

Number Toolkit

Mark Schemes

Question 1

Let $Q = \frac{30 \sin 2a}{8b}$, where $a = 45^\circ$ and $b = 2$.

(a) Calculate the exact value of Q .

(b) Give your answer from part (a) correct to

- (i) two decimal places
- (ii) two significant figures.

Nina estimates the value of Q to be 2.

(c) Calculate the percentage error in Nina's estimate.

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Nina estimates the value of Q to be 2.

(c) Calculate the percentage error in Nina's estimate.

a) Sub a and b into Q .

$$Q = \frac{30 \sin 2(45^\circ)}{8(2)}$$

$$Q = 1.875$$

b) i) $Q = 1.88$ (2dp)

ii) $Q = 1.9$ (2sf)

c) Percentage error formula.

$$E = \left| \frac{V_A - V_E}{V_E} \right| \times 100\% \quad (\text{in formula booklet})$$

where V_A is the approximated value and V_E is the exact value.

$$V_A = 2 \quad V_E = 1.875$$

Sub V_A and V_E into formula.

$$E = \left| \frac{2 - 1.875}{1.875} \right| \times 100\%$$

$$E = 6.666... \%$$

$$E = 6.67\% \quad (3sf)$$

Question 2

Let $R = \frac{4x}{6 \cos 5y}$, where $x = 1.25$ and $y = 36^\circ$.

(a) Find the value of R . Give your answer as a fraction.

[2]

(b) Give your answer from part (a) to

- (i) one decimal place
- (ii) three significant figures.

[2]

Kieran estimates the value of R to be -1 .

(c) Calculate the percentage error in Kieran's estimate.

[2]

Let $R = \frac{4x}{6 \cos 5y}$, where $x = 1.25$ and $y = 36^\circ$.

(a) Find the value of R . Give your answer as a fraction.

$$R = -\frac{5}{6}$$

[2]

(b) Give your answer from part (a) to

- (i) one decimal place
- (ii) three significant figures.

[2]

Kieran estimates the value of R to be -1 .

(c) Calculate the percentage error in Kieran's estimate.

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(b) Give your answer from part (a) to

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- (ii) three significant figures.

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Kieran estimates the value of R to be -1 .

(c) Calculate the percentage error in Kieran's estimate.

[2]

a) Sub x and y into R .

$$R = \frac{4(1.25)}{6 \cos 5(36^\circ)}$$

$$R = -\frac{5}{6} \text{ (fraction)}$$

b) $R = -0.83333\dots$

i) $R = -0.8$ (1dp)

ii) $R = -0.833$ (3sf)

c) Percentage error formula.

$$E = \left| \frac{V_A - V_E}{V_E} \right| \times 100\% \quad (\text{in formula booklet})$$

where V_E is the exact value and V_A is the approximated value.

$$V_A = -1 \quad V_E = -\frac{5}{6}$$

Sub V_A and V_E into formula.

$$E = \left| \frac{-1 - \left(-\frac{5}{6}\right)}{\left(-\frac{5}{6}\right)} \right| \times 100\%$$

$$E = 20\%$$

Question 3

Consider the numbers $a = 4.14 \times 10^6$ and $b = 2.54 \times 10^{-7}$.

(a) Calculate $C = \sqrt[10]{\left(\frac{a}{b}\right)^3}$. Give your answer correct to the nearest integer.

[2]

(b) Give your answer to part (a) in the form $a \times 10^k$, where $1 \leq a \leq 10$ and $k \in \mathbb{Z}$.

[2]

(c) Calculate the percentage error if C was approximated to be 9000.

[2]

a) Sub a and b into C.

$$C = \sqrt[10]{\left(\frac{4.14 \times 10^6}{2.54 \times 10^{-7}}\right)^3}$$

$$C = 9197.0804\dots$$

$$C = 9197 \text{ (nearest integer)}$$

Consider the numbers $a = 4.14 \times 10^6$ and $b = 2.54 \times 10^{-7}$.

(a) Calculate $C = \sqrt[10]{\left(\frac{a}{b}\right)^3}$. Give your answer correct to the nearest integer.

$$C = 9197 \text{ (nearest integer)}$$

[2]

(b) Give your answer to part (a) in the form $a \times 10^k$, where $1 \leq a \leq 10$ and $k \in \mathbb{Z}$.

[2]

(c) Calculate the percentage error if C was approximated to be 9000.

[2]

b) $C = 9.197 \times 10^3$

Consider the numbers $a = 4.14 \times 10^6$ and $b = 2.54 \times 10^{-7}$.

(a) Calculate $C = \sqrt[10]{\left(\frac{a}{b}\right)^3}$. Give your answer correct to the nearest integer.

$$C = 9197 \text{ (nearest integer)}$$

[2]

(b) Give your answer to part (a) in the form $a \times 10^k$, where $1 \leq a \leq 10$ and $k \in \mathbb{Z}$.

[2]

(c) Calculate the percentage error if C was approximated to be 9000.

[2]

c) Percentage error formula.

$$E = \left| \frac{V_A - V_E}{V_E} \right| \times 100\% \quad (\text{in formula booklet})$$

where V_E is the exact value and V_A is the approximated value.

$$V_A = 9000 \quad V_E = \sqrt[10]{\left(\frac{4.14 \times 10^6}{2.54 \times 10^{-7}}\right)^3} \quad (\text{exact answer for } C)$$

Sub V_A and V_E into formula.

$$E = \frac{\left| 9000 - \sqrt[10]{\left(\frac{4.14 \times 10^6}{2.54 \times 10^{-7}}\right)^3} \right|}{\sqrt[10]{\left(\frac{4.14 \times 10^6}{2.54 \times 10^{-7}}\right)^3}}$$

$$E = 2.1428\dots \%$$

$$E = 2.14 \%$$

Question 4

A cylinder has radius of 12.7 cm and height of 14.4 cm.

(a) Calculate the volume of the cylinder correct to

- (i) one decimal place
- (ii) three significant figures
- (iii) the nearest integer.

(b) Write your answer to part (a) (ii) in the form $a \times 10^k$, where $1 \leq a \leq 10$ and $k \in \mathbb{Z}$.

[3]

[2]

a) Volume of a cylinder formula

$$V = \pi r^2 h \quad (\text{in formula booklet})$$

$$r = 12.7 \quad h = 14.4$$

Sub r and h into formula.

$$V = \pi (12.7)^2 (14.4)$$

$$V = 7296.58\dots$$

i) $V = 7296.6 \text{ cm}^3$ (1dp)

ii) $V = 7300 \text{ cm}^3$ (3sf)

iii) $V = 7297$ (nearest integer)

A cylinder has radius of 12.7 cm and height of 14.4 cm.

(a) Calculate the volume of the cylinder correct to

- (i) one decimal place
- (ii) three significant figures
- (iii) the nearest integer.

$$V = 7300 \text{ cm}^3 \quad (3\text{sf})$$

[3]

(b) Write your answer to part (a) (ii) in the form $a \times 10^k$, where $1 \leq a \leq 10$ and $k \in \mathbb{Z}$.

[2]

b) $V = 7.3 \times 10^3 \text{ cm}^3$

Question 5

A rectangular field has length, L , of 25.2 m and width, W , of 21.4 m, each correct to 1 decimal place.

(a) Calculate the lower and upper bound for

(i) L .

(ii) W .

(b) Calculate the lower and upper bound for the

(i) perimeter, P .

(ii) area, A , of the field.

a) For L

Any value equal to or more than 25.15cm will be rounded up to 25.2cm (1dp).

Any value less than 25.25cm will be rounded down to 25.2cm (1dp).

For W

Any value equal to or more than 21.35cm will be rounded up to 21.4cm (1dp).

Any value less than 21.45cm will be rounded down to 21.4cm (1dp).

Write bounds as an inequality.

i) $25.15\text{m} \leq L < 25.25\text{m}$

ii) $21.35\text{m} \leq W < 21.45\text{m}$

[2]

[4]

A rectangular field has length, L , of 25.2 m and width, W , of 21.4 m, each correct to 1 decimal place.

(a) Calculate the lower and upper bound for

(i) L $25.15\text{m} \leq L < 25.25\text{m}$

(ii) W $21.35\text{m} \leq W < 21.45\text{m}$

(b) Calculate the lower and upper bound for the

(i) perimeter, P .

(ii) area, A , of the field.

b) For lower bound use

$$L = 25.15 \quad W = 21.35$$

$$P = 2(25.15) + 2(21.35) \quad A = (25.15)(21.35)$$

$$P = 93\text{m} \quad A = 536.9525\text{m}^2$$

For upper bound use

$$L = 25.25 \quad W = 21.45$$

$$P = 2(25.25) + 2(21.45) \quad A = (25.25)(21.45)$$

$$P = 93.4\text{m} \quad A = 541.6125\text{m}^2$$

i) $93\text{m} \leq P < 93.4\text{m}$

ii) $537\text{m}^2 \leq A < 541\text{m}^2$ (3sf)

[2]

[4]

Question 6

Calculate the following, giving your answer in the form $a \times 10^k$, where $1 \leq a \leq 10$ and $k \in \mathbb{Z}$.

- (i) $4 \times (6.2 \times 10^{-5})$
 (ii) $(4 \times 10^5) - (5 \times 10^4)$
 (iii) $(4321^{-1})(1.2 \times 10^{-1})$.

[6]

i) Input equation into calculator.

$$4 \times (6.2 \times 10^{-5}) = 0.00024$$

Rewrite into form $a \times 10^k$, where $1 \leq a \leq 10 \dots$

$$2.48 \times 10^{-4}$$

ii) Input equation into calculator.

$$(4 \times 10^5) - (5 \times 10^4) = 350\ 000$$

Rewrite into form $a \times 10^k$, where $1 \leq a \leq 10 \dots$

$$3.5 \times 10^5$$

iii) Input equation into calculator.

$$(4321^{-1})(1.2 \times 10^{-1}) = 0.0002777 \dots$$

Rewrite into form $a \times 10^k$, where $1 \leq a \leq 10 \dots$

$$2.78 \times 10^{-5} \text{ (3sf)}$$

Question 7

Consider the following four numbers.

$$a = 0.272 \quad b = 0.0272 \times 10^5 \quad c = e(10e)^{-1} \quad d = 2.72 \times 10^2$$

(a) Write down

- (i) the number that is in the form $a \times 10^k$, where $1 \leq a \leq 10$ and $k \in \mathbb{Z}$
 (ii) the largest of these numbers.

[2]

(b) (i) Find the value of $a + b - c + d$.

- (ii) Give your answer to part (b)(i) in the form $a \times 10^k$, where $1 \leq a \leq 10$ and $k \in \mathbb{Z}$.

[4]

a) i) $d = 2.72 \times 10^2$

$$a = 0.272 \quad b = 2720 \quad c = 0.1 \quad d = 272$$

ii) $b = 0.0272 \times 10^5$

Consider the following four numbers.

$$a = 0.272 \quad b = 0.0272 \times 10^5 \quad c = e(10e)^{-1} \quad d = 2.72 \times 10^2$$

(a) Write down

- (i) the number that is in the form $a \times 10^k$, where $1 \leq a \leq 10$ and $k \in \mathbb{Z}$
 (ii) the largest of these numbers.

(b) (i) Find the value of $a + b - c + d$.

- (ii) Give your answer to part (b)(i) in the form $a \times 10^k$, where $1 \leq a \leq 10$ and $k \in \mathbb{Z}$.

[2]

[4]

b) Sub a, b, c and d into equation.

$$0.272 + 0.0272 \times 10^5 - e(10e)^{-1} + 2.72 \times 10^2$$

$$0.272 + 2720 + 0.1 + 272$$

$$= 2992.172$$

i) 2990 (3sf)

ii) 2.99×10^3

Question 8

Five Olympic barbells labelled "2.2 m in length", were delivered to an Olympic weightlifting team. The coach measured each barbell to check its length, in metres, and recorded the following:

$$2.18, \quad 2.21, \quad 2.23, \quad 2.19, \quad 2.24$$

- (a) (i) Find the mean of the coach's recorded measurements.
 (ii) Calculate the percentage error between the mean and the stated length of 2.2 m.

[3]

The weights of the barbells are labelled 20 kg. The coach also weighed each barbell, in kg, and recorded the following:

$$20.3, \quad 19.9, \quad 20.3, \quad 20.4, \quad 20.1$$

- (b) (i) Find the mean of the coach's recorded weights.
 (ii) Calculate the percentage error between the mean and the stated weight of 20 kg.

[3]

a) i) Mean, \bar{x} , of a set of data

$$\bar{x} = \frac{\sum_{i=1}^k f_i x_i}{n} \quad \text{and} \quad n = \sum_{i=1}^k f_i \quad (\text{in formula booklet})$$

$$\bar{x} = \frac{2.18 + 2.21 + 2.23 + 2.19 + 2.24}{5}$$

$$\bar{x} = 2.21 \text{ m}$$

Alternatively you could input the values into your GDC.

ii) Percentage error formula.

$$E = \left| \frac{V_A - V_E}{V_E} \right| \times 100\% \quad (\text{in formula booklet})$$

$$V_A = 2.21 \quad V_E = 2.2$$

Sub V_A and V_E into formula.

$$E = \left| \frac{2.21 - 2.2}{2.2} \right| \times 100\%$$

$$E = 0.45454\dots$$

$$E = 0.455\% \quad (3sf)$$

Five Olympic barbells labelled, "2.2 m in length", were delivered to an Olympic weightlifting team. The coach measured each barbell to check its length, in metres, and recorded the following:

2.18, 2.21, 2.23, 2.19, 2.24

- (a) (i) Find the mean of the coach's recorded measurements.
(ii) Calculate the percentage error between the mean and the stated length of 2.2 m.

[3]

The weights of the barbells are labelled 20 kg. The coach also weighed each barbell, in kg, and recorded the following:

20.3, 19.9, 20.3, 20.4, 20.1

- (b) (i) Find the mean of the coach's recorded weights.
(ii) Calculate the percentage error between the mean and the stated weight of 20 kg.

[3]

b) i) Mean, \bar{x} , of a set of data

$$\bar{x} = \frac{\sum_{i=1}^k f_i x_i}{n} \quad \text{and} \quad n = \sum_{i=1}^k f_i \quad (\text{in formula booklet})$$

$$\bar{x} = \frac{20.3 + 19.9 + 20.3 + 20.4 + 20.1}{5}$$

$$\bar{x} = 20.2 \text{ kg}$$

Alternatively you could input the values into your GDC.

ii) Percentage error formula.

$$E = \left| \frac{V_A - V_E}{V_E} \right| \times 100\% \quad (\text{in formula booklet})$$

$$V_A = 20.2 \quad V_E = 20$$

Sub V_A and V_E into formula.

$$E = \left| \frac{20.2 - 20}{20} \right| \times 100\%$$

$$E = 1\%$$

Question 9

In a game show, there is a transparent box filled with identical cubes. Contestants must estimate the number of cubes in the box. The box is 60 cm wide, 80 cm long and 20 cm tall.

- (a) Find the volume of the box.

[2]

Monica estimates the volume of one cube is 300 cm³. She uses this value to estimate the number of cubes in the box.

- (b) Find Monica's estimated number of cubes in the box.

[2]

The actual number of cubes in the box is 280.

- (c) Find the percentage error in Monica's estimated number of cubes in the box.

[2]

a) Volume of a cuboid formula.

$$V = Lwh \quad (\text{in formula booklet})$$

where l is the length, w is the width and h is the height.

$$l = 80 \quad w = 60 \quad h = 20$$

Sub l , w and h into formula.

$$V = 80 \times 60 \times 20$$

$$V = 96\,000 \text{ cm}^3$$

In a game show, there is a transparent box filled with identical cubes. Contestants must estimate the number of cubes in the box. The box is 60 cm wide, 80 cm long and 20 cm tall.

(a) Find the volume of the box.

$$V = 96\,000 \text{ cm}^3$$

[2]

Monica estimates the volume of one cube is 300cm^3 . She uses this value to estimate the number of cubes in the box.

(b) Find Monica's estimated number of cubes in the box.

[2]

The actual number of cubes in the box is 280.

(c) Find the percentage error in Monica's estimated number of cubes in the box.

[2]

$$b) \quad N = \frac{\text{box volume}}{\text{cube volume}}$$

$$N = \frac{96\,000}{300}$$

$$N = 320 \text{ cubes}$$

In a game show, there is a transparent box filled with identical cubes. Contestants must estimate the number of cubes in the box. The box is 60 cm wide, 80 cm long and 20 cm tall.

(a) Find the volume of the box.

[2]

Monica estimates the volume of one cube is 300cm^3 . She uses this value to estimate the number of cubes in the box.

(b) Find Monica's estimated number of cubes in the box.

$$N = 320 \text{ cubes}$$

[2]

The actual number of cubes in the box is 280.

(c) Find the percentage error in Monica's estimated number of cubes in the box.

[2]

c) Percentage error formula.

$$E = \left| \frac{V_A - V_E}{V_E} \right| \times 100\% \quad (\text{in formula booklet})$$

where V_E is the exact value and V_A is the approximated value.

$$V_A = 320 \quad V_E = 280$$

Sub V_A and V_E into formula.

$$E = \left| \frac{320 - 280}{280} \right| \times 100\%$$

$$E = 14.2857\%$$

$$E = 14.3\%$$

Question 10

Solve the following systems of linear equations using technology.

(i)

$$\begin{aligned} 5x + 3y - 2z &= -12 \\ 3x - 4y - z &= 17 \\ 10x - 10y + z &= 65 \end{aligned}$$

(ii)

$$\begin{aligned} 4x - 5y + z &= 50 \\ 3x + y + 3z &= -16 \\ 6x - 2z &= 61 + y \end{aligned}$$

(i) Using your GDC

$$\begin{aligned} x &= 2 \\ y &= -4 \\ z &= 5 \end{aligned}$$

[6]

(ii) Using your GDC

$$\begin{aligned} x &= 6 \\ y &= -7 \\ z &= -9 \end{aligned}$$

Question 11

Solve the following systems of linear equations using technology.

(i)

$$\begin{aligned} 2x - 5y - 7z &= -21 \\ 3z + x - 4y &= 44 \\ x + z - y &= 12 \end{aligned}$$

(ii)

$$\begin{aligned} z - x - y &= -11 \\ 5x + 11z - 2y &= -28 \\ 3y - 4z + x &= 30 \end{aligned}$$

(i) Using your GDC

$$\begin{aligned} x &= -1 \\ y &= -6 \\ z &= 7 \end{aligned}$$

[6]

(ii) Using your GDC

$$\begin{aligned} x &= 3 \\ y &= 5 \\ z &= -3 \end{aligned}$$