Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 2

2 May 2024

Zone A morning | **Zone B** morning | **Zone C** morning

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2224 – 7205

11 pages

Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

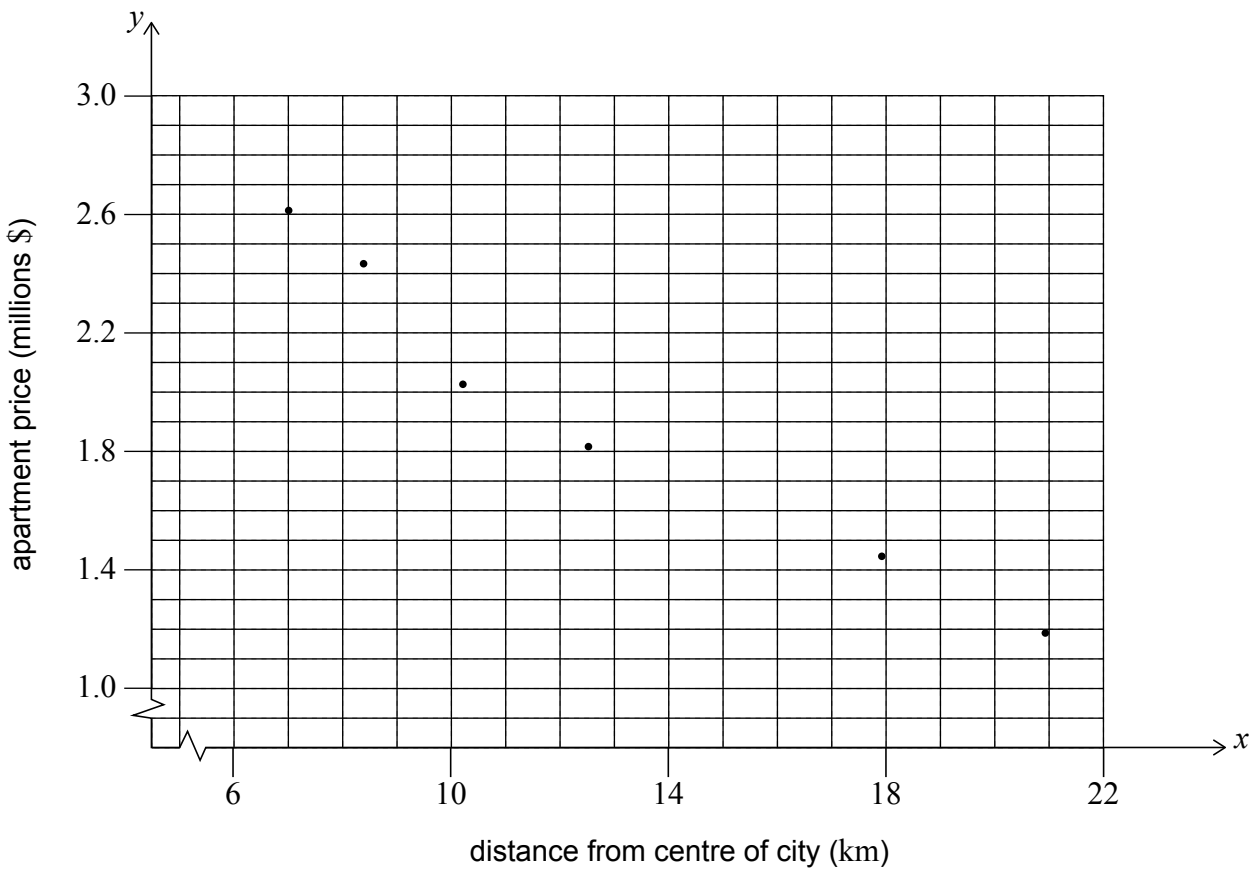
1. [Maximum mark: 19]

Xavie conducted a study to see if there is a relationship between the price of an apartment, y, and its distance, x, from the city centre of Melbourne.

They took a random sample of six typical apartments along a train line in the city. Xavie obtained the data shown in the following table.

x **(kilometres)**	7.0	8.4	10.3	12.5	17.8	20.9
y **(millions of dollars)**	2.61	2.44	2.03	1.81	1.45	1.18

A plot of these data is seen in the following graph.



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(Question 1 continued)

(a) Write down the value of the Spearman's rank correlation coefficient, r_s. [1]

(b) (i) Find the Pearson's product-moment correlation coefficient, r.

(ii) Use your value of r to state which **two** of the following would best describe the correlation between the variables.

Positive	Negative	Strong	Weak	No correlation

[4]

The relationship between the variables can be modelled by the regression equation $y = ax + b$.

(c) (i) Write down the value of a.

(ii) Write down the value of b.

(iii) According to this model, state in context what the value of b represents. [3]

(d) Xavie uses the regression equation to estimate the price of a typical apartment located $19.6\,\text{km}$ from the city centre.

(i) Find this estimated price.

(ii) State two reasons that Xavie might use to justify the validity of this estimate. [5]

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(Question 1 continued)

To verify whether this relationship applies in a different direction from the city centre, Xavie considers two locations, A and B, both an equal distance from the city centre. They take a random sample of seven apartments from each location and record the prices (in millions of dollars) in the following tables.

Apartment price in location A						
1.21	1.25	1.31	1.32	1.58	1.95	2.13

Apartment price in location B						
1.51	1.58	1.69	2.61	2.72	2.81	2.95

Xavie conducts a t-test, at the $5\ \%$ level of significance, to see if the mean apartment price in location A is different to the mean apartment price in location B. They assume the population variances are the same.

For this test, Xavie takes the null hypothesis to be $\mu_A = \mu_B$.

(e) Write down the alternative hypothesis. [1]

(f) Find the p-value for this test. [2]

(g) State the conclusion of the test. Justify your answer. [2]

(h) State one additional assumption Xavie has made about the distributions to conduct this test. [1]

2. [Maximum mark: 15]

The company Fred Express delivers packages. From past experience, the time taken, T, to deliver a package follows a normal distribution with mean 64 hours and standard deviation 12 hours.

(a) State $\mathrm{P}(T < 64)$. [1]

(b) Find $\mathrm{P}(44 < T < 64)$. [2]

$30\,\%$ of packages are delivered in less than k hours.

(c) (i) Sketch a diagram of this normal distribution, shading the region that represents $\mathrm{P}(T < k)$.

(ii) Find the value of k. [4]

For quality control, the manager randomly selects five outgoing packages. These selections are independent.

(d) Find the probability that exactly two of these packages are delivered in less than k hours. [3]

Fred Express charges a fixed amount of $\$4.50$ for any package weighing $1\,\mathrm{kg}$ or less. Heavier packages are charged an additional fee of $\$2.00$ per kg. This fee is applied for any weight **in excess of** $1\,\mathrm{kg}$. For example, a $1.5\,\mathrm{kg}$ package is charged an additional $\$1.00$.

(e) Write down an expression for the amount charged to deliver a package of weight $x\,\mathrm{kg}$, where $x > 1$. [2]

(f) Find the amount Fred Express charges for a $5.3\,\mathrm{kg}$ package. [1]

Meiling is charged $\$7.20$ for the delivery of a package.

(g) Find the weight of Meiling's package. [2]

3. [Maximum mark: 13]

A shop uses the following model to estimate n, the number of smoothies sold per day, in terms of x, the price of a single smoothie in pesos.

$$n = \frac{40\,000}{x^2}$$

The maximum number of smoothies the shop can make in a day is 400.

(a) Find the maximum price they could charge per smoothie for the shop to sell 400 in one day. [2]

(b) On a day when the shop sells smoothies at 50 pesos each, use the model to find

(i) the number of smoothies sold.

(ii) the total income from the smoothies sold. [2]

The cost of making each smoothie is 20 pesos. The profit per day (P) is the total income from the sale of smoothies that day minus the cost of making them.

(c) (i) Show that, according to the model, $P = \frac{40\,000}{x} - \frac{800\,000}{x^2}$.

(ii) Find $\frac{\mathrm{d}P}{\mathrm{d}x}$.

(iii) Find the value of x for which $\frac{\mathrm{d}P}{\mathrm{d}x} = 0$.

(iv) Find the number of smoothies sold when the profit is maximized. [9]

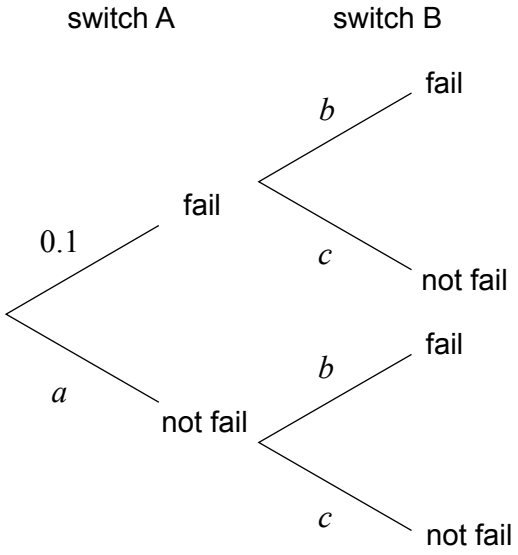
4. [Maximum mark: 12]

A type of generator will only function if a particular switch is working. The generator has a main switch, A, and a 'back up' switch, B.

The manufacturer claims the probability of switch A failing within one month of being fitted is 0.1 and the probability of the cheaper switch B failing within one month is 0.3. Whether or not a switch fails is independent of the state of the other switch.

If both switches fail, the generator needs to shut down to replace the switches. Both switches are replaced after a month of use (whether they have failed or not) or whenever the generator needs to be shut down.

The following tree diagram shows the probabilities of a switch failing within one month of them both being replaced, assuming the manufacturer's claim is correct.



(a) Write down the values of

(i) a.

(ii) b.

(iii) c. [2]

(b) Hence find the probability that the generator needs to shut down within one month of the switches being replaced. [1]

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(Question 4 continued)

The owner of the generator is suspicious of the switch manufacturer's claims, so they look back through the past 200 occasions when the switches were replaced. The records show whether no switches, one switch or two switches had failed.

The data the owner collected are shown in the following table.

No switch fails	One switch fails	Two switches fail
118	72	10

(c) Show that the expected value of no switches failing in the generator, during the last 200 occasions when the switches were replaced, is 126. [2]

(d) Perform a χ^2 goodness of fit test at the 5% significance level to test whether the manufacturer's claims are correct using the following hypotheses.

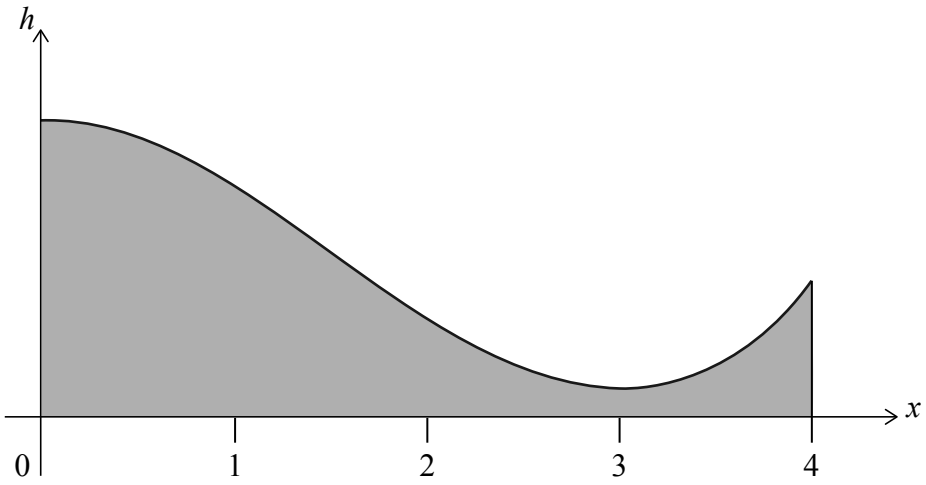
H_0: The manufacturer's claims are correct.
H_1: The manufacturer's claims are not both correct. [7]

5. [Maximum mark: 21]

Andrew designs a fibreglass water slide for a water park.

Let x be the horizontal distance, in metres, from the start of the water slide.
Let h be the height, in metres, of the water slide above the horizontal ground.

The following diagram shows a cross section of the water slide. The water slide will be supported by a concrete structure, represented by the shaded region in the diagram.



It is known that $h'(x) = 0.6x^2 - 1.8x$.

(a) Find the gradient of the water slide when $x = 2$. [2]

At the exit point of the water slide, $x = 4$ and $h = 1.4$.

(b) Find an expression for $h(x)$. [4]

(c) (i) Write down an integral that can be used to find the area of the shaded region.

(ii) Find the cross-sectional area of the concrete structure. [4]

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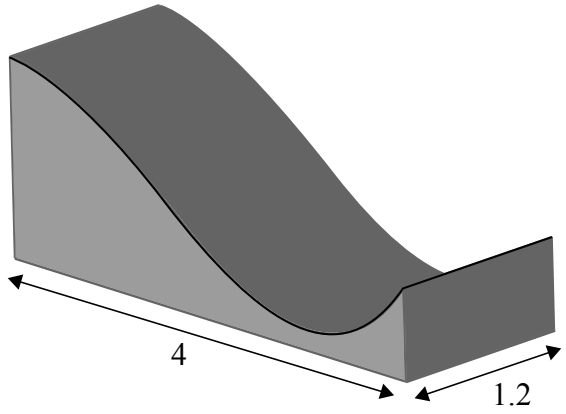
(Question 5 continued)

The water slide and concrete structure have a uniform cross section and a width of 1.2 metres, as shown in the following diagram.

diagram not to scale



(d) Find the volume of the concrete structure. [1]

To comply with safety regulations, Andrew updates this design. The start of the design will stay the same. Then, over the domain $2 \le x \le 4$, the updated design must satisfy the following conditions.

$$h(x) = ax^2 + bx + c \qquad h(4) = 1 \qquad h'(2) = -1.2 \qquad h'(3) = 0$$

(e) Write down **three** equations, in terms of a, b and c, using the given conditions. [6]

(f) Hence find the values of a, b and c. [2]

Andrew claims the updated design will still provide a smooth path for a person using the slide.

(g) Explain why Andrew's claim is correct at the point where $x = 2$. [2]