

IB Maths: AA HL

Matrix Transformations

Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AA HL Topic Questions

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| Course | IB Maths |
| Section | 3. Geometry & Trigonometry |
| Topic | 3.6 Matrix Transformations |
| Difficulty | Medium |

Level: IB Maths

Subject: IB Maths AA HL

Board: IB Maths

Topic: Matrix Transformations

Question 1

(a)

Write down the matrix that represents a rotation of 225° clockwise about the origin.

[2 marks]

(b)

Write down the matrix that represents a reflection in the y -axis.

[2 marks]

(c)

Find a single matrix that represents the composite transformation consisting of the transformation in part (a) followed by the transformation in part (b).

[2 marks]

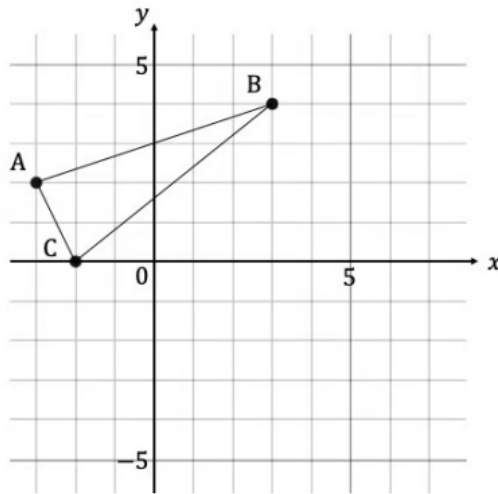
(d)

Hence find the coordinates of the image of the point $(4, -1)$ after a rotation of 225° clockwise about the origin followed by a reflection in the y -axis.

[2 marks]

Question 2

The diagram below shows a triangle .



- (a)
Write down the position matrix of the triangle ABC.

[2 marks]

The triangle ABC is to be mapped to triangle $A'B'C'$ by a single transformation defined by the transformation matrix $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$.

- (b)
Find the position matrix of the mapped image and draw triangle $A'B'C'$ on the diagram.

[3 marks]

- (c)
Describe fully the transformation that triangle ABC has undergone.

[2 marks]

Question 3

Points in a plane are subjected to a transformation T that transforms a point (x, y) to the point (x', y') , where T is defined by

$$T: \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 0.5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

(a)

Describe in words the transformation T .

[2 marks]

(b)

Find the matrix T^{-1} .

[2 marks]

(c)

Hence find the coordinates of the point (x, y) if $(x', y') = (12, -4)$.

[2 marks]

Question 4

A quadrilateral has vertices $(2, 1)$, $(4, 4)$, $(7, 1)$ and $(9, 4)$.

(a)

Find the area of the quadrilateral.

[2 marks]

The quadrilateral undergoes a transformation represented by the matrix $\begin{pmatrix} -2 & 1 \\ -4 & 3 \end{pmatrix}$.

(b)

Find the determinant of the transformation matrix.

[2 marks]

(c)

Hence find the area of the image.

[2 marks]

Question 5

An object undergoes a vertical stretch with scale factor 2 followed by a reflection in the x -axis. The position

matrix of the image is $\begin{pmatrix} 2 & 5 & 4 \\ 3 & -2 & -1 \end{pmatrix}$.

Use a matrix method to find the coordinates of the object before the transformation.

[5 marks]

Question 6

The points $A(7, -3)$ and $B(2, 6)$ are transformed to become the points $A'(18, 8)$ and $B'(12, 16)$ respectively.

(a)

Find the 2×2 matrix T that represents the linear transformation.

[5 marks]

(b)

Given that point $C(-4, 5)$ is transformed by T^2 , find the coordinates of the image point C' .

[3 marks]

Question 7

An object is reflected in the line $y = \frac{\sqrt{3}}{3}x$.

(a)

Write down the matrix that represents the transformation.

[3 marks]

P is a vertex of the object that is being reflected.

(b)

Find the coordinates of **P** if the coordinates of its image **P'** are $(2, 2\sqrt{3})$.

[3 marks]

Question 8

The triangle PQR, with vertices $P(-1, 1)$, $Q(5, 3)$ and $R(-9, -2)$, is translated by the vector $\begin{pmatrix} -7 \\ 2 \end{pmatrix}$ and then enlarged by a scale factor of 3 with the centre of enlargement at the origin.

(a)

Find a single transformation in the form $AX + b$ that maps PQR onto $P'Q'R'$.

[2 marks]

(b)

Hence determine the coordinates of $P'Q'R'$.

[3 marks]

Question 9

a)

Find the 2×2 transformation matrices that represent the following transformations:

(i)

R , a rotation of $\frac{\pi}{4}$ radians anti-clockwise

(ii)

S , a reflection in the line $y = x$

(iii)

T , a stretch with scale factor 5 parallel to the y -axis.

[3 marks]

(b)

Find a single transformation matrix that represents the composite transformation

(i)

RT^3

(ii)

R^8STS

[4 marks]

(c)

Find the coordinates of the image of the point $A(3, -1)$ after it has undergone the composite transformation specified in part (b)(i).

[2 marks]

(d)

State the name of the single transformation that is equivalent to the composite transformation specified in part (b)(ii).

[1 mark]

Question 10

The triangle PQR with position matrix T_0 has vertices P(5, 2), Q(-3, 1) and R(-5, -4).

The triangle is transformed by the transformation matrix $M = \begin{pmatrix} 0 & \frac{1}{2} \\ -\frac{1}{2} & 0 \end{pmatrix}$.

T_n denotes the position matrix of the image triangle after PQR has been transformed n times by matrix M .

(a)

By multiplying two appropriate single transformation matrices together, verify that the matrix M is an enlargement by scale factor $\frac{1}{2}$ followed by a 90° clockwise rotation.

[3 marks]

(b)

Explain why the area of the triangle with position matrix T_1 will be $\frac{1}{4}$ of the area of triangle PQR.

[2 marks]

(c)

Find T_2 , and hence the coordinates of the image triangle after triangle PQR is transformed twice by matrix M .

[3 marks]