Question 1

Two ships A and B are travelling so that their position relative to a fixed point O at time t, in hours, can be defined by the position vectors $\boldsymbol{r_A} = (2 - t)\boldsymbol{i} + (4 + 3t)\boldsymbol{j}$ and $\boldsymbol{r_B} = (t - 8)\boldsymbol{i} + (29 - 2t)\boldsymbol{j}$.

The unit vectors i and j are a displacement of 1 km due East and North of O respectively.

(a) Find the coordinates of the initial position of the two ships.

[2]

(b) Show that the two ships will collide and find the time at which this will occur.

[3]

(c) Find the coordinates of the point of collision.

[2]

a) The initial position is when $t = 0$.

$r_A = (2 - (0))i + (4 + 3(0))j = 2i + 4j$

$r_B = ((0) - 8)i + (29 - 2(0))j = -8i + 29j$

$\boxed{A(2,4) \text{ and } B(-8, 29)}$

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(b) Show that the two ships will collide and find the time at which this will occur.

[3]

(c) Find the coordinates of the point of collision.

[2]

b) $r_A = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + t\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ $r_B = \begin{pmatrix} -8 \\ 29 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$x = 2 - t = -8 + t \longrightarrow 10 = 2t$

$y = 4 + 3t = 29 - 2t \longrightarrow 25 = 5t$

$\boxed{\therefore \text{ the ships collide when } t = 5 \text{ hours}}$

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[2]

(b) Show that the two ships will collide and find the time at which this will occur.

$\therefore$ the ships collide when $t = 5$ hours

[3]

(c) Find the coordinates of the point of collision.

[2]

c) sub in $t = 5$ into r_A or r_B

$r_A = (2 - (5))i + (4 + 3(5))j = -3i + 19j$

$\therefore$ collision pt is $(-3, 19)$

Question 2

A car, moving at constant speed, takes 4 minutes to drive in a straight line from point A$(-3, 5)$ to point B$(7, 11)$.

At time t, in minutes, the position vector of the car relative to the origin can be given in the form $\boldsymbol{p} = a + tb$.

(a) Find the vectors a and b.

[3]

A cat has decided to take a nap at point X$(4, 9)$.

(b) Show that the cat does not lie on the route along which the car drives.

[3]

(c) Find the shortest distance between the car and the cat during the movement of the car.

[6]

a) vector a represents the initial position and vector b represents the direction vector.

$b = \frac{1}{4}\begin{pmatrix} 7 - (-3) \\ 11 - 5 \end{pmatrix}$

4 minutes to travel between A and B.

$\therefore a = \begin{pmatrix} -3 \\ 5 \end{pmatrix}, \quad b = \begin{pmatrix} \frac{5}{2} \\ \frac{3}{2} \end{pmatrix}$

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[6]

b) show that t is inconsistent at X.

$\begin{pmatrix} 4 \\ 9 \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \end{pmatrix} + t\begin{pmatrix} \frac{5}{2} \\ \frac{3}{2} \end{pmatrix}$

$4 = -3 + \frac{5}{2}t \qquad \therefore t = \frac{14}{5}$

$9 = 5 + \frac{3}{2}t \qquad \therefore t = \frac{8}{3}$

t is inconsistent, so $(4, 9)$ does not lie on the car's route.

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[6]

c) shortest distance is the perpendicular distance from X to the line

direction vector $= \begin{pmatrix} \frac{5}{2} \\ \frac{3}{2} \end{pmatrix}$

$\therefore$ perpendicular vector $= \begin{pmatrix} -\frac{3}{2} \\ \frac{5}{2} \end{pmatrix}$

$\therefore r = \begin{pmatrix} 4 \\ 9 \end{pmatrix} + \lambda \begin{pmatrix} -\frac{3}{2} \\ \frac{5}{2} \end{pmatrix}$ (equation from cat to route)

write both equations in parametric form and rearrange x or y for t or λ

$$\begin{cases} x = -3 + \frac{5}{2}t = 4 - \frac{3}{2}\lambda \longrightarrow t = \frac{14}{5} - \frac{3}{5}\lambda \\ y = 5 + \frac{3}{2}t = 9 + \frac{5}{2}\lambda \end{cases}$$

sub $t = \frac{14}{5} - \frac{3}{5}\lambda$ into y.

$$5 + \frac{3}{2}\left(\frac{14}{5} - \frac{3}{5}\lambda\right) = 9 + \frac{5}{2}\lambda \longrightarrow \lambda = \frac{1}{17}$$

$$x = 4 - \frac{3}{2}\left(\frac{1}{17}\right) = \frac{133}{34} \qquad y = 9 + \frac{5}{2}\left(\frac{1}{17}\right) = \frac{311}{34}$$

$$\text{distance} = \sqrt{\left(\frac{133}{34} - 4\right)^2 + \left(\frac{311}{34} - 9\right)^2} = \frac{\sqrt{34}}{34}$$

$\boxed{\text{distance} = 0.171 \text{ units (3 s.f.)}}$

Question 3

A bird takes off from a perch and flies at a constant speed in a straight line. The position of the bird in flight relative to its nest, (east, north and above/below the nest), can be described by the vector equation

$$r_1 = \begin{pmatrix} 18 \\ 4 \\ -2 \end{pmatrix} + t\begin{pmatrix} 272 \\ -360 \\ 225 \end{pmatrix}.$$

All displacements are given in metres and t is the time in minutes.

(a) Find the distance between the perch that the bird took flight from and its nest.

[2]

(b) Find the speed at which the bird is travelling. Give your answer in kmh^{-1}.

[3]

A second bird takes off at the same time as the first bird from a different perch and also flies in a straight line at a constant speed. The flight of the second bird, relative to the same nest, can be described by the vector equation

$$r_2 = \begin{pmatrix} 12 \\ -8 \\ -3 \end{pmatrix} + t\begin{pmatrix} -187 \\ -438 \\ 80 \end{pmatrix}.$$

(c) Find the distance between the two birds after 8 minutes of flying.

[4]

a)

Distance between two points (x_1, y_1, z_1) & (x_2, y_2, z_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

Distance $= \sqrt{18^2 + 4^2 + (-2)^2} = 2\sqrt{86} = 18.5472\ldots$

$\boxed{\text{Distance} = 18.5 \text{ m}}$

A bird takes off from a perch and flies at a constant speed in a straight line. The position of the bird in flight relative to its nest, (east, north and above/below the nest), can be described by the vector equation

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(c) Find the distance between the two birds after 8 minutes of flying.

[4]

b) The bird's speed is equal to the magnitude of the direction vector.

$$\text{speed} = \left|\begin{pmatrix} 272 \\ -360 \\ 225 \end{pmatrix}\right| = \sqrt{272^2 + (-360)^2 + 225^2} = 504.1914\ldots \text{m min}^{-1}$$

$$504.1914\ldots \text{m min}^{-1} \times \frac{\overset{60 \text{ mins in an hour}}{60}}{\underset{1000\text{m in a km}}{1000}} = 30.2514\ldots \text{kmh}^{-1}$$

$$\boxed{\text{speed} = 30.3 \text{ kmh}^{-1}}$$

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$$r_1 = \begin{pmatrix} 18 \\ 4 \\ -2 \end{pmatrix} + t\begin{pmatrix} 272 \\ -360 \\ 225 \end{pmatrix}.$$

All displacements are given in metres and t is the time in minutes.

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(b) Find the speed at which the bird is travelling. Give your answer in kmh^{-1}.

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A second bird takes off at the same time as the first bird from a different perch and also flies in a straight line at a constant speed. The flight of the second bird, relative to the same nest, can be described by the vector equation

$$r_2 = \begin{pmatrix} 12 \\ -8 \\ -3 \end{pmatrix} + t\begin{pmatrix} -187 \\ -438 \\ 80 \end{pmatrix}.$$

(c) Find the distance between the two birds after 8 minutes of flying.

[4]

c) Find the positions of each bird after 8 mins $(t=8)$.

$$r_1 = \begin{pmatrix} 18 \\ 4 \\ -2 \end{pmatrix} + 8\begin{pmatrix} 272 \\ -360 \\ 225 \end{pmatrix} = \begin{pmatrix} 2194 \\ -2876 \\ 1798 \end{pmatrix}$$

$$r_2 = \begin{pmatrix} 12 \\ -8 \\ -3 \end{pmatrix} + 8\begin{pmatrix} -187 \\ -438 \\ 80 \end{pmatrix} = \begin{pmatrix} -1484 \\ -3512 \\ 637 \end{pmatrix}$$

Distance between two points (x_1, y_1, z_1) & (x_2, y_2, z_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

$$\text{Distance} = \sqrt{(2194 + 1484)^2 + (-2876 + 3512)^2 + (1798 - 637)^2}$$

$$\boxed{\text{Distance} = 3908.9769\ldots = 3910 \text{ m}}$$

Question 4

A drone travels in a straight line and at a constant speed. It moves from an initial point $A(4, 5, -2)$ to a second point $B(7, -1, 0)$. The person controlling the drone is located at $C(2, 3, 2)$.

The x, y and z directions are due east, due north and vertically upwards respectively with all distances in metres.

(a) Write down an equation for the line along which the drone travels.

[2]

At some point P on its flight, the drone is vertically level with the person controlling the drone.

(b) Find the coordinates of point P.

[3]

(c) Find the distance between P and the person controlling the drone.

[2]

a) $r = \begin{pmatrix} 4 \\ 5 \\ -2 \end{pmatrix} + t\begin{pmatrix} 7-4 \\ -1-5 \\ 0+2 \end{pmatrix}$

$$\boxed{r = \begin{pmatrix} 4 \\ 5 \\ -2 \end{pmatrix} + t\begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix}}$$

N.B you could use the coordinates of B as the position vector.

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The x, y and z directions are due east, due north and vertically upwards respectively with all distances in metres.

(a) Write down an equation for the line along which the drone travels.

$$\boxed{r = \begin{pmatrix} 4 \\ 5 \\ -2 \end{pmatrix} + t\begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix}}$$

[2]

At some point P on its flight, the drone is vertically level with the person controlling the drone.

(b) Find the coordinates of point P.

[3]

(c) Find the distance between P and the person controlling the drone.

[2]

b) $C(2, 3, 2)$

$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4+3t \\ 5-6t \\ -2+2t \end{pmatrix}$ $\quad \therefore t = 2$

$$\boxed{P(10, -7, 2)}$$

A drone travels in a straight line and at a constant speed. It moves from an initial point A$(4, 5, -2)$ to a second point B$(7, -1, 0)$. The person controlling the drone is located at C$(2, 3, 2)$.

The x, y and z directions are due east, due north and vertically upwards respectively with all distances in metres.

(a) Write down an equation for the line along which the drone travels.

[2]

At some point P on its flight, the drone is vertically level with the person controlling the drone.

(b) Find the coordinates of point P.

$\boxed{P(10, -7, 2)}$

[3]

(c) Find the distance between P and the person controlling the drone.

[2]

c) Distance between two points (x_1, y_1, z_1) & (x_2, y_2, z_2) | $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

$P(10, -7, 2)$ $\quad$ $C(2, 3, 2)$

$d = \sqrt{(10-2)^2 + (-7-3)^2 + (2-2)^2} = 2\sqrt{41}$

$\boxed{d = 12.8 \text{ m } (3\text{s.f.})}$

Question 5

Two snails are taking part in a snail race starting from the same point and moving in a straight line. The position of the first snail S_1 is given by the equation

$$r = \begin{pmatrix} 5 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \end{pmatrix},$$

The displacement of the second snail S_2, relative to the finish point, is given by

$$s(t) = 8 - 3t^2.$$

All distances are in centimetres and time is in minutes.

(a) Write down the distance that the snails race.

[1]

(b) Find an expression for the velocity of S_2 at time t.

[2]

(c) Find the displacement of S_2 from the finishing point when the speed of the two snails is equal.

[5]

a) $s(0) = 8$

$\boxed{8 \text{ cm}}$

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[1]

(b) Find an expression for the velocity of S_2 at time t.

[2]

(c) Find the displacement of S_2 from the finishing point when the speed of the two snails is equal.

[5]

b) $s'(t) = v(t)$

$v(t) = (2) \times (-3t) = -6t$

$\boxed{v(t) = -6t}$

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All distances are in centimetres and time is in minutes.

(a) Write down the distance that the snails race.

[1]

(b) Find an expression for the velocity of S_2 at time t.

$\boxed{v(t) = -6t}$

[2]

(c) Find the displacement of S_2 from the finishing point when the speed of the two snails is equal.

[5]

c) Velocity of S_1 is equal to the magnitude of the direction vector.

$S_1 : |v| = \sqrt{1^2 + (-2)^2} = \sqrt{5}$ cm min^{-1}

$|-6t| = \sqrt{5}$ $\quad \therefore t = \frac{\sqrt{5}}{6} = 0.3726...$

$s\left(\frac{\sqrt{5}}{6}\right) = 8 - 3\left(\frac{\sqrt{5}}{6}\right)^2 = \frac{91}{12} = 7.5833...$

$\boxed{s\left(\frac{\sqrt{5}}{6}\right) = 7.58 \ (3s.f)}$

Question 6

A ball is pushed off the top of a 150 m tall skyscraper with an initial velocity of $\begin{pmatrix}1.5\\0\end{pmatrix}$ ms^{-1}.

The point at which the ball is pushed can be considered the origin of a Cartesian coordinate system. It is assumed that any effects of air resistance will be negligible and $g = 9.81\ ms^{-2}$.

(a) Find the velocity vector at time t.

[2]

(b) displacement vector of the ball at time t.

[2]

(c) Find the time at which the ball reaches the ground.

[2]

(d) Find the total horizontal distance travelled by the ball.

[2]

a) g is the force of gravity, which is an accelerative force.

$g = 9.81\ ms^{-2}$ means your downwards velocity increases by $9.81\ ms^{-1}$ every second.

$$\boxed{v = \begin{pmatrix}1.5\\-9.81t\end{pmatrix}}$$

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$$\boxed{v = \begin{pmatrix}1.5\\-9.81t\end{pmatrix}}$$

[2]

(b) displacement vector of the ball at time t.

[2]

(c) Find the time at which the ball reaches the ground.

[2]

(d) Find the total horizontal distance travelled by the ball.

[2]

b) $\int v(t) = s(t) \longrightarrow$ integrate

$$s = \begin{pmatrix}1.5t\\ \frac{-9.81}{2}t^2\end{pmatrix}$$

$$\boxed{s = \begin{pmatrix}1.5t\\-4.905t^2\end{pmatrix}}$$

A ball is pushed off the top of a 150 m tall skyscraper with an initial velocity of $\begin{pmatrix}1.5\\0\end{pmatrix}$ ms^{-1}.

The point at which the ball is pushed can be considered the origin of a Cartesian coordinate system. It is assumed that any effects of air resistance will be negligible and $g = 9.81\ ms^{-2}$.

(a) Find the velocity vector at time t.

[2]

(b) displacement vector of the ball at time t.

$$\boxed{s = \begin{pmatrix}1.5t\\-4.905t^2\end{pmatrix}}$$

[2]

(c) Find the time at which the ball reaches the ground.

[2]

(d) Find the total horizontal distance travelled by the ball.

[2]

c) $150 - 4.905t^2 = 0$ $\quad t = 5.5300...$

$$\boxed{t = 5.53\text{ s } (3\text{s.f.})}$$

A ball is pushed off the top of a 150 m tall skyscraper with an initial velocity of $\begin{pmatrix}1.5\\0\end{pmatrix}$ ms^{-1}.

The point at which the ball is pushed can be considered the origin of a Cartesian coordinate system. It is assumed that any effects of air resistance will be negligible and $g = 9.81\ ms^{-2}$.

(a) Find the velocity vector at time t.

[2]

(b) displacement vector of the ball at time t.

[2]

(c) Find the time at which the ball reaches the ground.

$$\boxed{t = 5.53\text{ s } (3\text{s.f.})}$$

[2]

(d) Find the total horizontal distance travelled by the ball.

[2]

d) $1.5(5.5300) = 8.2950...$

$$\boxed{8.30\text{ m}}$$

Question 7

Two aeroplanes are observed flying in straight lines, with respect to the airport located at $(0, 0, 0)$. The flightpaths l_A and l_B, of aeroplanes A and B respectively, can be defined by:

$$l_A: \mathbf{r} = \begin{pmatrix} 6 \\ 3 \\ -2 \end{pmatrix} + \alpha \begin{pmatrix} -1 \\ -4 \\ 3 \end{pmatrix}$$

$$l_B: \mathbf{s} = \begin{pmatrix} -7 \\ -1 \\ 5 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

where α and β is the time elapsed in minutes since the start of the observation for each aeroplane. All distances are in kilometres.

The flightpaths intersect at point P.

(a) Find the values of α and β and hence show that the two planes will not collide.

[4]

a) $r: \ x = 6 - \alpha \qquad s: \ x = -7 + \beta$

$y = 3 - 4\alpha \qquad y = -1 - 2\beta$

$z = -2 + 3\alpha \qquad z = 5 + \beta$

solve simultaneously $\rightarrow$ $6 - \alpha = -7 + \beta$

$3 - 4\alpha = -1 - 2\beta$

$-2 + 3\alpha = 5 + \beta$

$\boxed{\alpha = 5 \text{ mins}, \ \beta = 8 \text{ mins so they won't collide.}}$

(b) Find

(i) the coordinates of the point at which the flightpaths intersect,

(ii) the distance between the airport and the point P.

[4]

Two aeroplanes are observed flying in straight lines, with respect to the airport located at $(0, 0, 0)$. The flightpaths l_A and l_B, of aeroplanes A and B respectively, can be defined by:

$$l_A: \mathbf{r} = \begin{pmatrix} 6 \\ 3 \\ -2 \end{pmatrix} + \alpha \begin{pmatrix} -1 \\ -4 \\ 3 \end{pmatrix}$$

$$l_B: \mathbf{s} = \begin{pmatrix} -7 \\ -1 \\ 5 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

where α and β is the time elapsed in minutes since the start of the observation for each aeroplane. All distances are in kilometres.

The flightpaths intersect at point P.

(a) Find the values of α and β and hence show that the two planes will not collide.

$\boxed{\alpha = 5 \text{ mins}, \ \beta = 8 \text{ mins, so they won't collide.}}$

[4]

(b) Find

(i) the coordinates of the point at which the flightpaths intersect,

(ii) the distance between the airport and the point P.

[4]

b) sub $\alpha = 5$ mins or $\beta = 8$ mins into their respective equations

$x = 6 - (5) = 1$

$y = 3 - 4(5) = -17$ i) $\boxed{(1, -17, 13)}$

$z = -2 + 3(5) = 13$

ii) $\boxed{\text{dist} = \sqrt{(1)^2 + (-17)^2 + (13)^2} = 3\sqrt{51} = 21.4 \text{ km (3 s.f.)}}$

EXAM PAPERS PRACTICE

Question 8

A particle starts from a position at $(0, 0)$ and moves such that its velocity at time t, in seconds, is given by $v = \begin{pmatrix} 2e^{3t} \\ e^{3t} - 4 \end{pmatrix}$. All distances are in metres.

(a) Find an expression for the acceleration of the particle at time t.

[2]

(b) Find an expression for the position of the particle at time t.

[4]

(c) Find the value of t such that the speed of the particle is 6 ms^{-1}.

[3]

a) $v'(t) = a(t) \longrightarrow$ differentiate

$a = \begin{pmatrix} 3 \times 2e^{3t} \\ 3 \times e^{3t} \end{pmatrix}$

$\boxed{a = \begin{pmatrix} 6e^{3t} \\ 3e^{3t} \end{pmatrix}}$

A particle starts from a position at $(0, 0)$ and moves such that its velocity at time t, in seconds, is given by $v = \begin{pmatrix} 2e^{3t} \\ e^{3t} - 4 \end{pmatrix}$. All distances are in metres.

(a) Find an expression for the acceleration of the particle at time t.

[2]

(b) Find an expression for the position of the particle at time t.

[4]

(c) Find the value of t such that the speed of the particle is 6 ms^{-1}.

[3]

b) $\int v(t) = s(t) \longrightarrow$ integrate

$s = \begin{pmatrix} \frac{2}{3}e^{3t} + c_1 \\ \frac{1}{3}e^{3t} - 4t + c_2 \end{pmatrix}$

at $t = 0$, $s = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\therefore c_1 = -\frac{2}{3}$, $c_2 = -\frac{1}{3}$

$\boxed{s = \begin{pmatrix} \frac{2}{3}e^{3t} - \frac{2}{3} \\ \frac{1}{3}e^{3t} - \frac{1}{3} \end{pmatrix}}$

A particle starts from a position at $(0, 0)$ and moves such that its velocity at time t, in seconds, is given by $v = \begin{pmatrix} 2e^{3t} \\ e^{3t} - 4 \end{pmatrix}$. All distances are in metres.

(a) Find an expression for the acceleration of the particle at time t.

[2]

(b) Find an expression for the position of the particle at time t.

[4]

(c) Find the value of t such that the speed of the particle is 6 ms^{-1}.

[3]

c) $\sqrt{(2e^{3t})^2 + (e^{3t} - 4)^2} = 6$ solve with GDC

$\boxed{t = 0.3610\ldots = 0.361\text{s } (3sf)}$