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# Mathematics: applications and interpretation

## Standard level

### Paper 2

31 October 2023

Zone A afternoon | Zone B afternoon | Zone C afternoon

1 hour 30 minutes

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#### Instructions to candidates

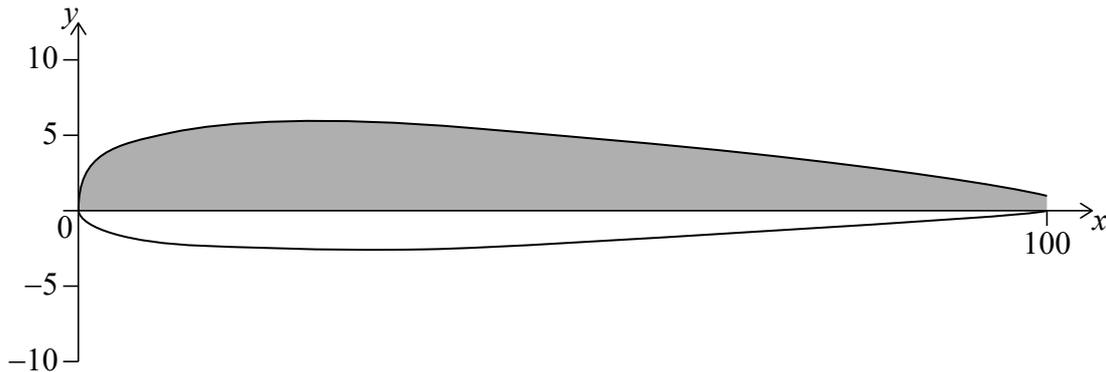
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

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Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 12]

Gabriel is investigating the shape of model airplane wings. A cross-section of one of the wings is shown, graphed on the coordinate axes.



The shaded part of the cross-section is the area between the  $x$ -axis and the curve with equation

$$y = 2\sqrt{x} - \frac{x}{5} + 1, \text{ for } 0 \leq x \leq 100$$

where  $x$  is the distance, in cm, from the front of the wing and  $y$  is the height, in cm, above the horizontal axis through the wing, as shown in the diagram.

- (a) Find the values of  $a$ ,  $b$  and  $c$ , shown in the table. [3]

<b><math>x</math> (cm)</b>	0	25	50	75	100
<b><math>y</math> (cm)</b>	1	$a$	$b$	$c$	1

Gabriel uses the trapezoidal rule with four intervals to estimate the shaded area of the cross-section of the wing.

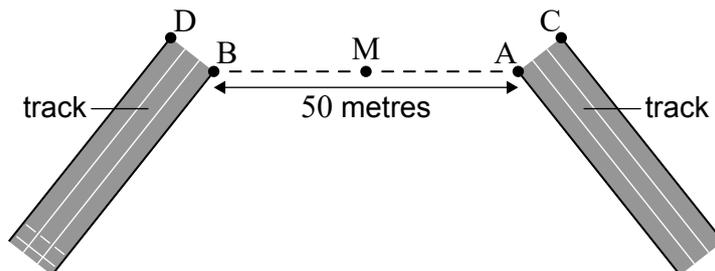
- (b) Find Gabriel's estimate of the shaded area of the cross-section. [3]
- (c) (i) Write down the integral that Gabriel can use to find the exact area of the shaded part of the cross-section.
- (ii) Hence, use your graphic display calculator to find the area of the shaded part of the cross-section. Give your answer correct to one decimal place. [4]
- (d) Calculate the percentage error of Gabriel's estimate in part (b). [2]

2. [Maximum mark: 15]

Madhu is designing a jogging track for the campus of her school. The following diagram shows an incomplete portion of the track.

Madhu wants to design the track such that the inner edge is a smooth curve from point A to point B, and the other edge is a smooth curve from point C to point D. The distance between points A and B is 50 metres.

diagram not to scale

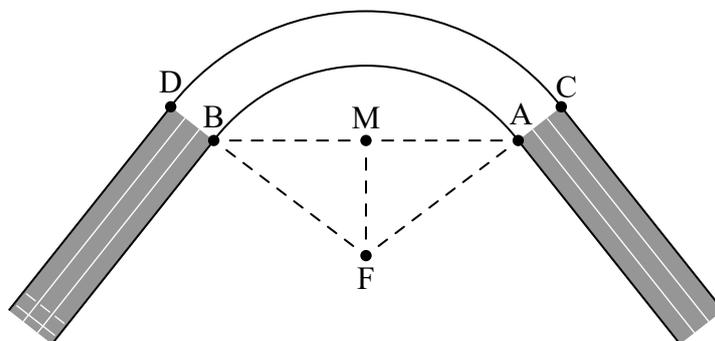


To create a smooth curve, Madhu first walks to M, the midpoint of [AB].

(a) Write down the length of [BM]. [1]

Madhu then walks 20 metres in a direction perpendicular to [AB] to get from point M to point F. Point F is the centre of a circle whose arc will form the smooth curve between points A and B on the track, as shown in the following diagram.

diagram not to scale



(b) (i) Find the length of [BF].  
(ii) Find  $\hat{BFM}$ . [4]

(c) Hence, find the length of arc AB. [3]

(This question continues on the following page)

**(Question 2 continued)**

The outer edge of the track, from C to D, is also a circular arc with centre F, such that the track is 2 metres wide.

- (d) Calculate the area of the curved portion of the track, ABDC. [4]

The base of the track will be made of concrete that is 12 cm deep.

- (e) Calculate the volume of concrete needed to create the curved portion of the track. [3]

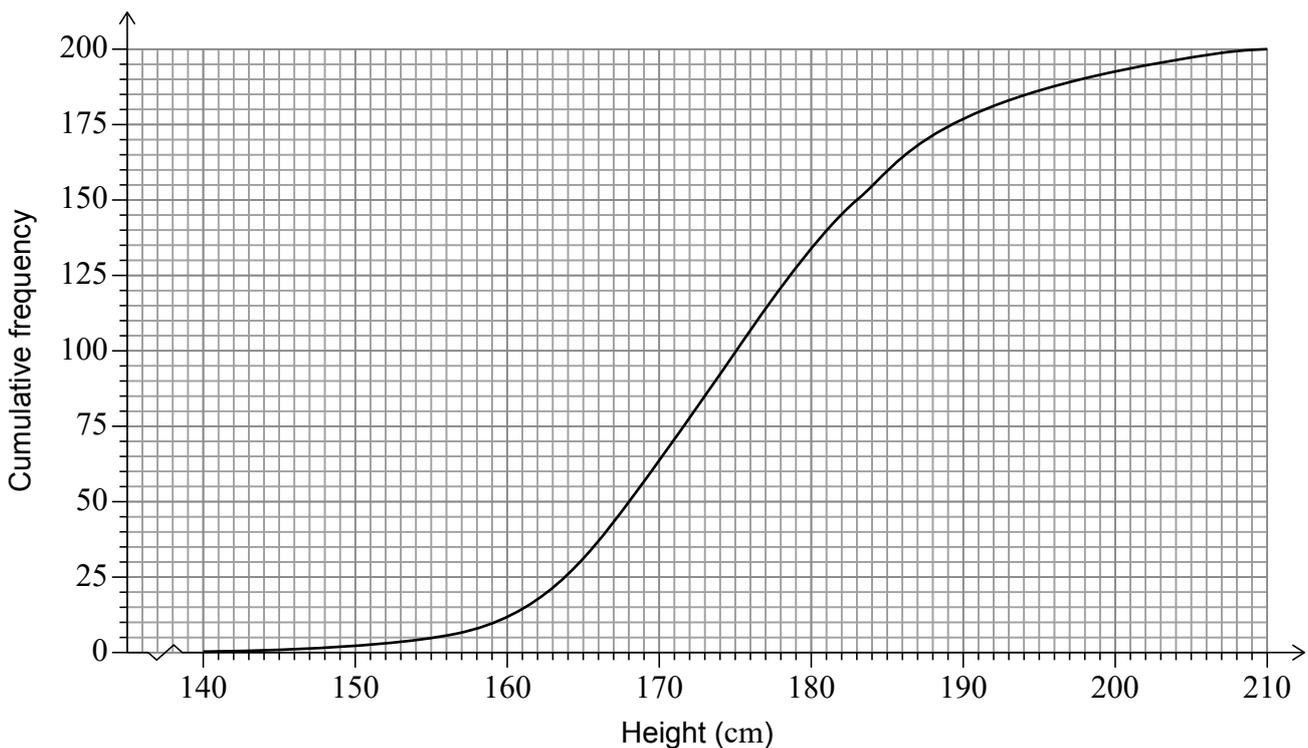
3. [Maximum mark: 18]

The heights,  $h$ , of 200 university students are recorded in the following table.

Height (cm)	Frequency
$140 \leq h < 160$	11
$160 \leq h < 170$	51
$170 \leq h < 180$	68
$180 \leq h < 190$	47
$190 \leq h < 210$	23

- (a) (i) Write down the mid-interval value of  $140 \leq h < 160$ .
- (ii) Calculate an estimate of the mean height of the 200 students. [3]

This table is used to create the following cumulative frequency graph.



- (b) Use the cumulative frequency curve to estimate the interquartile range. [2]

Laszlo is a student in the data set and his height is 204 cm.

- (c) Use your answer to part (b) to estimate whether Laszlo's height is an outlier for this data. Justify your answer. [3]

(This question continues on the following page)

**(Question 3 continued)**

It is believed that the heights of university students follow a normal distribution with mean 176 cm and standard deviation 13.5 cm.

It is decided to perform a  $\chi^2$  goodness of fit test on the data to determine whether this sample of 200 students could have plausibly been drawn from an underlying distribution  $N(176, 13.5^2)$ .

(d) Write down the null and the alternative hypotheses for the test. [2]

As part of the test, the following table is created.

Height of student (cm)	Observed frequency	Expected frequency
$h < 160$	11	23.6
$160 \leq h < 170$	51	42.1
$170 \leq h < 180$	68	$a$
$180 \leq h < 190$	47	46.7
$190 \leq h$	23	$b$

(e) (i) Find the value of  $a$  and the value of  $b$ .  
(ii) Hence, perform the test to a 5% significance level, clearly stating the conclusion in context. [8]

4. [Maximum mark: 16]

Tiffany wants to buy a house for a price of 285 000 US Dollars (USD). She goes to a bank to get a loan to buy the house. To be eligible for the loan, Tiffany must make an initial down payment equal to 15 % of the price of the house.

The bank offers her a 30-year loan for the remaining balance, with a 4 % nominal interest rate per annum, compounded monthly. Tiffany will pay the loan in fixed payments at the end of each month.

- (a) (i) Find the original amount of the loan after the down payment is paid.  
Give the exact answer.
- (ii) Calculate Tiffany's monthly payment for this loan, to two decimal places. [5]
- (b) Using your answer from part (a)(ii), calculate the total amount Tiffany will pay over the life of the loan, to the nearest dollar. Do **not** include the initial down payment. [2]

Tiffany would like to repay the loan faster and increases her payments such that she pays 1300 USD each month.

- (c) Find the total number of monthly payments she will need to make to pay off the loan. [2]
- This strategy will result in Tiffany's final payment being less than 1300 USD.
- (d) Determine the amount of Tiffany's final payment, to two decimal places. [4]
  - (e) Hence, determine the total amount Tiffany will save, to the nearest dollar, by making the higher monthly payments. [3]

5. [Maximum mark: 19]

Howell Industries is an international company that sells travel coffee mugs. Their market research predicts that the average number of mugs they will sell each month is modelled by the equation

$$n = 20\,000 - 1000x$$

where  $x$  represents the selling price, in euro (EUR), of each mug.

A salesperson suggests that Howell Industries should sell the mugs for 16 EUR each.

- (a) Find the average number of mugs that this model predicts Howell Industries will sell at this price. [2]
- (b) Calculate Howell Industries' average monthly income, before any expenses, at this selling price. [2]
- (c) Hence, write down the function  $R(x)$  that can be used to predict Howell Industries' average monthly income, before expenses, at any selling price,  $x$ . [1]

Howell Industries has 10 000 EUR of fixed monthly operational costs. Additionally, Howell Industries must pay their mug supplier 10 EUR for each coffee mug.

- (d) Calculate Howell Industries' average monthly profit if they sell each mug at a price of 16 EUR. [3]
- (e) Show that the average monthly profit for any selling price,  $x$ , can be found using the function  $P(x) = -1000x^2 + 30\,000x - 210\,000$ . [2]
- (f) (i) Find  $P'(x)$ .  
(ii) Show that the salesperson's selling price does not maximize their average monthly profit. [4]

Howell Industries negotiates a new deal with their mug supplier. Under the new deal, the supplier agrees to discount the cost of each mug based on the number of mugs purchased by Howell Industries. The cost charged by the supplier for each mug can be found using the function

$$C(n) = 10 - 0.0001n$$

where  $n$  represents the number of mugs sold by Howell Industries.

- (g) Find the function that can be used to find Howell Industries' average monthly profit using the new deal from the supplier. [3]
- (h) Hence, find the selling price, per mug, that Howell Industries should choose in order to maximize their average monthly profit under the new deal. [2]