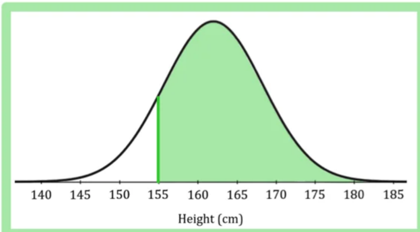
## Question 1

The random variable, $X$, is seen on the following diagram which shows the distribution of heights, in cm, of adult women in the UK:




The distribution of heights follows a normal distribution, with a mean of 162 cm and a standard deviation of 6.3 cm.

$X \sim N(162, 6.3^2)$

(a) On the diagram above, shade in the region representing $P(X > 155)$.

[2]

(b) (i) Find the probability that a randomly selected woman has a height of more than 155cm.

(ii) Use your answer from part (b)(i) to find the probability that a randomly selected woman has a height of more than 169cm.

[4]

(c) Suggest a range of heights within which the height of approximately

(i) 68%

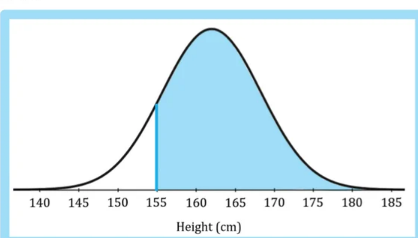
(ii) 95%

(iii) 99.7%

of adult women in the UK will fall.

[3]

The random variable, $X$, is seen on the following diagram which shows the distribution of heights, in cm, of adult women in the UK:




The distribution of heights follows a normal distribution, with a mean of 162 cm and a standard deviation of 6.3 cm.

$\sigma = 6.3$ $\mu = 162$

(a) On the diagram above, shade in the region representing $P(X > 155)$.

[2]

(b) (i) Find the probability that a randomly selected woman has a height of more than 155cm.

(ii) Use your answer from part (b)(i) to find the probability that a randomly selected woman has a height of more than 169cm.

[4]

(c) Suggest a range of heights within which the height of approximately

(i) 68%

(ii) 95%

(iii) 99.7%

of adult women in the UK will fall.

[3]

(b)(i) Using the normal distribution on your calculator

Lower = 155

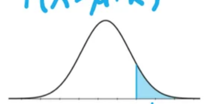
Upper = 999...

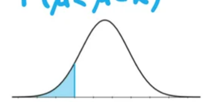
$P(X > 155) = 0.866739...$

$\boxed{P(X > 155) = 0.8667 \ (4dp)}$

(ii) The normal distribution is symmetrical about its mean.

$P(X > \mu + k) = P(\mu < \mu - k)$




$P(X > 169) = P(X < 155)$

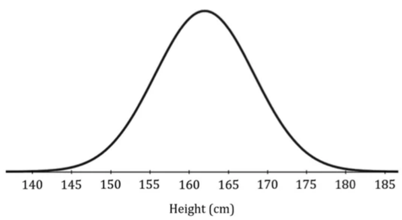
$= 1 - P(X > 155)$

$= 1 - 0.866739...$

$= 0.133260...$

$\boxed{P(X > 169) = 0.1333 \ (4dp)}$

The random variable, $X$, is seen on the following diagram which shows the distribution of heights, in cm, of adult women in the UK:




The distribution of heights follows a normal distribution, with a mean of 162 cm and a standard deviation of 6.3 cm.

(a) On the diagram above, shade in the region representing $P(X > 155)$.

[2]

(b) (i) Find the probability that a randomly selected woman has a height of more than 155cm.

(ii) Use your answer from part (b)(i) to find the probability that a randomly selected woman has a height of more than 169cm.

[4]

(c) Suggest a range of heights within which the height of approximately

(i) 68%

(ii) 95%

(iii) 99.7%

of adult women in the UK will fall.

[3]

(c)(i) 68% of data lies between $\mu \pm \sigma$

$\mu + \sigma = 162 + 6.3 = 168.3$

$\mu - \sigma = 162 - 6.3 = 155.7$

$\boxed{\text{68\% of heights lie in range 155.7cm to 168.3cm}}$

95 % of data lies between $\mu \pm 2\sigma$

$\mu + 2\sigma = 162 + 2(6.3) = 174.6$

$\mu - 2\sigma = 162 - 2(6.3) = 149.4$

$\boxed{\text{95\% of heights lie in range 149.4 cm to 174.6cm}}$

99.7 % of data lies between $\mu \pm 3\sigma$

$\mu + 3\sigma = 162 + 3(6.3) = 180.9$

$\mu - 3\sigma = 162 - 3(6.3) = 143.1$

$\boxed{\text{99.7\% of heights lie in range 143.1 cm to 180.9cm}}$

## Question 2

(a) For the random variable $X \sim N(23, 4^2)$ find the following probabilities:

(i) $P(X < 20)$

(ii) $P(X \geq 29)$

(iii) $P(20 \leq X < 29)$

[3]

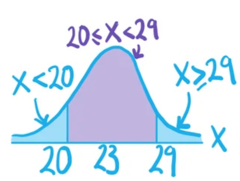
(b) For the random variable $Y \sim N(100, 225)$ find the following probabilities:

(i) $P(Y \leq 90)$

(ii) $P(Y > 140)$

(iii) $P(85 \leq Y \leq 115)$

[3]




$X \sim N(23, 4^2)$

$\mu = 23$

$\sigma^2 = 4^2$

$\sigma = 4$

(a) For a normal distribution $P(X<k) = P(X \leq k)$

For probabilities use 4dp or 3sf (whichever is more accurate)

(i) Lower $= -999...$

Upper $= 20$

$P(X<20) = 0.226627...$

$\boxed{P(X<20) = 0.2266 \text{ (4dp)}}$

(ii) Lower $= 29$

Upper $= 999...$

$P(X \geq 29) = 0.066807...$

$\boxed{P(X \geq 29) = 0.0668 \text{ (4dp)}}$

(iii) Lower $= 20$

Upper $= 29$

$P(20 \leq X < 29) = 0.706565...$

$\boxed{P(20 \leq X < 29) = 0.7066 \text{ (4dp)}}$

(a) For the random variable $X \sim \mathrm{N}(23,\ 4^2)$ find the following probabilities:

(i) $\mathrm{P}(X < 20)$

(ii) $\mathrm{P}(X \geq 29)$

(iii) $\mathrm{P}(20 \leq X < 29)$

[3]

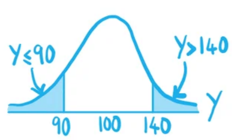
(b) For the random variable $Y \sim \mathrm{N}(100,\ 225)$ find the following probabilities:

(i) $\mathrm{P}(Y \leq 90)$

(ii) $\mathrm{P}(Y > 140)$

(iii) $\mathrm{P}(85 \leq Y \leq 115)$

[3]




$Y \sim N(100, 225)$
$\mu = 100$
$\sigma^2 = 225$
$\sigma = 15$

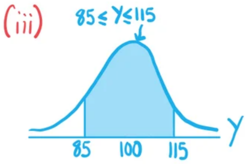
(b) (i) Lower = −999...
Upper = 90 $\quad P(Y \leq 90) = 0.252492...$

$\boxed{P(Y \leq 90) = 0.2525 \quad (4dp)}$

(ii) Lower = 140
Upper = 999... $\quad P(Y > 140) = 0.003830 \ ...$

$\boxed{P(Y > 140) = 0.00383 \ (3sf)}$

(iii) $85 \leq Y \leq 115$




Lower = 85
Upper = 115

$P(85 \leq Y \leq 115) = 0.682689...$

$\boxed{P(85 \leq Y \leq 115) = 0.6827 \ (4dp)}$

## Question 3

The weight, $W$ g, of a chocolate bar produced by a certain manufacturer is modelled as $W \sim \mathrm{N}(200, 1.75^2)$.

(a) Find:

(i) $\mathrm{P}(W < 195)$

(ii) $\mathrm{P}(W > 203)$

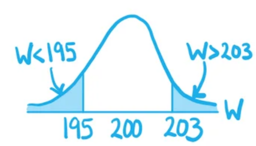
[2]

Heledd buys a pack containing 12 of the chocolate bars. It may be assumed that the 12 bars in the pack represent a random sample.

(b) Find the probability that all of the bars in the pack have a weight of at least 195 g.

[2]

(a) $W \sim N(200, 1.75^2)$
$\mu = 200$
$\sigma^2 = 1.75^2$
$\sigma = 1.75$




(i) Lower = −999...
Upper = 195

$P(W < 195) = 0.002137...$

$\boxed{P(W < 195) = 0.00214 \quad (3sf)}$

(ii) Lower = 203
Upper = 999...

$P(W > 203) = 0.043238...$

$\boxed{P(W > 203) = 0.0432 \quad (4dp)}$

The weight, $W$ g, of a chocolate bar produced by a certain manufacturer is modelled as $W \sim \mathrm{N}(200, 1.75^2)$.

(a) Find:

(i) $\mathrm{P}(W < 195)$

(ii) $\mathrm{P}(W > 203)$

[2]

Heledd buys a pack containing 12 of the chocolate bars. It may be assumed that the 12 bars in the pack represent a random sample.

(b) Find the probability that all of the bars in the pack have a weight of at least 195 g.

[2]

(b) Let $X$ be the number of chocolate bars in the sample that have a weight of at least 195g then $X \sim B(12, p)$

$p = P(W \geq 195)$
$= 1 - P(W < 195)$
$= 1 - 0.002137...$
$= 0.997863...$

Use full answer to avoid rounding errors.

$\therefore X \sim B(12, 0.997863)$

If all of the sample exceed 195g then $X = 12$

$P(X = 12) = (0.997863)^{12}$
$= 0.974655...$

$\boxed{P(X=12) = 0.9747 \ (4dp)}$

## Question 4

The random variable $X \sim \mathrm{N}(330,\ 10^2)$.

(a) Find the value of $a$, to 2 decimal places, such that:

(i) $\mathrm{P}(X < a) = 0.25$

(ii) $\mathrm{P}(X > a) = 0.25$

(iii) $\mathrm{P}(315 \leq X \leq a) = 0.5$

[4]

The random variable $Y \sim \mathrm{N}(10,\ 10)$.

(b) Find the value of $b$ and the value of $c$, each to 2 decimal places, such that:

(i) $\mathrm{P}(Y < b) = 0.4$

(ii) $\mathrm{P}(Y > c) = 0.25$

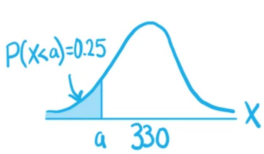
[2]

(c) Use a sketch of the distribution of $Y$ to explain why $\mathrm{P}(b \leq Y \leq c) = 0.35$.

[2]

(a) $X \sim N(330, 10^2)$ $\mu = 330$ $\sigma^2 = 10^2$ $\sigma = 10$

(i)

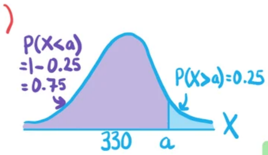



Inverse Normal
Area = 0.25

$a = 323.255...$

$\boxed{a = 323.26 \ (2dp)}$

(ii)

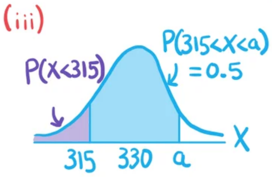



Inverse Normal
Area = 0.75 ← Area uses P(X<a)

$a = 336.744...$

$\boxed{a = 336.74 \ (2dp)}$

(iii)




$P(X < 315) = 0.066807...$

$P(X < a) = P(X < 315) + P(315 < X < a)$

$P(X < a) = 0.066807... + 0.5$
$= 0.566807...$

Inverse Normal
Area = 0.566807

$a = 331.682...$

$\boxed{a = 331.68 \ (2dp)}$

The random variable $X \sim \mathrm{N}(330,\ 10^2)$.

(a) Find the value of $a$, to 2 decimal places, such that:

(i) $\mathrm{P}(X < a) = 0.25$

(ii) $\mathrm{P}(X > a) = 0.25$

(iii) $\mathrm{P}(315 \le X \le a) = 0.5$

[4]

The random variable $Y \sim \mathrm{N}(10,\ 10)$.

(b) Find the value of $b$ and the value of $c$, each to 2 decimal places, such that:

(i) $\mathrm{P}(Y < b) = 0.4$

(ii) $\mathrm{P}(Y > c) = 0.25$

[2]

(c) Use a sketch of the distribution of $Y$ to explain why $\mathrm{P}(b \le Y \le c) = 0.35$.

[2]

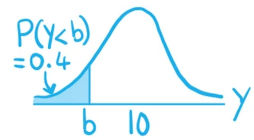
(b) $Y \sim N(10,10)$

$\mu = 10$

$\sigma^2 = 10$

$\sigma = \sqrt{10}$

(i)

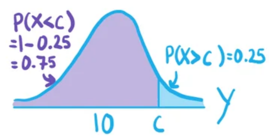



Inverse Normal
Area = 0.4

$b = 9.19884\ldots$

$\boxed{b = 9.20 \quad (2dp)}$

(ii)




Inverse Normal
Area = 0.75

$c = 12.1329\ldots$

$\boxed{c = 12.13 \ (2dp)}$

The random variable $X \sim \mathrm{N}(330,\ 10^2)$.

(a) Find the value of $a$, to 2 decimal places, such that:

(i) $\mathrm{P}(X < a) = 0.25$

(ii) $\mathrm{P}(X > a) = 0.25$

(iii) $\mathrm{P}(315 \le X \le a) = 0.5$

[4]

The random variable $Y \sim \mathrm{N}(10,\ 10)$.

(b) Find the value of $b$ and the value of $c$, each to 2 decimal places, such that:

(i) $\mathrm{P}(Y < b) = 0.4$

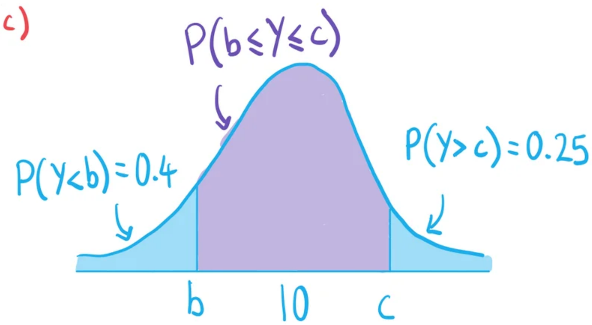
(ii) $\mathrm{P}(Y > c) = 0.25$

[2]

(c) Use a sketch of the distribution of $Y$ to explain why $\mathrm{P}(b \le Y \le c) = 0.35$.

[2]

(c)




The total area is 1

$$\boxed{\begin{aligned} &P(Y<b) + P(b \le Y \le c) + P(Y>c) = 1 \\ &P(b \le Y \le c) = 1 - P(Y<b) - P(Y>c) \\ &\qquad = 1 - 0.4 - 0.25 \\ &\qquad = 0.35 \end{aligned}}$$

## Question 5

The test scores, $X$, of a group of RAF recruits in an aptitude test are modelled as a normal distribution with $X \sim \mathrm{N}(210, 27.8^2)$.

(a) (i) Find the values of $a$ and $b$ such that $\mathrm{P}(X < a) = 0.25$ and $\mathrm{P}(X > b) = 0.25$.

(ii) Hence find the interquartile range of the scores.

[3]

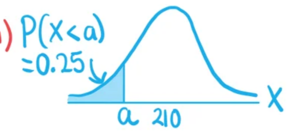
Those who score in the top 30% on the test move on to the next stage of training.

(b) One of the recruits, Amelia, achieves a score of 231. Determine whether Amelia will move on to the next stage of training.

[2]

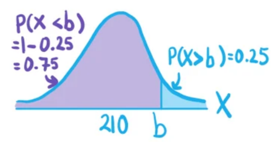
(a) $X \sim N(210\ 27.8^2)$

$\mu = 210$
$\sigma^2 = 27.8^2$
$\sigma = 27.8$

(i) 


Inverse Normal
Area = 0.25
a = 191.249...

$\boxed{a = 191 \ (3sf)}$




Inverse Normal
Area = 0.75
a = 228.750...

$\boxed{a = 229 \ (3sf)}$

(ii) $P(X < LQ) = 0.25$ and $P(X > UQ) = 0.25$

$IQR = 228.750... - 191.249...$
$= 37.501...$

$\boxed{IQR = 37.5 \ (3sf)}$

The test scores, $X$, of a group of RAF recruits in an aptitude test are modelled as a normal distribution with $X \sim \mathrm{N}(210, 27.8^2)$.

(a) (i) Find the values of $a$ and $b$ such that $\mathrm{P}(X < a) = 0.25$ and $\mathrm{P}(X > b) = 0.25$.

(ii) Hence find the interquartile range of the scores.

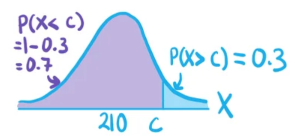
[3]

Those who score in the top 30% on the test move on to the next stage of training.

(b) One of the recruits, Amelia, achieves a score of 231. Determine whether Amelia will move on to the next stage of training.

[2]

(b) In the top 30% means $X \geq c$ where $P(X \geq c) = 0.3$




Inverse Normal
Area = 0.7

$c = 224.578...$

$231 > 224.578...$

$\boxed{\text{Amelia is in the top 30\% and will move on to the next stage of training.}}$

## Question 6

A machine is used to fill cans of a particular brand of soft drink. The volume, $V$ ml, of soft drink in the cans is normally distributed with mean 330 ml and standard deviation $\sigma$ ml.

It is known that approximately 16% of the cans contain more than 333.28 ml of soft drink.

(a) Using the properties of the normal distribution, explain why 3.28 ml would provide a good approximation for the value of $\sigma$.

[2]

(b) Using $\sigma = 3.28$ ml, find $P(320 \le V \le 340)$.

[1]

Six cans of the soft drink are chosen at random.

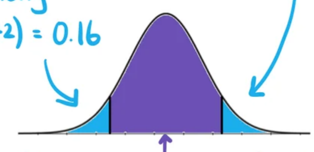
(c) Again using $\sigma = 3.28$ ml, find the probability that all of the cans contain less than 329 ml of soft drink.

[3]

(a) $V \sim N(330, \sigma^2)$

By symmetry
$P(V < 326.72) = 0.16$

$P(V > 333.28) = 0.16$



$P(326.72 < V < 333.28) = 0.68$

68% of data lies between $\mu \pm \sigma$

$\mu + \sigma = 333.28$

$330 + \sigma = 333.28$

$\boxed{\sigma = 3.28 \text{ as } \mu + \sigma = 333.28}$

A machine is used to fill cans of a particular brand of soft drink. The volume, $V$ ml, of soft drink in the cans is normally distributed with mean 330 ml and standard deviation $\sigma$ ml.

$\mu = 330$

It is known that approximately 16% of the cans contain more than 333.28 ml of soft drink.

(a) Using the properties of the normal distribution, explain why 3.28 ml would provide a good approximation for the value of $\sigma$.

[2]

(b) Using $\sigma = 3.28$ ml, find $P(320 \le V \le 340)$.

[1]

Six cans of the soft drink are chosen at random.

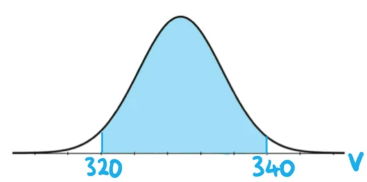
(c) Again using $\sigma = 3.28$ ml, find the probability that all of the cans contain less than 329 ml of soft drink.

[3]

(b) Using the normal distribution on your calculator.

Lower = 320

Upper = 340




$P(320 \le V \le 340) = 0.99770227...$

$\boxed{P(320 \le V \le 340) = 0.9977 \text{ (4dp)}}$

A machine is used to fill cans of a particular brand of soft drink. The volume, $V$ ml, of soft drink in the cans is normally distributed with mean 330 ml and standard deviation $\sigma$ ml.

It is known that approximately 16% of the cans contain more than 333.28 ml of soft drink.

(a) Using the properties of the normal distribution, explain why 3.28 ml would provide a good approximation for the value of $\sigma$.

[2]

(b) Using $\sigma = 3.28$ ml, find $\mathrm{P}(320 \leq V \leq 340)$.

[1]

Six cans of the soft drink are chosen at random.

(c) Again using $\sigma = 3.28$ ml, find the probability that all of the cans contain less than 329 ml of soft drink.

[3]

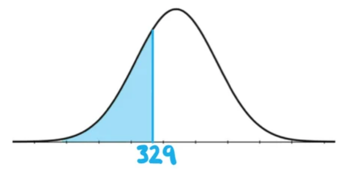
(c) Let $X$ be the number of the 6 cans that contain less than 329ml.

$X \sim B(6, p)$

$p = P(V < 329)$

Lower $= -999\ldots$

Upper $= 329$




$p = 0.38022951\ldots$

$P(X=6) = p^6$

$P(X=6) = (0.38022951\ldots)^6$

$= 0.00302186\ldots$

$\boxed{P(X=6) = 0.00302 \ (3sf)}$