Question 1

The total cost, C, in New Zealand dollars (NZD), of a premium gym membership at Cityfitness can be modelled by the function

$$C = 16.99t + 49, \qquad t \geq 0$$

where t is the time in weeks.

(a) Calculate the cost of the gym membership for a year. Give your answer correct to 2 decimal places.

[1]

(b) Find the number of weeks it takes for the total cost to exceed 2000 NZD.

[2]

At Les Mills the initial payment is 20 NZD lower than Cityfitness, however the weekly cost is 8.51 NZD higher than Cityfitness

(c) Write a cost function for a gym membership at Les Mills using an appropriate model.

[1]

(d) Calculate how many weeks it will take for the cost of a Les Mills gym membership be more than the cost of a Cityfitness gym membership.

[3]

a) 52 weeks in a year.

Sub $t = 52$ into C.

$C = 16.99(52) + 49$

$\boxed{C = 932.48 \text{ NZD (2dp)}}$

The total cost, C, in New Zealand dollars (NZD), of a premium gym membership at Cityfitness can be modelled by the function

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[1]

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[3]

b) Set $C = 2000$ and rearrange for t.

$16.99t + 49 = 2000$ $\quad -49$

$16.99t = 1951$ $\quad \div 16.99$

$t \approx 114.8$

$\boxed{\therefore 115 \text{ weeks}}$

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[1]

(d) Calculate how many weeks it will take for the cost of a Les Mills gym membership be more than the cost of a Cityfitness gym membership.

[3]

c) Initial payment is 20 NZD less.

Initial payment $= 49 - 20$
$= 29$

Weekly cost is 8.51 NZD more.

Weekly cost $= 16.99 + 8.51$
$= 25.50$

$\boxed{C = 25.50t + 29}$

The total cost, C, in New Zealand dollars (NZD), of a premium gym membership at Cityfitness can be modelled by the function

$$C = 16.99t + 49, \qquad t \geq 0$$

where t is the time in weeks.

(a) Calculate the cost of the gym membership for a year. Give your answer correct to 2 decimal places.

[1]

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(c) Write a cost function for a gym membership at Les Mills using an appropriate model.

[1]

(d) Calculate how many weeks it will take for the cost of a Les Mills gym membership be more than the cost of a Cityfitness gym membership.

[3]

d) Les Mills > Cityfitness

$25.50t + 29 > 16.99t + 49$ -29

$25.50t > 16.99t + 20$ $-16.99t$

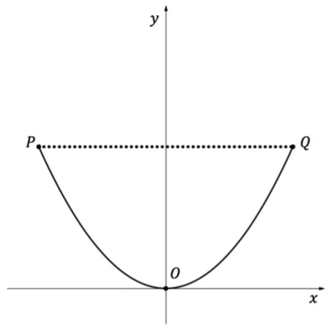
$8.51t > 20$ $\div 8.51$

$t > 2.35$

$\boxed{\therefore 3 \text{ weeks}}$

Question 2

The front view of the edge of a water tank is drawn on a set of axes below. The edge is modelled by $y = ax^2 + c$.



Point P has coordinates $(-4, 4)$, point O has coordinates $(0, 0)$ and point Q has coordinates $(4, 4)$.

(a) (i) Find the value of c.

(ii) Find the value of a.

(iii) Hence, write down the equation of the quadratic function which models the edge of the water tank.

[4]

(b) Given that 1 unit represents 1 m, find the width of the water tank when its height is 2.25 m.

[2]

a) i) c represents the y-intercept.

$\boxed{c = 0}$

ii) Sub point Q* into y.

$4 = a(4)^2$ expand

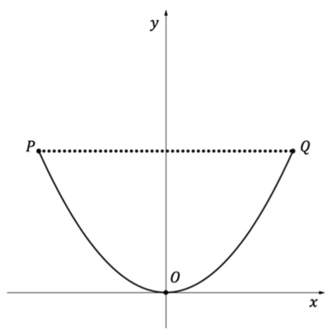
$4 = 16a$ $\div 16$

$\boxed{a = \frac{1}{4}}$

*N.B you could also use point P.

iii) $\boxed{y = \frac{1}{4}x^2}$

The front view of the edge of a water tank is drawn on a set of axes below. The edge is modelled by $y = ax^2 + c$.



Point P has coordinates $(-4, 4)$, point O has coordinates $(0, 0)$ and point Q has coordinates $(4, 4)$.

(a) (i) Find the value of c.

(ii) Find the value of a.

(iii) Hence, write down the equation of the quadratic function which models the edge of the water tank.

$\boxed{y = \frac{1}{4}x^2}$

[4]

(b) Given that 1 unit represents 1 m, find the width of the water tank when its height is 2.25 m.

[2]

b) y represents the height of the water tank.

x represents half the width of the water tank.

Set $y = 2.25$ and rearrange for x.

$2.25 = \frac{1}{4}x^2$ $\times 4$

$9 = x^2$ $\sqrt{\ }$

$3 = x$ (reject $x = -3$)

The width of the water tank is 6m.

Question 3

The number of German words, W, that Helen remembers after completing a German language course decreases exponentially over time when she does not practice her German. This decrease can be modelled by the function

$$W(t) = a \times b^{-t} + 320, \qquad t \geq 0$$

Where a and b are positive constants and t is the time in years since Helen completed the German language course.

Helen can remember 2400 German words as soon as she completes the German language course.

(a) Find the value of a.

[2]

After 2 years Helen has not practiced her German and can only remember 1020 German words.

(b) Find the value of b.

[3]

The number of German words Helen remembers never decreases below a certain number of words, c.

(c) Find the value of c.

[1]

a) When $t = 0$, $W(t) = 2400$.

$W(0) = 2400$

$a \times \underbrace{b^{-(0)}}_{=1} + 320 = 2400$) $b^{-(0)} = 1$

$a + 320 = 2400$) -320

$\boxed{a = 2080}$

The number of German words, W, that Helen remembers after completing a German language course decreases exponentially over time when she does not practice her German. This decrease can be modelled by the function

$$W(t) = a \times b^{-t} + 320, \qquad t \geq 0$$

Where a and b are positive constants and t is the time in years since Helen completed the German language course.

Helen can remember 2400 German words as soon as she completes the German language course.

(a) Find the value of a. $\boxed{a = 2080}$

[2]

After 2 years Helen has not practiced her German and can only remember 1020 German words.

(b) Find the value of b.

[3]

The number of German words Helen remembers never decreases below a certain number of words, c.

(c) Find the value of c.

[1]

b) When $t = 2$, $W(t) = 1020$.

$W(2) = 1020$

$2080 \times b^{-(2)} + 320 = 1020$) -320

$2080 \times b^{-(2)} = 700$) $\div 2080$

$b^{-(2)} = \dfrac{700}{2080}$) reciprocate

$b^2 = \dfrac{2080}{700}$) $\sqrt{}$

$\boxed{b \approx 1.72}$

The number of German words, W, that Helen remembers after completing a German language course decreases exponentially over time when she does not practice her German. This decrease can be modelled by the function

$$W(t) = a \times b^{-t} + 320, \qquad t \geq 0$$

Where a and b are positive constants and t is the time in years since Helen completed the German language course.

Helen can remember 2400 German words as soon as she completes the German language course.

(a) Find the value of a. $\boxed{a = 2080}$

[2]

After 2 years Helen has not practiced her German and can only remember 1020 German words.

(b) Find the value of b. $\boxed{b \approx 1.72}$

[3]

The number of German words Helen remembers never decreases below a certain number of words, c.

(c) Find the value of c.

[1]

c) As t tends towards infinity (∞),

$2080 \times 1.72^{-(\infty)}$ tends towards zero

$W(t) = 2080 \times 1.72^{-t} + 320$

$\lim_{x \to \infty} W(t) = 0 + 320$
$= 320$

$\boxed{\therefore c = 320}$

Question 4

In a trial for a new drug, scientists found that the amount of the drug in the bloodstream decreased over time, according to the model

$$D(t) = 1.4 \times 0.77^t, \qquad t \geq 0$$

where D is the amount of the drug in the bloodstream in mg per litre (mg L^{-1}) and t is the time in hours.

(a) Write down the amount of the drug in the bloodstream at $t = 0$.

[1]

a) Sub $t = 0$ into $D(t)$.

$D(0) = 1.4 \times 0.77^{(0)}$

$\boxed{D(0) = 1.4 \text{ mg } L^{-1}}$

(b) Calculate the amount of the drug in the bloodstream after four hours.

[2]

(c) Calculate the time, in hours, for the amount of the drug in the bloodstream to decrease to 0.22 mg L^{-1}.

[3]

The scientists found that some of the test subjects had an elevated heart rate for 45 minutes after ingesting the drug.

(d) Find the amount of the drug in the bloodstream when the heart rates from the effected test subjects returned to normal.

[2]

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(b) Calculate the amount of the drug in the bloodstream after four hours.

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The scientists found that some of the test subjects had an elevated heart rate for 45 minutes after ingesting the drug.

(d) Find the amount of the drug in the bloodstream when the heart rates from the effected test subjects returned to normal.

[2]

b) Sub $t = 4$ into $D(t)$.

$D(4) = 1.4 \times 0.77^{(4)}$

$\boxed{D(4) = 0.492 \text{ mg L}^{-1}}$

In a trial for a new drug, scientists found that the amount of the drug in the bloodstream decreased over time, according to the model

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where D is the amount of the drug in the bloodstream in mg per litre (mg L^{-1}) and t is the time in hours.

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[1]

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[3]

The scientists found that some of the test subjects had an elevated heart rate for 45 minutes after ingesting the drug.

(d) Find the amount of the drug in the bloodstream when the heart rates from the effected test subjects returned to normal.

[2]

c) Set $D(t) = 0.22$ and solve for t on your GDC.

$1.4 \times 0.77^t = 0.22$

$\boxed{t = 7.08 \text{ hours}}$

In a trial for a new drug, scientists found that the amount of the drug in the bloodstream decreased over time, according to the model

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where D is the amount of the drug in the bloodstream in mg per litre (mg L^{-1}) and t is the time in hours.

(a) Write down the amount of the drug in the bloodstream at $t = 0$.

[1]

(b) Calculate the amount of the drug in the bloodstream after four hours.

[2]

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[3]

The scientists found that some of the test subjects had an elevated heart rate for 45 minutes after ingesting the drug.

(d) Find the amount of the drug in the bloodstream when the heart rates from the effected test subjects returned to normal.

[2]

d) 45 minutes is 0.75 hours.

Sub $t = 0.75$ into $D(t)$.

$D(0.75) = 1.4 \times 0.77^{(0.75)}$

$\boxed{D(0.75) = 1.15 \text{ mg } L^{-1}}$

Question 5

The number of bacteria in a Petri dish is modelled by the function

$$N(t) = 75 \times 2^{0.5t}, \quad t \geq 0$$

where N is the number of bacteria and t is the time in hours.

(a) Write down the number of bacteria in the Petri dish at $t = 0$.

[1]

(b) Calculate the number of bacteria present after 10 hours.

[2]

(c) Calculate the time, in hours, for the number of bacteria to reach 10 000.

[3]

a) Sub $t = 0$ into $N(t)$.

$N(0) = 75 \times 2^{0.5(0)}$

$\boxed{N(0) = 75 \text{ bacteria}}$

The number of bacteria in a Petri dish is modelled by the function

$$N(t) = 75 \times 2^{0.5t}, \quad t \geq 0$$

where N is the number of bacteria and t is the time in hours.

(a) Write down the number of bacteria in the Petri dish at $t = 0$.

[1]

(b) Calculate the number of bacteria present after 10 hours.

[2]

(c) Calculate the time, in hours, for the number of bacteria to reach 10 000.

[3]

b) Sub $t = 10$ into $N(t)$.

$N(10) = 75 \times 2^{0.5(10)}$

$\boxed{N(10) = 2400 \text{ bacteria}}$

The number of bacteria in a Petri dish is modelled by the function

$$N(t) = 75 \times 2^{0.5t}, \quad t \geq 0$$

where N is the number of bacteria and t is the time in hours.

(a) Write down the number of bacteria in the Petri dish at $t = 0$.

[1]

(b) Calculate the number of bacteria present after 10 hours.

[2]

(c) Calculate the time, in hours, for the number of bacteria to reach 10 000.

[3]

c) Set $N(t) = 10\,000$ and solve for t on your GDC.

$75 \times 2^{0.5t} = 10\,000$

$\boxed{t = 14.1 \text{ hours}}$

Question 6

A remote-controlled sailboat's velocity is dependent on the wind speed. The sailboat's velocity is lower during very high and very low wind speeds.

The sailboat's velocity can be modelled by the function

$$V(w) = 0.0025w(2 - w)(w - 35), \quad 2 \leq w \leq 35$$

where V is the sailboat's velocity, in km h^{-1}, and w is the wind speed, in km h^{-1}.

(a) Find the sailboat's velocity when the windspeed is 20 km h^{-1}.

[1]

(b) Find the windspeed when the sailboat's velocity is 5.94 km h^{-1}.

[2]

(c) Show that $V(w) = -0.0025w^3 + 0.0925w^2 - 0.175w$.

[2]

(d) Using your graphics display calculator find the maximum velocity of the sailboat and the windspeed required for this.

[3]

a) Sub $w = 20$ into $V(w)$.

$V(20) = 0.0025(20)(2 - 20)(20 - 35)$

$\boxed{V(20) = 13.5 \text{ km h}^{-1}}$

A remote-controlled sailboat's velocity is dependent on the wind speed. The sailboat's velocity is lower during very high and very low wind speeds.

The sailboat's velocity can be modelled by the function

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(a) Find the sailboat's velocity when the windspeed is 20 km h^{-1}.

[1]

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[2]

(c) Show that $V(w) = -0.0025w^3 + 0.0925w^2 - 0.175w$.

[2]

(d) Using your graphics display calculator find the maximum velocity of the sailboat and the windspeed required for this.

[3]

b) Set $V(w) = 5.94$ and solve for t on your GDC.

$0.0025w(2 - w)(20 - w) = 5.94$

$\boxed{w = 11 \text{ km h}^{-1}}$

A remote-controlled sailboat's velocity is dependent on the wind speed. The sailboat's velocity is lower during very high and very low wind speeds.

The sailboat's velocity can be modelled by the function

$$V(w) = 0.0025w(2 - w)(w - 35), \qquad 2 \leq w \leq 35$$

where V is the sailboat's velocity, in km h^{-1}, and w is the wind speed, in km h^{-1}.

(a) Find the sailboat's velocity when the windspeed is 20 km h^{-1}.

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(c) Show that $V(w) = -0.0025w^3 + 0.0925w^2 - 0.175w$.

[2]

(d) Using your graphics display calculator find the maximum velocity of the sailboat and the windspeed required for this.

[3]

c) Expand $V(w)$.

$V(w) = 0.0025w(2 - w)(w - 35)$

Expand first bracket.

$V(w) = (0.005w - 0.0025w^2)(w - 35)$

Expand second bracket.

$V(w) = 0.005w^2 - 0.175w - 0.0025w^3 + 0.0875w^2$

Rearrange and collect like terms.

$\boxed{V(w) = -0.0025w^3 + 0.0925w^2 - 0.175w}$

A remote-controlled sailboat's velocity is dependent on the wind speed. The sailboat's velocity is lower during very high and very low wind speeds.

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(c) Show that $V(w) = -0.0025w^3 + 0.0925w^2 - 0.175w$.

[2]

(d) Using your graphics display calculator find the maximum velocity of the sailboat and the windspeed required for this.

[3]

d) Graph $V(w)$ on your GDC and find its maximum.

maximum = (23.7, 14.5)

maximum velocity = 14.5 km h^{-1}
when the windspeed = 23.7 km h^{-1}

Question 7

A Ferris wheel rotates at a constant speed, the height of a particular seat above the ground is modelled by the function

$$H(t) = -14\cos(10° \times t) + 16, \quad t \geq 0$$

where H is the height of the seat above the ground, in metres, and t is the elapsed time, in seconds, since the start of the ride.

(a) Write down

(i) the minimum height of the seat

(ii) the maximum height of the seat.

[4]

(b) Calculate the number of seconds it takes for the Ferris wheel to do one full rotation.

[2]

a) Graph $H(t)$ on your GDC. and find its minimum and maximum.

i) $\boxed{H_{min} = 2\,m}$

ii) $\boxed{H_{max} = 30\,m}$

A Ferris wheel rotates at a constant speed, the height of a particular seat above the ground is modelled by the function

$$H(t) = -14\cos(10° \times t) + 16, \quad t \geq 0$$

Where H is the height of the seat above the ground, in metres, and t is the elapsed time, in seconds, since the start of the ride.

(a) Write down the minimum height of the seat.

[2]

(b) Write down the maximum height of the seat.

[2]

(c) Calculate the number of seconds it takes for the Ferris wheel to do one full rotation.

[2]

c) $H(t)$ is in the form $a\cos bx + c$.

one full rotation = period of $H(t)$.

Period formula

$\text{Period} = \dfrac{360°}{b}$ (not in formula booklet)

$\text{Period} = \dfrac{360°}{10°}$

$\text{Period} = 36$

$\boxed{\text{one full rotation} = 36\,s}$

Question 8

The water depth, D, in metres, at a port can be modelled by the function

$$D(t) = 5\sin(30° \times t) + 15, \quad 0 \leq t \leq 24$$

where t is the elapsed time, in hours, since midnight.

(a) Write down the depth of the water at midnight.

[1]

(b) The cycle of water depths repeats every P hours. Find the value of P.

[2]

(c) (i) Calculate the maximum and minimum depths.

(ii) Find the times at which the maximum and minimum depths occur during the day.

[4]

a) $t = 0$ at midnight.

Sub $t = 0$ into $D(t)$.

$D(0) = 5\sin(30° \times (0)) + 15$

$\boxed{D(0) = 15\,m}$

The water depth, D, in metres, at a port can be modelled by the function

$$D(t) = 5\sin(30° \times t) + 15, \qquad 0 \le t \le 24$$

where t is the elapsed time, in hours, since midnight.

(a) Write down the depth of the water at midnight. [1]

(b) The cycle of water depths repeats every P hours. Find the value of P. [2]

(c) (i) Calculate the maximum and minimum depths.

(ii) Find the times at which the maximum and minimum depths occur during the day. [4]

b) $D(t)$ is in the form $a \sin bx + c$.

P is equal to the period of $D(t)$.

Period formula

Period $= \frac{360°}{b}$ (not in formula booklet)

$P = \frac{360°}{30°}$

$\boxed{P = 12 \text{ hours}}$

The water depth, D, in metres, at a port can be modelled by the function

$$D(t) = 5\sin(30° \times t) + 15, \qquad 0 \le t \le 24$$

where t is the elapsed time, in hours, since midnight.

(a) Write down the depth of the water at midnight. [1]

(b) The cycle of water depths repeats every P hours. Find the value of P. [2]

(c) (i) Calculate the maximum and minimum depths.

(ii) Find the times at which the maximum and minimum depths occur during the day. [4]

c) Graph $D(t)$ on your GDC and find its maximums and minimums within $0 \le t \le 24$.

maximums $= (3, 20)$ and $(15, 20)$

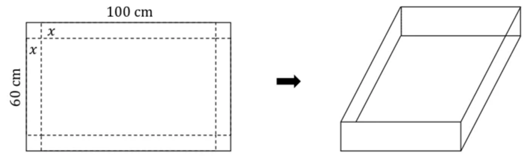
minimums $= (9, 10)$ and $(21, 10)$

i) $\boxed{\begin{array}{l}\text{maximum depth} = 20\text{ m} \\ \text{minimum depth} = 10\text{ m}\end{array}}$

ii) $\boxed{\begin{array}{l}\text{maximum depth at } 3{:}00 \text{ and } 15{:}00 \\ \text{minimum depth at } 9{:}00 \text{ and } 21{:}00\end{array}}$

Question 9

A rectangular sheet of cardboard 60 cm by 100 cm has square sides of x cm cut from each corner. It is folded to make an open box as shown.



(a) Show that the volume of the box can be modelled by the function

$$V = 4x^3 - 320x^2 + 6000x.$$

[3]

(b) State the domain of V.

[2]

(c) Using your graphics display calculator find the maximum value of V and the value of x which gives this volume.

[2]

a) The shape of the box is a cuboid.

Volume of a cuboid formula

$V = lwh$ $\quad l$ = length, w = width, h = height

$l = 100 - 2x \quad w = 60 - 2x \quad h = x$

Sub l, w and h into formula.

$V = (100 - 2x)(60 - 2x)x$

Expand brackets.

$V = (6000 - 320x + 4x^2)x$

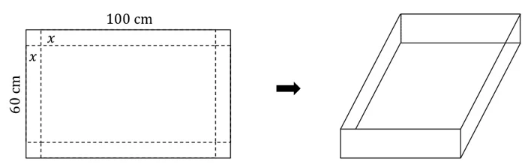
Expand fully.

$V = 6000x - 320x^2 + 4x^3$

Rearrange into the form given.

$\boxed{V = 4x^3 - 320x^2 + 6000x}$

A rectangular sheet of cardboard 60 cm by 100 cm has square sides of x cm cut from each corner. It is folded to make an open box as shown.



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$$V = 4x^3 - 320x^2 + 6000x.$$

[3]

(b) State the domain of V.

[2]

(c) Using your graphics display calculator find the maximum value of V and the value of x which gives this volume.

[2]

b) Dimensions of the box are

$l = 100 - 2x \quad w = 60 - 2x \quad h = x$

$\therefore$ the volume of the box can be given by

$V = (100 - 2x)(60 - 2x)x$

If $x = 0$

then $h = 0$ and so $V = 0$.

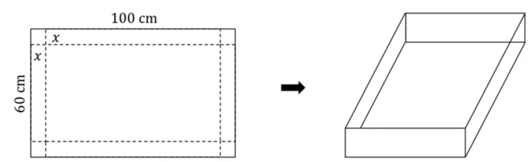
If $x = 30$

then $w = 60 - 2(30) = 0$ and so $V = 0$.

$\boxed{\text{Domain is } \{x \mid 0 < x < 30\}}$

EXAM PAPERS PRACTICE

A rectangular sheet of cardboard 60 cm by 100 cm has square sides of x cm cut from each corner. It is folded to make an open box as shown.



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$$V = 4x^3 - 320x^2 + 6000x\,.$$

[3]

(b) State the domain of V.

[2]

(c) Using your graphics display calculator find the maximum value of V and the value of x which gives this volume.

[2]

c) Graph V on your GDC and find its maximum.

maximum = (12.1, 32 800) (3sf)

$\boxed{\therefore V_{max} = 32\,800 \text{ cm}^3 \text{ when } x = 12.1 \text{ cm}}$

Question 10

Grace leaves a cup of hot tea to cool and measures its temperature every minute. Her results are shown in the table below.

Time, t (minutes)	0	1	2	3	4
Temperature, y (°C)	88	58	43	35.5	k

(a) Write down the decrease in temperature of the tea

(i) during the first minute

(ii) during the second minute

(iii) during the third minute.

[3]

(b) Assuming the pattern in the answers to part (a) continues, find the value of k. Give your answer correct to 2 decimal places.

[2]

The function that models the change in temperature of the tea is $y = a(2^{-t}) + b$, where b represents the temperature the tea tends towards and $a + b$ is the initial temperature.

(c) Write down two equations relating a and b.

[2]

(d) Find the value of a and b.

[2]

a) i) During the first minute.

88 - 58 = 30

$\boxed{30°C}$

ii) During the second minute.

58 - 43 = 15

$\boxed{15°C}$

iii) During the third minute.

43 - 35.5 = 7.5

$\boxed{7.5°C}$

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Temperature, y (°C)	88	58	43	35.5	k

(a) Write down the decrease in temperature of the tea

(i) during the first minute $\boxed{30°C}$

(ii) during the second minute $\boxed{15°C}$

(iii) during the third minute. $\boxed{7.5°C}$

[3]

(b) Assuming the pattern in the answers to part (a) continues, find the value of k. Give your answer correct to 2 decimal places.

[2]

b) The decrease in the temperature of the tea in any given minute is half the decrease from the previous minute.

$\frac{15}{30} = \frac{7.5}{15} = \frac{1}{2}$

$\therefore$ the decrease during the fourth minute is

$7.5 \times \frac{1}{2} = 3.75$

$\therefore k = 35.5 - 3.75$

$\boxed{k = 31.75°C}$

The function that models the change in temperature of the tea is $y = a(2^{-t}) + b$, where b represents the temperature the tea tends towards and $a + b$ is the initial temperature.

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[2]

(d) Find the value of a and b.

[2]

c) $a + b$ is the initial temperature ($t = 0$).

$\boxed{88 = a + b}$

When $t = 1$, $y = 58$.

$58 = a(2^{-(1)}) + b$

$\boxed{58 = \frac{1}{2}a + b}$

Grace leaves a cup of hot tea to cool and measures its temperature every minute. Her results are shown in the table below.

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Temperature, y (°C)	88	58	43	35.5	k

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(iii) during the third minute.

[3]

(b) Assuming the pattern in the answers to part (a) continues, find the value of k. Give your answer correct to 2 decimal places.

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The function that models the change in temperature of the tea is $y = a(2^{-t}) + b$, where b represents the temperature the tea tends towards and $a + b$ is the initial temperature.

(c) Write down two equations relating a and b.

$\boxed{88 = a + b}$ $\boxed{58 = \frac{1}{2}a + b}$

[2]

(d) Find the value of a and b.

[2]

d) Simultaneous equations

① $88 = a + b$

② $58 = \frac{1}{2}a + b$

① - ②

$$\begin{array}{rl} & 88 = a + b \\ - & 58 = \frac{1}{2}a + b \\ \hline & 30 = \frac{1}{2}a \end{array}$$

$\boxed{\therefore a = 60}$

Sub a into ①.

$88 = 60 + b$

$\boxed{\therefore b = 28}$

Alternatively you could solve the simultaneous equations on your GDC.