Question 1

A company manufactures food tins in the shape of cylinders which must have a constant volume of 150π cm^3. To lessen material costs the company would like to minimise the surface area of the tins.

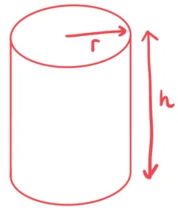
(a) By first expressing the height h of the tin in terms of its radius r, show that the surface area of the cylinder is given by $S = 2\pi r^2 + \frac{300\pi}{r}$.

[2]

(b) Use calculus to find the minimum value for the surface area of the tins. Give your answer correct to 2 decimal places.

[4]

a)



The surface area for a cylinder is:

$S = 2\pi r^2 + h2\pi r$

Find h in terms of r to eliminate h.

$Vol = 150\pi = \pi r^2 h$

$h = \frac{150}{r^2}$

$S = 2\pi r^2 + \frac{150(2\pi \cancel{r})}{r^{\cancel{2}}}$

$$\boxed{S = 2\pi r^2 + \frac{300\pi}{r}}$$

A company manufactures food tins in the shape of cylinders which must have a constant volume of 150π cm^3. To lessen material costs the company would like to minimise the surface area of the tins.

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[2]

(b) Use calculus to find the minimum value for the surface area of the tins. Give your answer correct to 2 decimal places.

[4]

$\frac{dS}{dr} = 0$

b) $S = 2\pi r^2 + 300\pi r^{-1}$

Minimum point occurs when gradient, $\left(\frac{dS}{dr}\right) = 0$.

Find r at that minimum.

$\frac{dS}{dr} = 4\pi r - 300\pi r^{-2} = 0$

$4r = \frac{300}{r^2}$

$r = \sqrt[3]{75}$

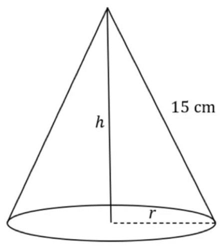
Sub $r = \sqrt[3]{75}$ into eqn to find minimum value of S.

$S = 2\pi\left(\sqrt[3]{75}\right)^2 + \frac{300\pi}{\left(\sqrt[3]{75}\right)} = \boxed{335.23\text{ cm}^2}$

(2dp)

Question 2

A right-angled triangle of height h, base r and hypotenuse 15 cm has been rotated about its vertical axis to form a cone.



(a) Write an expression for r in terms of h.

[2]

(b) Show that the volume of the cone, V cm^3, can be expressed as:

$$V = \frac{\pi}{3}(225h - h^3)$$

[3]

(c) Find the value of h which provides the cone with its maximum volume.

[3]

a) Pythagoras

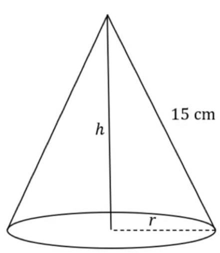
$r^2 + h^2 = 15^2$

$r^2 + h^2 = 225$

$r^2 = 225 - h^2$

$\boxed{r = \sqrt{225 - h^2}}$

A right-angled triangle of height h, base r and hypotenuse 15 cm has been rotated about its vertical axis to form a cone.



(a) Write an expression for r in terms of h. $\boxed{r = \sqrt{225 - h^2}}$

[2]

(b) Show that the volume of the cone, V cm^3, can be expressed as:

$$V = \frac{\pi}{3}(225h - h^3)$$

[3]

(c) Find the value of h which provides the cone with its maximum volume.

[3]

b) Volume of a cone

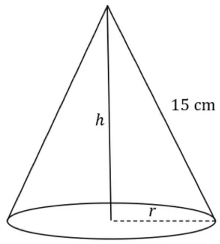
$V = \frac{1}{3}\pi r^2 h$ (in formula booklet)

$V = \frac{1}{3}\pi \left(\sqrt{225 - h^2}\right)^2 h$

$V = \frac{\pi}{3}(225 - h^2)h$

$\boxed{V = \frac{\pi}{3}(225h - h^3)}$

A right-angled triangle of height h, base r and hypotenuse 15 cm has been rotated about its vertical axis to form a cone.



(a) Write an expression for r in terms of h.

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$$V = \frac{\pi}{3}(225h - h^3)$$

[3]

(c) Find the value of h which provides the cone with its maximum volume.

[3]

c) Graph V on your GDC and find its maximum.

$h = 8.66025...$

$\boxed{h = 8.66 \text{ cm} \quad (3sf)}$

Question 3

A wire of length 1 m is cut into two pieces. The first piece is bent into the shape of a square. The second piece is bent into a rectangle which has a length l twice as long as its width w. Let x cm be the perimeter of the square.

(a) Find an expression for the area of the square.

[3]

(b) Show that the width of the rectangle $w = \frac{100-x}{6}$.

[3]

(c) Find an expression for the sum of the area of the two shapes, S.

[3]

(d) Find the value of x such that the sum of the areas, S, is a minimum.

[4]

a) if square perimeter $= x$

$\therefore$ square sides $= \frac{x}{4}$

$\therefore$ square area $= \left(\frac{x}{4}\right)^2$

$\boxed{\therefore \text{ square area } = \frac{x^2}{16}}$

A wire of length 1 m is cut into two pieces. The first piece is bent into the shape of a square. The second piece is bent into a rectangle which has a length l twice as long as its width w. Let x cm be the perimeter of the square.

(a) Find an expression for the area of the square. [3]

(b) Show that the width of the rectangle $w = \frac{100-x}{6}$. [3]

(c) Find an expression for the sum of the area of the two shapes, S. [3]

(d) Find the value of x such that the sum of the areas, S, is a minimum. [4]

b) $1\text{ m} = 100\text{ cm}$

Rectangle perimeter $= 2l + 2w$

Rectangle perimeter $= 100 - x$

$100 - x = 2l + 2w$ $\quad (l = 2w)$

$100 - x = 2(2w) + 2w$

$100 - x = 6w$

$$\boxed{w = \frac{100 - x}{6}}$$

A wire of length 1 m is cut into two pieces. The first piece is bent into the shape of a square. The second piece is bent into a rectangle which has a length l twice as long as its width w. Let x cm be the perimeter of the square.

(a) Find an expression for the area of the square.

$$\boxed{\therefore \text{ square area} = \frac{x^2}{16}}$$

[3]

(b) Show that the width of the rectangle $w = \frac{100-x}{6}$. [3]

(c) Find an expression for the sum of the area of the two shapes, S. [3]

(d) Find the value of x such that the sum of the areas, S, is a minimum. [4]

c) S = rectangle area + square area

$S = lw + \frac{x^2}{16}$ $\quad (l = 2w)$

$S = 2w^2 + \frac{x^2}{16}$

$S = 2\left(\frac{100 - x}{6}\right)^2 + \frac{x^2}{16}$

$S = \cancel{2}\frac{(100 - x)^2}{\cancel{36}\,18} + \frac{x^2}{16}$

$$\boxed{S = \frac{(100 - x)^2}{18} + \frac{x^2}{16}}$$

A wire of length 1 m is cut into two pieces. The first piece is bent into the shape of a square. The second piece is bent into a rectangle which has a length l twice as long as its width w. Let x cm be the perimeter of the square.

(a) Find an expression for the area of the square.

[3]

(b) Show that the width of the rectangle $w = \frac{100-x}{6}$.

[3]

(c) Find an expression for the sum of the area of the two shapes, S.

[3]

(d) Find the value of x such that the sum of the areas, S, is a minimum.

[4]

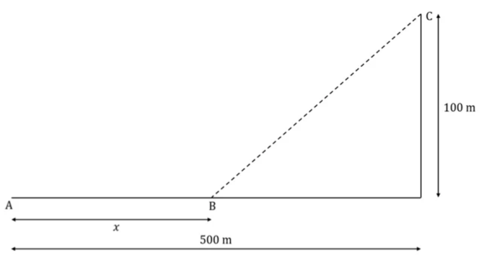
d) Graph S and find its minimum.

$x = 47.05882...$

$\boxed{x = 47.1 \text{ cm } (3sf)}$

Question 4

Liam, a keen runner and swimmer, enters a competition which requires the competitors to run from point A along a straight beach, before diving into the water and swimming directly to point C. Liam is able to run at a speed of 8 m/s along the beach and swim at 2 m/s in the water.



Let x represent the distance between A and B, the distance that Liam runs along the beach before entering the water and swimming along the line BC.

(a) Find an expression for the time taken for Liam to run x metres between A and B.

[2]

(b) Show that the length of $BC = \sqrt{10000 + (500 - x)^2}$.

[2]

(c) Find an expression for the total time taken for Liam to finish the race.

[2]

(d) Find the value of x that will allow Liam to complete the race in the quickest time.

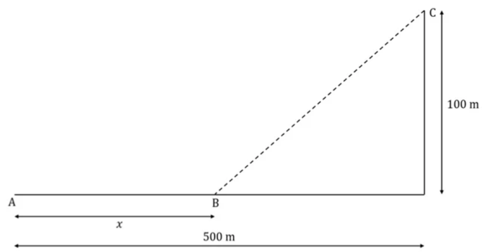
[3]

a) Speed = $\frac{\text{dist } (d)}{\text{time } (t)}$ (not in formula booklet)

$8 = \frac{x}{t_r}$

$\boxed{t_r = \frac{x}{8}}$

Liam, a keen runner and swimmer, enters a competition which requires the competitors to run from point A along a straight beach, before diving into the water and swimming directly to point C. Liam is able to run at a speed of 8 m/s along the beach and swim at 2 m/s in the water.



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(a) Find an expression for the time taken for Liam to run x metres between A and B.

[2]

(b) Show that the length of $BC = \sqrt{10000 + (500 - x)^2}$.

[2]

(c) Find an expression for the total time taken for Liam to finish the race.

[2]

(d) Find the value of x that will allow Liam to complete the race in the quickest time.

[3]

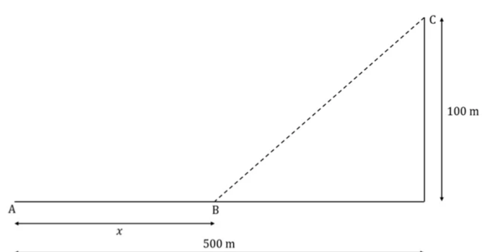
b) Pythagoras

$BC^2 = 100^2 + (500 - x)^2$

$BC^2 = 10\,000 + (500 - x)^2$

$$\boxed{BC = \sqrt{10\,000 + (500 - x)^2}}$$

Liam, a keen runner and swimmer, enters a competition which requires the competitors to run from point A along a straight beach, before diving into the water and swimming directly to point C. Liam is able to run at a speed of 8 m/s along the beach and swim at 2 m/s in the water.



Let x represent the distance between A and B, the distance that Liam runs along the beach before entering the water and swimming along the line BC.

(a) Find an expression for the time taken for Liam to run x metres between A and B.

$$\boxed{t_r = \frac{x}{8}}$$

[2]

(b) Show that the length of $BC = \sqrt{10000 + (500 - x)^2}$.

[2]

(c) Find an expression for the total time taken for Liam to finish the race.

[2]

(d) Find the value of x that will allow Liam to complete the race in the quickest time.

[3]

c) $\text{Speed} = \dfrac{\text{dist } (d)}{\text{time } (t)}$ (not in formula booklet)

Let T be the total time, t_r be the time running and t_s be the time swimming.

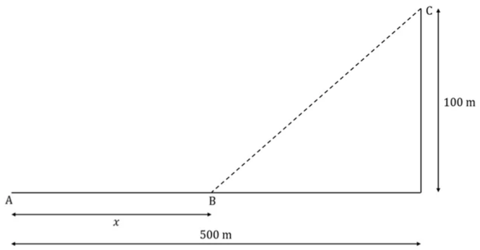
$2 = \dfrac{\sqrt{10\,000 + (500 - x)^2}}{t_s}$

$t_s = \dfrac{\sqrt{10\,000 + (500 - x)^2}}{2}$

$T = t_r + t_s$

$$\boxed{T = \frac{x}{8} + \frac{\sqrt{10\,000 + (500 - x)^2}}{2}}$$

Liam, a keen runner and swimmer, enters a competition which requires the competitors to run from point A along a straight beach, before diving into the water and swimming directly to point C. Liam is able to run at a speed of 8 m/s along the beach and swim at 2 m/s in the water.



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(a) Find an expression for the time taken for Liam to run x metres between A and B.

[2]

(b) Show that the length of $BC = \sqrt{10000 + (500 - x)^2}$.

[2]

(c) Find an expression for the total time taken for Liam to finish the race.

$\boxed{T = \frac{x}{8} + \frac{\sqrt{10\,000 + (500-x)^2}}{2}}$

[2]

(d) Find the value of x that will allow Liam to complete the race in the quickest time.

[3]

d) Graph T and find its minimum.

$x = 474.1801\ldots$

$\boxed{x = 474 \text{ m} \quad (3sf)}$

Question 5

A small cylindrical drum, closed at the top but open at the bottom, has a radius r cm and a height h cm. The volume of the drum is 1000 cm^3.

The material to make the top skin of the drum costs 25 cents per cm^2 and the curved surface of the drum costs 20 cents per cm^2.

(a) Find an expression for h in terms of r.

[2]

a) Volume of a cylinder

$V = \pi r^2 h$ (in formula booklet)

$1000 = \pi r^2 h$

$\boxed{h = \frac{1000}{\pi r^2}}$

(b) Show that the total cost of the material to make the drum is $C = 25\pi r^2 + \frac{40000}{r}$.

[4]

(c) Find $\frac{dC}{dr}$.

[2]

The function $C(r)$ has a local minimum at the point (p, q).

(d) Find the value of p.

[4]

(e) State, to the nearest dollar, the minimum cost required to make the drum.

[1]

(f) Find $\frac{d^2C}{dr^2}$ and hence, describe the concavity of the function $C(r)$ at $x = p$.

[4]

A small cylindrical drum, closed at the top but open at the bottom, has a radius r cm and a height h cm. The volume of the drum is 1000 cm^3.

The material to make the top skin of the drum costs 25 cents per cm^2 and the curved surface of the drum costs 20 cents per cm^2.

(a) Find an expression for h in terms of r. [2]

$\boxed{h = \frac{1000}{\pi r^2}}$

(b) Show that the total cost of the material to make the drum is $C = 25\pi r^2 + \frac{40000}{r}$. [4]

(c) Find $\frac{dC}{dr}$. [2]

The function $C(r)$ has a local minimum at the point (p, q).

(d) Find the value of p. [4]

(e) State, to the nearest dollar, the minimum cost required to make the drum. [1]

(f) Find $\frac{d^2C}{dr^2}$ and hence, describe the concavity of the function $C(r)$ at $x = p$. [4]

b) Top skin = cost × circle area

Top skin = $25\pi r^2$

Curved surface area of a cone

$A = 2\pi r h$ (in formula booklet)

Curved surface = cost × area

Curved surface = $20 \times 2\pi r h$ $\left(h = \frac{1000}{\pi r^2}\right)$

Curved surface = $20 \times 2\cancel{\pi}\cancel{r}\left(\frac{1000}{\cancel{\pi} r^{\cancel{2}}}\right)$

Curved surface = $\frac{40\,000}{r}$

$\boxed{\therefore\ C = 25\pi r^2 + \frac{40\,000}{r}}$

A small cylindrical drum, closed at the top but open at the bottom, has a radius r cm and a height h cm. The volume of the drum is 1000 cm^3.

The material to make the top skin of the drum costs 25 cents per cm^2 and the curved surface of the drum costs 20 cents per cm^2.

(a) Find an expression for h in terms of r. [2]

(b) Show that the total cost of the material to make the drum is $C = 25\pi r^2 + \frac{40000}{r}$. [4]

(c) Find $\frac{dC}{dr}$. [2]

The function $C(r)$ has a local minimum at the point (p, q).

(d) Find the value of p. [4]

(e) State, to the nearest dollar, the minimum cost required to make the drum. [1]

(f) Find $\frac{d^2C}{dr^2}$ and hence, describe the concavity of the function $C(r)$ at $x = p$. [4]

c) Derivative of x^n formula (in formula booklet)

$f(x) = y = x^n \longrightarrow f'(x) = \frac{dy}{dx} = nx^{n-1}$

$C = 25\pi r^2 + 40\,000r^{-1}$

$\frac{dC}{dr} = 50\pi r - 40\,000r^{-2}$

$\boxed{\frac{dC}{dr} = 50\pi r - \frac{40\,000}{r^2}}$

A small cylindrical drum, closed at the top but open at the bottom, has a radius r cm and a height h cm. The volume of the drum is 1000 cm^3.

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(a) Find an expression for h in terms of r.

[2]

(b) Show that the total cost of the material to make the drum is $C = 25\pi r^2 + \frac{40000}{r}$.

[4]

(c) Find $\frac{dC}{dr}$.

$$\boxed{\frac{dC}{dr} = 50\pi r - \frac{40\,000}{r^2}}$$

[2]

The function $C(r)$ has a local minimum at the point (p, q).

(d) Find the value of p.

[4]

(e) State, to the nearest dollar, the minimum cost required to make the drum.

[1]

(f) Find $\frac{d^2C}{dr^2}$ and hence, describe the concavity of the function $C(r)$ at $x = p$.

[4]

d) Graph C and find its minimum

$r = p = 6.3384\ldots$

$\boxed{p = 6.34 \text{ cm } (3sf)}$

A small cylindrical drum, closed at the top but open at the bottom, has a radius r cm and a height h cm. The volume of the drum is 1000 cm^3.

The material to make the top skin of the drum costs 25 cents per cm^2 and the curved surface of the drum costs 20 cents per cm^2.

(a) Find an expression for h in terms of r.

[2]

(b) Show that the total cost of the material to make the drum is $C = 25\pi r^2 + \frac{40000}{r}$.

[4]

(c) Find $\frac{dC}{dr}$.

[2]

The function $C(r)$ has a local minimum at the point (p, q).

(d) Find the value of p.

[4]

(e) State, to the nearest dollar, the minimum cost required to make the drum.

[1]

(f) Find $\frac{d^2C}{dr^2}$ and hence, describe the concavity of the function $C(r)$ at $x = p$.

[4]

e) Use your graph from part (d).

$C_{min} = 9466.1027\ldots$ cents

$\boxed{C_{min} = \$95 \text{ (nearest dollar)}}$

A small cylindrical drum, closed at the top but open at the bottom, has a radius r cm and a height h cm. The volume of the drum is 1000 cm^3.

The material to make the top skin of the drum costs 25 cents per cm^2 and the curved surface of the drum costs 20 cents per cm^2.

(a) Find an expression for h in terms of r.

[2]

(b) Show that the total cost of the material to make the drum is $C = 25\pi r^2 + \frac{40000}{r}$.

[4]

(c) Find $\frac{dC}{dr}$.

$$\boxed{\frac{dC}{dr} = 50\pi r - \frac{40\,000}{r^2}}$$

[2]

The function $C(r)$ has a local minimum at the point (p, q).

(d) Find the value of p.

$\boxed{p = 6.34 \text{ cm } (3sf)}$

[4]

(e) State, to the nearest dollar, the minimum cost required to make the drum.

[1]

(f) Find $\frac{d^2C}{dr^2}$ and hence, describe the concavity of the function $C(r)$ at $x = p$.

[4]

f) First derivative = gradient

Second derivative = concavity

Derivative of x^n formula (in formula booklet)

$f(x) = y = x^n \longrightarrow f'(x) = \frac{dy}{dx} = nx^{n-1}$

$\frac{dC}{dr} = 50\pi r - 40\,000 r^{-2}$

$\frac{d^2C}{dr^2} = 50\pi + 80\,000 r^{-3}$

$$\boxed{\frac{d^2C}{dr^2} = 50\pi + \frac{80\,000}{r^3}}$$

$50\pi + \frac{80000}{r^3} > 0$ when $x = 6.34$

$\boxed{\therefore C \text{ is concave up at } x = p.}$

Question 6

The daily cost function of a company producing pairs of running shoes is modelled by the cubic function

$$C(x) = 1225 + 11x - 0.009x^2 - 0.0001x^3, \qquad 0 \le x < 160$$

where x is the number of pairs of running shoes produced and C the cost in USD.

(a) Write down the daily cost to the company if no pairs of running shoes are produced.

[1]

a) $x = 0$ when no shoes are produced.

$C(0) = 1225 + 11(0) - 0.009(0)^2 - 0.0001(0)^3$

$\boxed{C(0) = 1225 \text{ USD}}$

The marginal cost of production is the cost of producing one additional unit. This can be approximated by the gradient of the cost function.

(b) Find an expression for the marginal cost, $C'(x)$, of producing pairs of running shoes.

[2]

(c) Find the marginal cost of producing

(i) 40 pairs of running shoes

(ii) 90 pairs of running shoes.

[2]

The optimum level of production is when marginal revenue equals marginal cost. The marginal revenue, $R'(x)$, is equal to 4.5.

(d) Find the optimum level of production.

(3)

The daily cost function of a company producing pairs of running shoes is modelled by the cubic function

$$C(x) = 1225 + 11x - 0.009x^2 - 0.0001x^3, \qquad 0 \leq x < 160$$

where x is the number of pairs of running shoes produced and C the cost in USD.

(a) Write down the daily cost to the company if no pairs of running shoes are produced.

[1]

The marginal cost of production is the cost of producing one additional unit. This can be approximated by the gradient of the cost function.

(b) Find an expression for the marginal cost, $C'(x)$, of producing pairs of running shoes.

[2]

(c) Find the marginal cost of producing

(i) 40 pairs of running shoes

(ii) 90 pairs of running shoes.

[2]

The optimum level of production is when marginal revenue equals marginal cost. The marginal revenue, $R'(x)$, is equal to 4.5.

(d) Find the optimum level of production.

(3)

b) Derivative of x^n formula (in formula booklet)

$f(x) = y = x^n \longrightarrow f'(x) = \frac{dy}{dx} = nx^{n-1}$

$C(x) = 1225 + 11x - 0.009x^2 - 0.0001x^3$

Apply formula

$\boxed{C'(x) = 11 - 0.018x - 0.0003x^2}$

The daily cost function of a company producing pairs of running shoes is modelled by the cubic function

$$C(x) = 1225 + 11x - 0.009x^2 - 0.0001x^3, \qquad 0 \leq x < 160$$

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(a) Write down the daily cost to the company if no pairs of running shoes are produced.

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(b) Find an expression for the marginal cost, $C'(x)$, of producing pairs of running shoes.

$\boxed{C'(x) = 11 - 0.018x - 0.0003x^2}$

[2]

(c) Find the marginal cost of producing

(i) 40 pairs of running shoes

(ii) 90 pairs of running shoes.

[2]

The optimum level of production is when marginal revenue equals marginal cost. The marginal revenue, $R'(x)$, is equal to 4.5.

(d) Find the optimum level of production.

(3)

c) i) Sub $x = 40$ into $C'(x)$

$C'(40) = 11 - 0.018(40) - 0.0003(40)^2$

$\boxed{C'(40) = 9.80 \text{ USD}}$

ii) Sub $x = 90$ into $C'(x)$

$C'(90) = 11 - 0.018(90) - 0.0003(90)^2$

$\boxed{C'(90) = 6.95 \text{ USD}}$

The daily cost function of a company producing pairs of running shoes is modelled by the cubic function

$$C(x) = 1225 + 11x - 0.009x^2 - 0.0001x^3, \qquad 0 \leq x < 160$$

where x is the number of pairs of running shoes produced and C the cost in USD.

(a) Write down the daily cost to the company if no pairs of running shoes are produced.

[1]

The marginal cost of production is the cost of producing one additional unit. This can be approximated by the gradient of the cost function.

(b) Find an expression for the marginal cost, $C'(x)$, of producing pairs of running shoes.

$\boxed{C'(x) = 11 - 0.018x - 0.0003x^2}$

[2]

(c) Find the marginal cost of producing

(i) 40 pairs of running shoes

(ii) 90 pairs of running shoes.

[2]

The optimum level of production is when marginal revenue equals marginal cost. The marginal revenue, $R'(x)$, is equal to 4.5.

(d) Find the optimum level of production.

(3)

d) Optimum level of production is when $R'(x) = C'(x)$

$4.5 = 11 - 0.018x - 0.0003x^2$

Solve for x on your GDC

$x = 120.222...$

$\boxed{\text{120 pairs of running shoes.}}$

Question 7

A cyclist riding over a hill can be modelled by the function

$$h(t) = -\frac{1}{24}t^2 + 3t + 12, \qquad 0 \leq t \leq 70$$

where h is the cyclist's altitude above mean sea level, in metres, and t is the elapsed time, in seconds.

(a) Calculate the cyclist's altitude after a minute.

[2]

(b) Find $h'(t)$.

[2]

(c) Calculate the cyclist's maximum altitude and the time it takes to reach this altitude.

[3]

a) Sub $t = 60$ into $h(t)$.

$h(60) = -\frac{1}{24}(60)^2 + 3(60) + 12$

$\boxed{h(60) = 42 \text{ m}}$

A cyclist riding over a hill can be modelled by the function

$$h(t) = -\frac{1}{24}t^2 + 3t + 12, \qquad 0 \le t \le 70$$

where h is the cyclist's altitude above mean sea level, in metres, and t is the elapsed time, in seconds.

(a) Calculate the cyclist's altitude after a minute. [2]

(b) Find $h'(t)$. [2]

(c) Calculate the cyclist's maximum altitude and the time it takes to reach this altitude. [3]

b) Derivative of x^n formula (in formula booklet)

$f(x) = y = x^n \longrightarrow f'(x) = \frac{dy}{dx} = nx^{n-1}$

$h(t) = -\frac{1}{24}t^2 + 3t + 12$

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(a) Calculate the cyclist's altitude after a minute. [2]

(b) Find $h'(t)$. $\boxed{h'(t) = -\frac{1}{12}t + 3}$ [2]

(c) Calculate the cyclist's maximum altitude and the time it takes to reach this altitude. [3]

c) Set $h'(t) = 0$ and solve for t.

$-\frac{1}{12}t + 3 = 0$

$\boxed{t = 36\text{s}}$

Sub $t = 36$ into $h(t)$.

$h(36) = -\frac{1}{24}(36)^2 + 3(36) + 12$

$\boxed{h(36) = 66\text{m}}$

Question 8

A company produces and sells cricket bats. The company's daily cost, C, in hundreds of Australian dollars (AUD), changes based on the number of cricket bats they produce per day. The daily cost function of the company can be modelled by

$$C(x) = 6x^3 - 10x^2 + 10x + 4$$

where x hundred cricket bats is the number of cricket bats produced on a particular day.

(a) Find the cost to the company for any day zero cricket bats are produced.

[1]

The company's daily revenue, in hundreds of AUD, from selling x hundred cricket bats is given by the function $R(x) = 42x$.

(b) Given that profit = revenue – cost, determine a function for the profit, $P(x)$, in hundreds of AUD from selling x hundred cricket bats.

[2]

(c) Find $P'(x)$.

[2]

The derivative of $P(x)$ gives the marginal profit. The production of bats will reach its profit maximising level when the marginal profit equals zero and $P(x)$ is positive.

(d) Find the profit maximising production level and the expected profit.

[3]

a) Sub $x = 0$ into $C(x)$.

$C(0) = 6(0)^3 - 10(0)^2 + 10(0) + 4$

$C(0) = 4$

$\boxed{400 \text{ AUD}}$

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[3]

b) $P(x) = R(x) - C(x)$

$P(x) = 42x - (6x^3 - 10x^2 + 10x + 4)$

$\boxed{P(x) = -6x^3 + 10x^2 + 32x - 4}$

EXAM PAPERS PRACTICE

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The derivative of $P(x)$ gives the marginal profit. The production of bats will reach its profit maximising level when the marginal profit equals zero and $P(x)$ is positive.

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[3]

c) Derivative of x^n formula (in formula booklet)

$f(x) = y = x^n \longrightarrow f'(x) = \frac{dy}{dx} = nx^{n-1}$

$P(x) = -6x^3 + 10x^2 + 32x - 4$

$\boxed{P'(x) = -18x^2 + 20x + 32}$

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$\boxed{P(x) = -6x^3 + 10x^2 + 32x - 4}$

[2]

(c) Find $P'(x)$.

$\boxed{P'(x) = -18x^2 + 20x + 32}$

[2]

The derivative of $P(x)$ gives the marginal profit. The production of bats will reach its profit maximising level when the marginal profit equals zero and $P(x)$ is positive.

(d) Find the profit maximising production level and the expected profit.

[3]

d) Set $P'(x) = 0$ and solve for x.

$-18x^2 + 20x + 32 = 0$

$x = 2$ $\quad \cancel{x = -0.889}$

Reject as $P(x) > 0$ and $x \geq 0$

Sub $x = 2$ into $P(x)$.

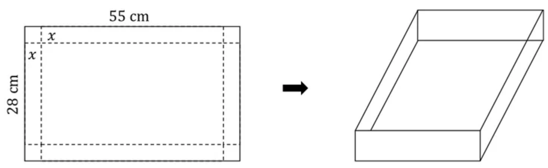
$P(2) = -6(2)^3 + 10(2)^2 + 32(2) - 4$

$P(2) = 52$

$\boxed{\text{∴ The profit maximising production level is 200 bats and the expected profit is 5200 AUD.}}$

Question 9

Dora decides to build a cardboard container for when she goes strawberry picking from a rectangular piece of cardboard, 55 cm $\times$ 28 cm. She cuts squares with side length x cm from each corner as shown in the diagram below.



(a) Show that the volume, V cm^3, of the container is given by

$$V = 4x^3 - 166x^2 + 1540x$$

[2]

(b) Find $\frac{dV}{dx}$.

[2]

(c) Find

(i) the value of x that maximises the volume of the container

(ii) the maximum volume of the container. Give your answer in the form $a \times 10^k$, where $1 \leq a \leq 10$ and $k \in \mathbb{Z}$.

[4]

a) Volume of a cuboid formula.

$V = lwh$ (in formula booklet)

where l is the length, w is the width and h is the height.

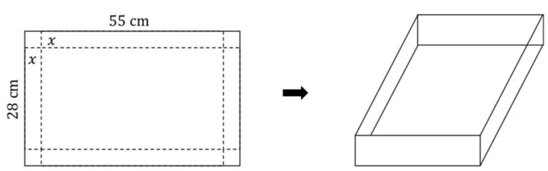
$l = 55 - 2x \quad w = 28 - 2x \quad h = x$

Sub l, w and h into formula.

$V = (55 - 2x)(28 - 2x)x$ expand and rearrange

$$\boxed{V = 4x^3 - 166x^2 + 1540x}$$

Dora decides to build a cardboard container for when she goes strawberry picking from a rectangular piece of cardboard, 55 cm $\times$ 28 cm. She cuts squares with side length x cm from each corner as shown in the diagram below.



(a) Show that the volume, V cm^3, of the container is given by

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[4]

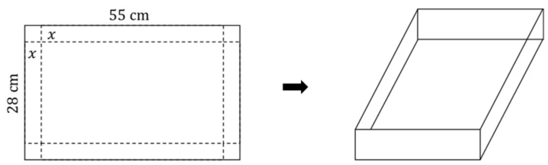
b) Derivative of x^n formula (in formula booklet)

$f(x) = y = x^n \longrightarrow f'(x) = \frac{dy}{dx} = nx^{n-1}$

$V = 4x^3 - 166x^2 + 1540x$

$$\boxed{\frac{dV}{dx} = 12x^2 - 332x + 1540}$$

Dora decides to build a cardboard container for when she goes strawberry picking from a rectangular piece of cardboard, 55 cm $\times$ 28 cm. She cuts squares with side length x cm from each corner as shown in the diagram below.



(a) Show that the volume, V cm^3, of the container is given by

$$V = 4x^3 - 166x^2 + 1540x$$

[2]

(b) Find $\frac{dV}{dx}$.

$$\boxed{\frac{dV}{dx} = 12x^2 - 332x + 1540}$$

[2]

(c) Find

(i) the value of x that maximises the volume of the container

(ii) the maximum volume of the container. Give your answer in the form $a \times 10^k$, where $1 \leq a \leq 10$ and $k \in \mathbb{Z}$.

[4]

c)i) Set $\frac{dV}{dx} = 0$ and solve for x.

$12x^2 - 332x + 1540 = 0$

$x = 5.8943...$ ~~$x = 21.772...$~~ Reject as $l > 0$

$$\boxed{x = 5.89\text{cm} \quad (3sf)}$$

ii) Sub $x = 5.8943...$ into V.

$V = 4(5.8943)^3 - 166(5.8943)^2 + 1540(5.8943)$

$V = 4129.059...$

$= 4130 \quad (3sf)$

$$\boxed{V = 4.13 \times 10^3 \text{ cm}^3}$$