

Markscheme

May 2025

Mathematics: applications and interpretation

Higher level

Paper 1



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Instructions to Examiners

Abbreviations

- **M** Marks awarded for attempting to use a correct **Method**.
- **A** Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding **M** marks.
- **R** Marks awarded for clear **Reasoning**.
- **AG** Answer given in the question and so no marks are awarded.
- **FT** Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg M1, A2.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award MO followed by A1, as A mark(s) depend on the preceding M mark(s), if any.
- Where **M** and **A** marks are noted on the same line, e.g. **M1A1**, this usually means **M1** for an **attempt** to use an appropriate method (e.g. substitution into a formula) and **A1** for using the **correct** values.
- Where there are two or more **A** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award **A0A1A1**.
- Where the markscheme specifies A3, M2 etc., do not split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the **AG** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this
 working is incorrect and/or suggests a misunderstanding of the question. This will encourage a
 uniform approach to marking, with less examiner discretion. Although some candidates may be
 advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere
 too.
- An exception to the previous rule is when an incorrect answer from further working is used in a subsequent part. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award *FT* marks as appropriate but do not award the final *A1* in the first part.

Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	8√2	5.65685 (incorrect decimal value)	No. Last part in question.	Award A1 for the final mark (condone the incorrect further working)
2.	$\frac{35}{72}$	0.468111 (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award A0 for the final mark (and full FT is available in subsequent parts)

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3 Implied marks

Implied marks appear in **brackets e.g.** (M1), and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (*FT*) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award *FT* marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then *FT* marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is **(M1)A1**, it is possible to award full marks for *their* correct answer, **without working being seen**. For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further **A** marks can be awarded for work which uses the error, but **M** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer *FT* marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (e.g. probability greater than 1, $\sin \theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.
- If the candidate's answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any *FT* marks in the subsequent parts. This includes when candidates fail to complete a "show that" question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these *FT* rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was "Hence".

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (*MR*). A candidate should be penalized only once for a particular misread. Use the *MR* stamp to indicate that this has been a misread and do not award the first mark, even if this is an *M* mark, but award all others as appropriate.

- If the question becomes much simpler because of the **MR**, then use discretion to award fewer marks.
- If the *MR* leads to an inappropriate value (*e.g.* probability greater than 1, $\sin \theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- **MR** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by METHOD 1, METHOD 2, etc.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, M marks and intermediate
 A marks can be scored, when presented using calculator notation, provided the evidence clearly
 reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

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8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "from the use of 3 sf values".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an $\bf A$ mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An

exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$.

However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so x(x+1) and $x^2 + x$ are both acceptable.

Please note: intermediate **A** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10 Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) attempt to calculate area of a circle using 4 or 2.8

M1

recognition of multiplication by
$$\frac{1}{5}$$
 or $\frac{72}{360}$

M1

$$A = \frac{1}{5} \left(\pi (4)^2 - \pi (2.8)^2 \right)$$

$$5.13 \ (m^2) \ (5.12707... \ (m^2), \ \frac{204\pi}{125} \ (m^2), \ 1.632\pi \ (m^2))$$

A1

5.12707...<6 OR Maureen has enough

AG

Note: Do not award the final *A1* for the area unless a comparison to 6 is shown **OR** a statement of having enough is seen.

[3 marks]

(b)
$$(C =) 2\pi(4)$$
 (25.1327...)

(A1)

recognition to multiply by
$$\frac{3}{5}$$
 or $\frac{216}{360}$

(M1)

$$\frac{3}{5} \times 2\pi(4) + 2 \times 4$$

23.1 (m)
$$(23.0796...(m), \frac{24\pi}{5} + 8)$$

A1

[3 marks]

[Total: 6 marks]

2. (a) $d = 5.75 \left(d = \frac{23}{4} \right)$

A1A1

Note: Award A1 for 5.75 seen, A1 for completely correct equation.

[2 marks]

(b) attempt to substitute into the axis of symmetry formula using *their* 5.75 **OR** attempt to substitute the coordinates (3, 5.1) or (8.5, 5.1) into the function

(M1)

$$5.75 = \frac{-b}{2 \times (-0.2)}$$
 OR $5.1 = -0.2(3)^2 + 3b$ OR $5.1 = -0.2(8.5)^2 + 8.5b$

(b =) 2.3

A1

(M1)

[2 marks]

(c) indication of finding zero or attempt to use symmetry $h(d) = 0 \quad \text{OR} \quad 0 = -0.2d^2 + 2.3d \quad \text{OR} \quad 2 \times 5.75 \quad \text{OR} \quad 3 - 0 = d - 8.5 \quad \text{OR}$ resketch of graph with zero indicated

A1

$$(d=) 11.5 (m)$$

Note: Award *A0* for a FT answer that is less than 8.5 (contradicts the given information).

[2 marks]

(d) **EITHER**

the dolphin is below the surface of the water / the dolphin is underwater / the dolphin is in the water

R1

OR

the value of h cannot be negative since the definition of the model only applies to the jump

R1

[1 mark]

[Total: 7 marks]

3. (a) (i)
$$N = 72$$

 $I\% = 7.15$
 $PV = \pm 42695$
 $FV = 0$
 $P/Y = 12$

C/Y=12

(M1)(A1)

Note: Award *(M1)* for attempt to use finance app in GDC and *(A1)* for the first five entries correct.

(PMT =) 730.99 (USD)

A1

Note: Do not award final *A1* if answer is not given to 2 d.p. or if given as a negative value.

(ii) attempt to find 80% of 42695

(M1)

$$N = 72$$

 $I\% = 6.40$
 $PV = \pm 34156$
 $FV = 0$
 $P/Y = 12$
 $C/Y = 12$

(PMT =) 572.54 (USD)

A1

(A1)

Note: Do not award final *A1* if answer is not given to 2 d.p. or if given as a negative value.

[6 marks]

(b) (i) does not have (enough) money for initial payment / does not want to make initial payment

R1

(ii) lower monthly payments / will pay less over the life of the loan / less interest will be paid over life of loan

R1 [2 marks]

[Total 8 marks]

4. (a) 15 p

[1 mark]

A1

(b) (p-5) or (n-15) seen

A1

adding a cost expression (multiplying a p and an n) to their 15p

(M1)

$$C(n) = 15p + (p-5)(n-15)$$
 OR $C(n) = np - 5n + 75$ (or equivalent)

A1 [3 marks]

(c) (i) $(25.73 \times 100 =) 2573$ (EUR)

A1

Note: Answer must be exact.

(ii) correctly substituting *their* total cost and n = 100 into *their* cost function 2573 = 15 p + (p-5)(100-15)

(A1)

$$(p =) 29.98 \text{ (EUR)}$$

A1

Note: Answer must be written to 2 dp.

[3 marks]

[Total: 7 marks]

5. (a) (i) 170° (A1)

attempt to substitute into the cosine rule formula

$$(DC^2) = 2^2 + 13^2 - 2(2)(13)\cos(170^\circ)$$
(A1)

$$(DC =) 15.0 (14.9736...) (km)$$

Note: Do not accept a 2sf answer of 15. Award **MOAOAO** for a 2sf answer of 15 with no working shown OR 2+13=15.

(ii) correct substitution into sine rule or cosine rule

(A1)

(M1)

$$\frac{\sin B\hat{D}C}{13} = \frac{\sin 170^{\circ}}{14.9736...} \quad \text{OR} \quad \cos(B\hat{D}C) = \frac{14.9736...^{2} + 2^{2} - 13^{2}}{2 \times 14.9736... \times 2}$$

$$(\hat{BDC} =) 8.67^{\circ} (8.67097...^{\circ})$$
 (A1)

(bearing =)
$$098.7^{\circ}$$
 ($098.6709...^{\circ}$)

[7 marks]

(b) (distance between points D and E = 13 (km)

A1

points $B,\,C,\,D,$ and E make a parallelogram $\mbox{\bf OR}$ DB and EC are parallel $\mbox{\bf and}$ equal in length

R1

Note: It is possible to award A1R0 for this question part.

[2 marks] [Total 9 marks]

6.	(a)	(i)	f(x) = 0.7x OR $f(x) = x - 0.3x$	A1
		(ii)	g(x) = x - 150	A1 [2 marks]
	(b)	atten	npt to discuss the order in which the reduction/discount are applied	(M1)
$(f \circ g)(x)$ describes when the 15 the 30% discount.			g(x) describes when the 150 CAD reduction is applied before 30% discount.	A1 [2 marks]
	(c)	EITF evide	IER ence of solving $(f \circ g)(x)$ and $(g \circ f)(x)$ for a price at 450 CAD	R1
		OR (<i>f</i> ∘	$g(x) = 0.7x - 105$ and $(g \circ f)(x) = 0.7x - 150$ seen	R1
		OR statir	ng that it is better for 30% to be deducted from a higher price	R1
		OR the r	numbers 165, 210 are seen	R1
		THE	N	

Note: Do not award ROA1.

 $(g \circ f)(x)$ is better for the customer

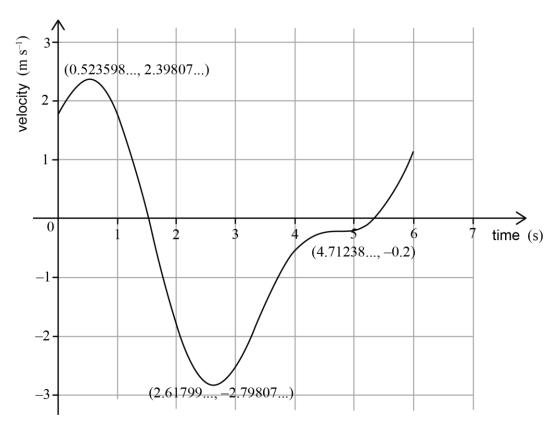
[2 marks] [Total 6 marks]

A1

7. (a) (i) correct general shape curve begins at (0, 1.8) and does not extend beyond t = 6

A1 A1

Note: Do not award the second *A1* if the first *A1* is not awarded.



(ii) (0.524, 2.40) **A1**

(2.62, -2.80)

(4.71, -0.2)

Note: Award **A1** for each correct coordinate. Award marks even if the labels are not shown on the graph near the points.

[5 marks]

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(b) (i) **EITHER**

$$\int_{0}^{6} |2\cos t + \sin 2t - 0.2| dt$$

A1A1

Note: Award A1 for correct boundaries and use of dt and A1 for completely correct expression.

OR

$$\left| \int_{0}^{1.52074...} (2\cos t + \sin 2t - 0.2) dt \right| + \left| \int_{1.52074...}^{5.31535...} (2\cos t + \sin 2t - 0.2) dt \right| + \left| \int_{5.31535...}^{6} (2\cos t + \sin 2t - 0.2) dt \right|$$

A1A1

Note: Award *A1* for 1.52074... and 5.31535... as boundaries and *A1* for completely correct expression.

Allow + - + instead of modulus signs

(ii) 7.76 (7.76487...) (m)

A1

[3 marks]

[Total 8 marks]

8. (a) f(x) = 0 seen **OR** sketch of the graph with all three *x*-intercepts marked **OR** evidence of using sum or product of roots

(M1)

(a =) 3

A1 [2 marks]

(b) evidence of using an integral to find the area

(M1)

$$\int f(x), \int_1^3$$
, etc.

A1

(area =) 4

[2 marks]

(c) attempt to equate two integrals

(M1)

$$\int_{1}^{b} f(x) dx = \int_{1}^{b} g(x) dx$$

evidence of using their GDC to solve the equation \mathbf{OR} attempt to expand both sides of the equation into expressions in terms of b (M1)

$$(b=)$$
 2.57 $(2.56949...)$

A1

[3 marks]

[Total 7 marks]

9. (a) Evidence of use of normal cdf with 45 **OR** sketch of bell curve with area above 45 labelled.

(M1)

0.0338556... (A1)

recognition of binomial distribution with probability of their 0.0338556...

(M1)

 $X \sim B(180, 0.0338556...)$

 $P(X \ge 10)$ **OR** $1-P(X \le 9)$

0.0869 (0.0869379...)

A1

[4 marks]

(b) defining event as being late and lateness marked correctly

 $0.0338556...\times(1-0.35) = 0.0220061...$

(A1)

(M1)

Y is the number of occasions of being marked late

 $Y \sim B (180, 0.0220061)$ AND $P(Y \ge 4)$ OR P(Y > 3) OR $1 - P(Y \le 3)$

0.561 (0.561005...) **A1**

[3 marks]

[Total 7 marks]

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10. (a) correct use of one log law x^{-2} **OR** x^2 **OR** (2-x)(-2-x)

$$(g(x) =) \frac{x^2 - 4}{x^2}$$
 OR $\frac{(2 - x)(-2 - x)}{x^2}$ OR $1 - 4x^{-2}$ OR equivalent

[2 marks]

(b)
$$f^{-1}(x) < -2$$
 OR $(-\infty, -2)$

Note: Award **A1** for strictness with -2 using either < or parenthesis and **A2** for completely correct inequality. Award **A2** for y < -2.

[2 marks]

(c) attempt to exchange variables (seen anywhere) $x = \ln\left(\frac{y^2 - 4}{y^2}\right)$ **OR** $x = \ln\left(1 - 4y^{-2}\right)$

$$e^x = \frac{y^2 - 4}{y^2}$$
 OR $e^x = 1 - 4y^{-2}$ (A1)

$$y^2(e^x-1)=-4$$
 (A1)

$$(y =) \pm \frac{2}{\sqrt{1 - e^x}}$$

(since domain of f(x) is x < -2) a = -2

A1

[5 marks]

[Total 9 marks]

11. (a) (i) $(u_2 =) 6 + 6i$

(ii) $(u_3 =) 8i$

[2 marks]

A1

(b) attempt to find |r|

$$|r| = \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2}$$

$$|r| = \frac{2\sqrt{2}}{3}$$

statement that multiplying a number by a complex number changes the modulus of the number by a factor of |r| (may be seen at any stage)

Note: Award at most *(M1)A1A0R1* if common ratio is given as 0.942809......

therefore it is a geometric sequence

AG [4 marks]

R1

(c) substitution of their r and 9 into the sum formula for infinite sequence $S_{\infty} = \frac{9}{1 - \frac{2\sqrt{2}}{3}}$

$$(S_{\infty} =) 157 (157.367..., 81 + 54\sqrt{2})$$

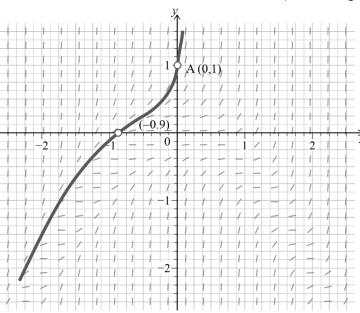
Note: No **FT** if their $|r| \ge 1$.

[2 marks] [Total 8 marks] **12.** (a) correct shape that goes through point (0, 1)

A1

crosses the *x*-axis is at $x = -0.9 (\pm 1 \text{ small grid box})$

A1



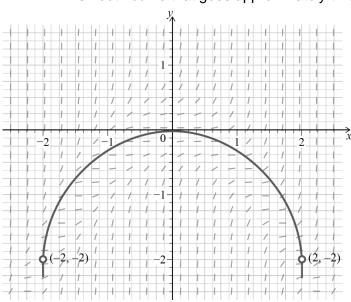
[2 marks]

(b) correct shape with maximum at (0, 0)

A1

smooth curve that goes approximately through points (-2, -2) and (2, -2)

A1



[2 marks]

(c) **EITHER**

substituting
$$y = ax^2$$
 into differential equation (M1)
$$0 = x^4 + 4x^2(ax^2) + 4(ax^2)^2$$

choosing an appropriate method to solve for
$$a$$
 (M1) $0 = x^4 (1 + 4a + 4a^2)$

$$a = -\frac{1}{2}$$

ΩR

$$x^4 + 4x^2y + 4y^2 = (x^2 + 2y)^2$$
(M1)

$$(x^2 + 2y)^2 = 0 (M1)$$

$$y = -\frac{1}{2}x^2$$
 (so $a = -\frac{1}{2}$)

[3 marks]

[Total 7 marks]

13. (a) an anticlockwise/counter-clockwise rotation of $\frac{\pi}{3}$ (about the origin)

A1A1

Note: Award A1 for rotation and A1 for correct angle/direction detail.

[2 marks]

(b) attempt to multiply A and B in correct order

(M1)

$$\mathbf{BA} = \begin{bmatrix} -0.8 & 0.6 \\ 0.6 & 0.8 \end{bmatrix} \begin{bmatrix} \cos\frac{\pi}{3} & -\sin\frac{\pi}{3} \\ \sin\frac{\pi}{3} & \cos\frac{\pi}{3} \end{bmatrix} \begin{bmatrix} -0.8 & 0.6 \\ 0.6 & 0.8 \end{bmatrix} \begin{bmatrix} 0.5 & -0.866025... \\ 0.866025... & 0.5 \end{bmatrix}$$

$$\begin{bmatrix} 0.119615... & 0.992820... \\ 0.992820... & -0.119615... \end{bmatrix}$$
 (A1)

attempt to create an expression with either
$$\tan 2\theta$$
 or $\sin 2\theta$ or $\cos 2\theta$ (M1) $\tan 2\theta = \frac{0.992820...}{0.119615...}$

$$\theta = 0.725446...$$
 (A1)

$$y = \tan(0.725446...)x$$

$$a = 0.887 \ (0.886751...)$$

Note: Award at most *(M1)(A1)(M1)(A1)A0* if the extraneous value of *a* is given alongside a correct answer.

[5 marks]

[Total 7 marks]

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14. (a)
$$L_1(x) = \text{Im} \left(14.8 e^{0.35xi} e^{1.27i} \right)$$

$$L_2(x) = \text{Im}\left(18.5e^{0.35xi}e^{-2.36i}\right)$$

[2 marks]

A1

(b)
$$L_3 = \text{Im} \left(14.8 e^{0.35xi} e^{1.27i} + 18.5 e^{0.35xi} e^{-2.36i} \right)$$

$$L_3 = \text{Im}\left(e^{0.35xi}\left(14.8e^{1.27i} + 18.5e^{-2.36i}\right)\right)$$
 (A1)

$$L_3 = \text{Im}\left(e^{0.35xi}\left(8.81559...e^{3.01604...i}\right)\right)$$
 (A1)

$$L_3 = 8.81559...\sin(0.35x + 3.01604...)$$

 $q = 8.82 (8.81559...), r = 3.02 (3.01604...)$

A1A1

Note: Award both marks if last line missing, but values are seen in equation for $L_{\rm 3}$.

[4 marks]

[Total 6 marks]

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15. (a) (i)
$$x_{n+1} = x_n + 0.1 \left(\frac{dx}{dt}\right)_n$$

$$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)_{n+1} = \left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)_n + 0.1 \left(\frac{\mathrm{d}^2x}{\mathrm{d}t^2}\right)_n \tag{M1}$$

$$\frac{d^2x}{dt^2} = 4.9 - 1.2 \frac{dx}{dt} - 4x \tag{A1}$$

$$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)_{n+1} = \left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)_n + 0.1\left(4.9 - 1.2\frac{\mathrm{d}x}{\mathrm{d}t} - 4x\right)_n \tag{A1}$$

(when
$$t = 1$$
 or $n = 10$) $x = 4.36$ (4.36332...) (m)

(ii)
$$(t=)$$
 2 (seconds)

Note: Award **A0** for either n = 20 or t = 20.

[5 marks]

(b)
$$x = 3.845$$
 seen **OR** $x = 3.855$ seen **(A1)**

substitution into percentage error formula using their value for part (a)(i) and 3.845 or 3.855 or 3.85

$$\left| \frac{4.36332...-3.845}{3.845} \right| \times 100$$

Note: Accept methods using centimetres instead of metres as long as both values use the same unit.

the same unit.

13.5 (13.4803...)(%)

[3 marks]

A1

[Total 8 marks]