Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 2

2 May 2024

Zone A morning | **Zone B** morning | **Zone C** morning

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2224 – 7210

11 pages

Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 16]

In a given week, the number of students in a particular primary school that were absent due to headlice (H), influenza (I) and/or chickenpox (C) were recorded as follows.

The primary school has 500 students.

35 students had headlice only
20 students had influenza only
5 students had chickenpox only
4 students had headlice and influenza but not chickenpox
2 students had headlice and chickenpox but not influenza
3 students had influenza and chickenpox but not headlice
1 student had headlice, influenza and chickenpox

(a) Draw a Venn diagram to represent this information. [3]

(b) Calculate the number of students who did not have headlice or influenza or chickenpox. [2]

A student is chosen at random from all the students in the school.

(c) Find the probability that this student has

(i) headlice.

(ii) influenza given that the student has headlice. [4]

(This question continues on the following page)

(Question 1 continued)

Diego is a teacher in the school. He believes that the number of students, n, who have had influenza during the first t days of the school year, can be modelled by the function

$$n(t) = 250 - 240(2)^{kt},\ k \in \mathbb{R}.$$

(d) Use Diego's model to calculate the number of students who started the school year with influenza. [2]

It is known that 130 students have had influenza during the first 10 days of the school year.

(e) Find the value of k. [2]

(f) Using this model, calculate how many days it will take for 200 students to have had influenza since the start of the school year. [2]

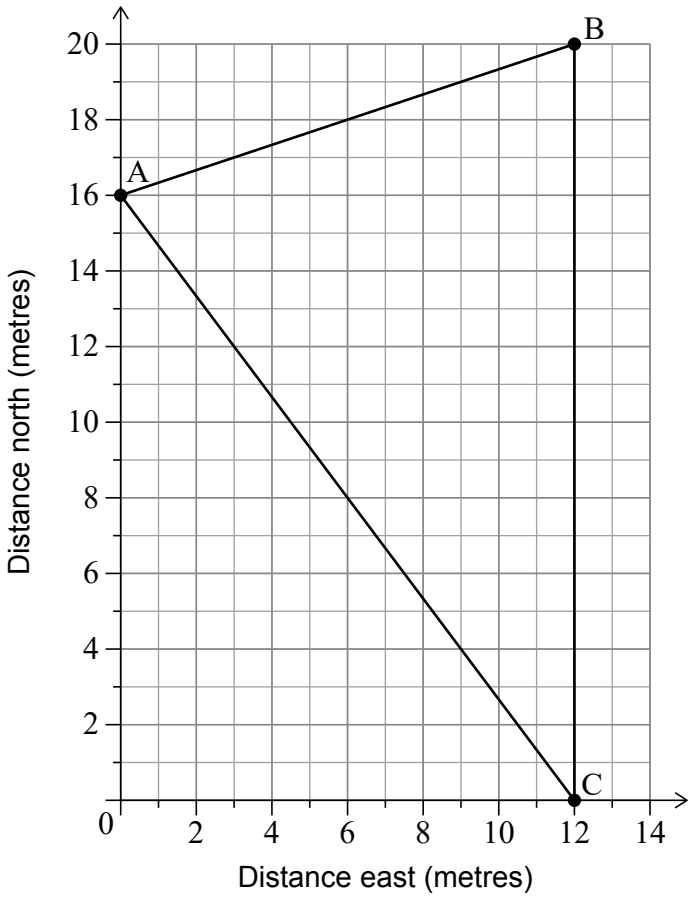
By the last day of the school year, it is known that 300 students have had influenza.

(g) Comment on the appropriateness of Diego's model. [1]

2. [Maximum mark: 14]

Mai is at an amusement park. A map of part of the amusement park is represented on the following coordinate axes.

Mai's favourite three attractions are positioned at $A(0, 16)$, $B(12, 20)$ and $C(12, 0)$.
All measurements are in metres.



(a) Write down the distance between B and C. [1]

(b) Calculate the distance between A and B. [2]

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(Question 2 continued)

Mai is standing at the attraction at B and wants to walk directly to the attraction at A.

(c) Calculate the bearing of A from B. [3]

A drinking fountain is to be installed at a point that is an equal distance from each of the attractions at A, B and C.

(d) (i) Write down the gradient of $[AC]$.

(ii) Write down the mid-point of $[AC]$.

(iii) Hence calculate the coordinates of the drinking fountain. [8]

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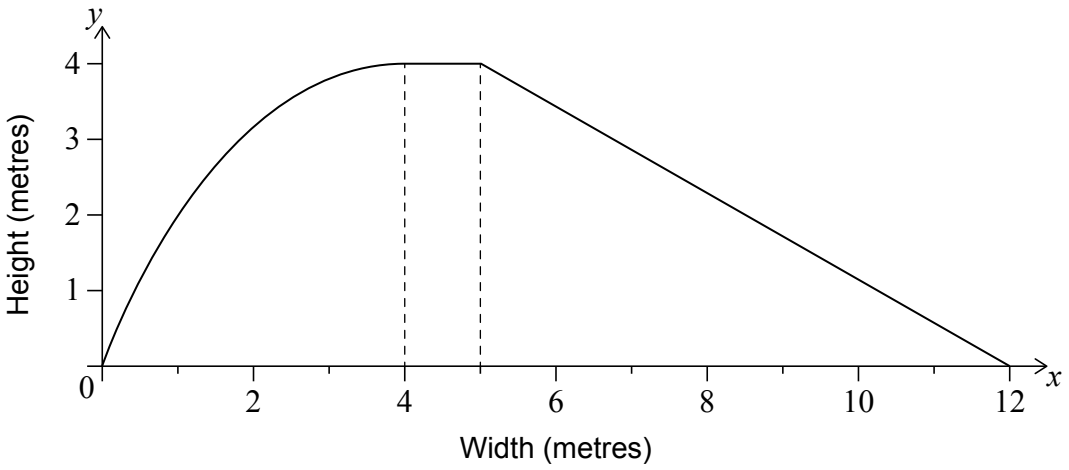
3. [Maximum mark: 15]

The following diagram shows a model of the side view of a water slide. All lengths are measured in metres.



The curved edge of the slide is modelled by

$$f(x) = -\frac{1}{4}x^2 + 2x \text{ for } 0 \le x \le 4.$$

The remainder of the slide is modelled by

$$g(x) = \begin{cases} 4, \text{ for } 4 \le x \le 5 \\ \frac{48}{7} - \frac{4x}{7}, \text{ for } 5 \le x \le 12 \end{cases}$$

(a) Use the trapezoidal rule with an interval width of 1 to calculate the approximate area under the model of the slide in the interval $0 \le x \le 4$. [5]

(b) Find $\int\left(-\frac{1}{4}x^2 + 2x\right)\mathrm{d}x$. [3]

(c) Calculate the exact area under the entire model of the slide, for $0 \le x \le 12$. [4]

(d) Find the percentage error in the **total** area under the entire model of the slide when using the approximate value from part (a). [3]

4. [Maximum mark: 19]

A recent study found that the heights of Dutch women can be modelled by a normal distribution with mean $170.7\,\text{cm}$ and standard deviation $6.3\,\text{cm}$.

A Dutch woman is chosen at random.

(a) Calculate the probability that her height is

(i) less than $160\,\text{cm}$.

(ii) between $160\,\text{cm}$ and $170\,\text{cm}$. [4]

$27\,\%$ of Dutch women have a height of more than h metres.

(b) Calculate the value of h. [2]

Janneke selects a random sample of 200 Dutch women from Amsterdam and measures their heights. She wants to determine whether this sample could have been chosen from a normally distributed population with mean of $170.7\,\text{cm}$ and standard deviation of $6.3\,\text{cm}$.

She performs a χ^2 goodness of fit test at the $5\,\%$ significance level. She begins by creating the following frequency table.

Height, h	Observed frequency	Expected frequency
$h \leq 160$	6	8.943
$160 < h \leq 170$	85	a
$170 < h \leq 180$	92	b
$h > 180$	17	13.99

(c) Calculate, correct to four significant figures, the value of

(i) a.

(ii) b. [2]

(This question continues on the following page)

(Question 4 continued)

The hypotheses for Janneke's test are

H_0: the heights are drawn from a normally distributed population with mean $170.7\,\text{cm}$ and standard deviation $6.3\,\text{cm}$

H_1: the heights are not drawn from a normally distributed population with mean $170.7\,\text{cm}$ and standard deviation $6.3\,\text{cm}$

(d) Write down the degrees of freedom for this test. [1]

The critical value for this test is 7.815.

(e) Perform the χ^2 goodness of fit test and state your conclusion, justifying your reasoning. [4]

Gundega claims that, on average, Latvian women are taller than Dutch women.

Random samples of 10 Latvian women and 10 Dutch women are chosen, and their heights are measured.

Heights of Latvian women (cm)									
171	163	180	159	169	182	166	168	171	170

Heights of Dutch women (cm)									
173	182	181	166	175	161	169	165	172	169

Gundega performs a t-test at the $5\,\%$ significance level. It is assumed that the populations are normally distributed and have equal variances.

(f) Write down the null and alternative hypotheses for this test. [2]

(g) Perform the t-test and state the conclusion, justifying your reasoning. [4]

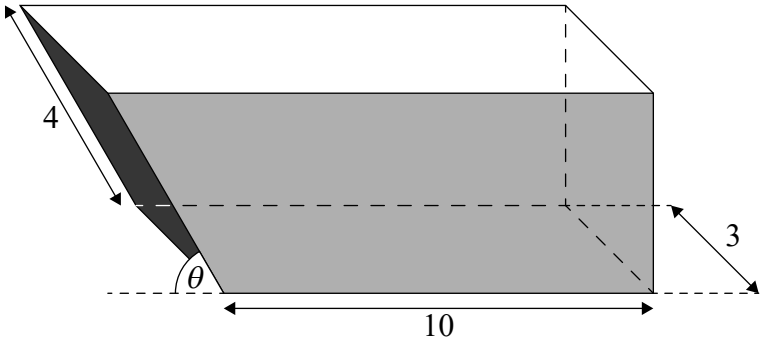
5. [Maximum mark: 16]

A skip is a container used to carry garbage away from a construction site. For safety reasons the garbage must not extend beyond the top of the skip. The maximum volume of garbage to be removed is therefore equal to the volume of the skip.

A particular design of skip can be modelled as a prism with a trapezoidal cross section. For the skip to be transported, it must have a rectangular base of length $10\,\text{m}$ and width $3\,\text{m}$. The length of the sloping edge is fixed at $4\,\text{m}$, and makes an angle of θ with the horizontal.

The following diagram shows such a skip.

diagram not to scale



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(Question 5 continued)

(a) Find the volume of this skip,

(i) if the length of the top edge of the skip is $11\,\text{m}$.

(ii) if the height of the skip is $3.2\,\text{m}$.

(iii) if θ is $60°$. [9]

(b) Show that the volume, $V\,\text{m}^3$, of the skip is given by

$$24\sin(\theta)(5+\cos(\theta)).$$ [2]

(c) Explain, in context, why $\theta \neq 0$. [1]

(d) (i) Sketch the graph of $V = 24\sin(\theta)(5+\cos(\theta))$, $0 < \theta < 90°$.

(ii) Find the maximum volume of the skip and the value of θ for which this maximum volume occurs. [4]

References:

5. Andyqwe, n.d. *Dumpster truck* [image online] Available at: https://www.gettyimages.co.uk/detail/photo/dumpster-truck-royalty-free-image/157611454 [Accessed 18 April 2023] Source adapted.