t)$.



[3]

(b) Find, to the nearest minute, the time when the vaccine volume, V_1 reaches a maximum value.

[2]

(c) Rebecca experienced side-effects from the vaccine between the times when the volume reached its maximum value until it had dropped to half of its maximum value. Find, to the nearest minute, the length of time that Rebecca experienced side-effects from taking the vaccine.

[3]

(d) The vaccine is medically determined to be no longer in Rebecca's bloodstream when it drops down to 1% of its maximum value. Find the time that the vaccine is no longer in Rebecca's bloodstream.

[2]

(e) Rebecca's friend, Zara, also had the vaccine on the same day. The volume in Zara's bloodstream can be modelled by an equation of the form of $V_2(t) = 1.766te^{-1.3t}$.
Calculate, to the nearest minute, how much faster V_2 took to reach a maximum volume compared to V_1.

[1]

d) Maximum : $(0.8, 0.5)$

Find t when $V(t) = 0.005$

$0.005 = 1.7te^{-1.25t}$

$t = 6.11126$ days

$0.11126 \times 24 = 2.67024$

$0.67024 \times 60 = 40.2 \approx 40$ mins

$t = 6$ days $+ 2$hrs $+ 40$ mins

11:40am on Sunday.

Rebecca recently had the COVID-19 vaccine. The volume, V, of the vaccine in her blood over time can be modelled by an equation of the form $V_1(t) = 1.7te^{-1.25t}$, where V is the concentration (in mg) of the vaccine in the bloodstream and t is time measured in days after 9am on Monday.

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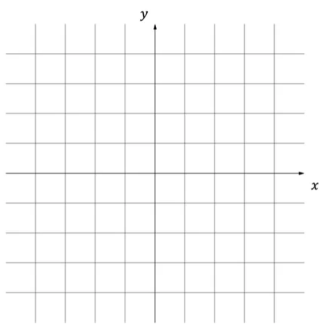

[3]

(b) Find, to the nearest minute, the time when the vaccine volume, V_1 reaches a maximum value.

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(e) Rebecca's friend, Zara, also had the vaccine on the same day. The volume in Zara's bloodstream can be modelled by an equation of the form of $V_2(t) = 1.766te^{-1.3t}$.
Calculate, to the nearest minute, how much faster V_2 took to reach a maximum volume compared to V_1.

[1]

e) Graph $V_2(t)$ and find its maximum.

maximum : $(0.769, 0.5)$

time $= 0.8 - 0.769$
$= 0.031$ days

$0.031 \times 24 = 0.744$ hrs

$0.744 \times 60 = 44.64$ mins

45 minutes

Question 10

Let $f(x) = e^x + 1$ and $g(x) = 4x + a$, where $x \in \mathbb{R}$ and a is a constant.

(a) Find $(g \circ f)(x)$. [2]

(b) Given that $(g \circ f)(0) = 2$, find the value of a. [2]

(c) Solve the equation $(g \circ f)(x) = 0$. [3]

(a) $f(x) \rightarrow g(x)$

$(g \circ f)(x) = 4(e^x + 1) + a$

$\boxed{(g \circ f)(x) = 4e^x + 4 + a}$

Let $f(x) = e^x + 1$ and $g(x) = 4x + a$, where $x \in \mathbb{R}$ and a is a constant.

(a) Find $(g \circ f)(x)$. $\boxed{(g \circ f)(x) = 4e^x + 4 + a}$ [2]

(b) Given that $(g \circ f)(0) = 2$, find the value of a. [2]

(c) Solve the equation $(g \circ f)(x) = 0$. [3]

(b) $(g \circ f)(0) = 2 = 4e^0 + 4 + a$

$2 = 4 + 4 + a$

$\boxed{a = -6}$

Let $f(x) = e^x + 1$ and $g(x) = 4x + a$, where $x \in \mathbb{R}$ and a is a constant.

(a) Find $(g \circ f)(x)$. $\boxed{(g \circ f)(x) = 4e^x + 4 + a}$ [2]

(b) Given that $(g \circ f)(0) = 2$, find the value of a. $\boxed{a = -6}$ [2]

(c) Solve the equation $(g \circ f)(x) = 0$. [3]

(c) $4e^x + 4 - 6 = 0$

$4e^x = 2$

$e^x = \frac{1}{2}$

$x = \ln\left(\frac{1}{2}\right)$

$\boxed{x = -\ln 2}$

EXAM PAPERS PRACTICE

Question 11

Let $f(x) = ab^x$, where $x, a, b \in \mathbb{R}$ and $x \geq 0$, $a, b > 1$.

The graph of f contains the points $(0, 3)$ and $(2, 75)$.

(a) Find the values of a and b. [3]

(b) Find an expression for $f^{-1}(x)$. [3]

(b) Find the value of $f^{-1}(375)$. [2]

(a) Substitute the coordinates into $f(x)$ to form two equations involving a and b

$$3 = ab^0$$

$$\boxed{a = 3}$$

$$75 = (3)b^2$$

$$25 = b^2$$

$$\boxed{b = 5}$$

Let $f(x) = ab^x$, where $x, a, b \in \mathbb{R}$ and $x \geq 0$, $a, b > 1$.

The graph of f contains the points $(0, 3)$ and $(2, 75)$.

(a) Find the values of a and b. $\boxed{a = 3}$ $\boxed{b = 5}$ [3]

(b) Find an expression for $f^{-1}(x)$. [3]

(b) Find the value of $f^{-1}(375)$. [2]

(b) $f(x) = (3)(5)^x$

Write in terms of x and y and rearrange to make x the subject

$$y = 3 \times 5^x$$

$$\frac{y}{3} = 5^x$$

$$\log_5\left(\frac{y}{3}\right) = x$$

Replace x with $f^{-1}(x)$ and y with x

$$\boxed{f^{-1}(x) = \log_5\left(\frac{x}{3}\right)}$$

Let $f(x) = ab^x$, where $x, a, b \in \mathbb{R}$ and $x \geq 0$, $a, b > 1$.

The graph of f contains the points $(0, 3)$ and $(2, 75)$.

(a) Find the values of a and b. [3]

(b) Find an expression for $f^{-1}(x)$. $\boxed{f^{-1}(x) = \log_5\left(\frac{x}{3}\right)}$ [3]

(b) Find the value of $f^{-1}(375)$. [2]

(c) $f^{-1}(375) = y = \log_5\left(\frac{375}{3}\right)$

$$y = \log_5(125)$$

$$5^y = 125$$

$$y = 3$$

$$\boxed{f^{-1}(375) = 3}$$

EXAM PAPERS PRACTICE

Question 12

Consider $f(x) = \ln(\sqrt{x^2 - 16})$.

(a) Find the largest possible domain D_f for f to be a function.

[2]

(a) You cannot take a log of a number that is less than or equal to zero

$x^2 - 16 > 0$

$x^2 > 16$

$x > 4$ or $x < -4$

$\boxed{D_f = -\infty \le x < -4,\ 4 < x \le \infty}$

Let $f(x) = \ln(\sqrt{x^2 - 16})$, for $x \in D_f$.

(b) Explain why

(i) f is an even function

(ii) the inverse function f^{-1} does not exist.

[3]

Consider $f(x) = \ln(\sqrt{x^2 - 16})$.

(a) Find the largest possible domain D_f for f to be a function.

[2]

Let $f(x) = \ln(\sqrt{x^2 - 16})$, for $x \in D_f$.

(b) Explain why

(i) f is an even function

(ii) the inverse function f^{-1} does not exist.

[3]

(b) (i) $\boxed{f(x) = f(-x) \text{ for all of } D_f}$

(ii) $f^{-1}(x)$ does not exist for $x \in D_f$ because $f(x)$ is not a one-to-one function

EXAM PAPERS PRACTICE

Question 13

Let $f(x) = \frac{2(x+1)}{x-1}$, for $x \neq 1$, and $g(x) = x + 1$, for $x \in \mathbb{R}$.

The graphs of f and g intersect at points A and B.

(a) Find the coordinates of A and B.

[5]

(b) Find the equation of the straight line that passes through A and B, giving your answer in the form $ax + by + d = 0$.

[3]

(c) Write down the gradient of the line that is perpendicular to the line passing through A and B.

[2]

a) $f(x) = g(x)$

$$\frac{2(x+1)}{x-1} = x+1$$

$$2x + 2 = (x+1)(x-1) = x^2 - 1$$

$$x^2 - 2x - 3 = (x-3)(x+1) = 0 \quad \therefore x = -1, 3$$

Sub $x = -1, 3$ into $f(x)$ or $g(x)$.

$g(-1) = (-1) + 1 = 0 \quad \therefore (-1, 0)$

$g(3) = (3) + 1 = 4 \quad \therefore (3, 4)$

$\boxed{A(-1,0) \text{ and } B(3,4)}$

Let $f(x) = \frac{2(x+1)}{x-1}$, for $x \neq 1$, and $g(x) = x + 1$, for $x \in \mathbb{R}$.

The graphs of f and g intersect at points A and B.

(a) Find the coordinates of A and B.

$\boxed{A(-1,0) \text{ and } B(3,4)}$

[5]

(b) Find the equation of the straight line that passes through A and B, giving your answer in the form $ax + by + d = 0$.

[3]

(c) Write down the gradient of the line that is perpendicular to the line passing through A and B.

[2]

b) Find the gradient, $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$m_{AB} = \frac{0-4}{-1-3} = \frac{-4}{-4} = 1$$

Use point-gradient formula: $y - y_1 = m(x - x_1)$

$m = 1 \quad B(\overset{x_1}{3}, \overset{y_1}{4}) \rightarrow$ you could use $A(-1, 0)$

$$y - 4 = (1)(x - 3)$$

$\boxed{x - y + 1 = 0}$

Let $f(x) = \frac{2(x+1)}{x-1}$, for $x \neq 1$, and $g(x) = x + 1$, for $x \in \mathbb{R}$.

The graphs of f and g intersect at points A and B.

(a) Find the coordinates of A and B.

[5]

(b) Find the equation of the straight line that passes through A and B, giving your answer in the form $ax + by + d = 0$.

$\boxed{x - y + 1 = 0}$

[3]

(c) Write down the gradient of the line that is perpendicular to the line passing through A and B.

[2]

c) Perpendicular gradient: $m_\perp = -\frac{1}{m}$

$m_\perp = -\frac{1}{(1)}$

$\boxed{m_\perp = -1}$