Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

15 May 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

2225 – 7306

25 pages

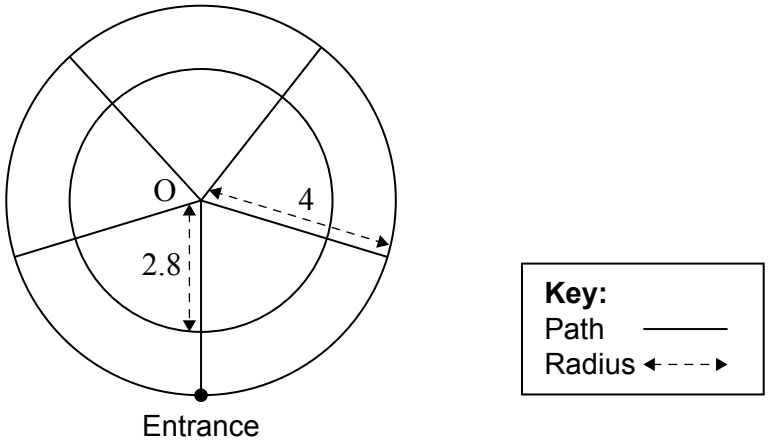
Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 6]

A child's game is played by making paths in the snow. First, two circular paths are made using the same centre, O. The radius of the smaller circle is $2.8\,\mathrm{m}$, and the radius of the larger circle is $4\,\mathrm{m}$. Additional paths are then made from O to the outer edge of the larger circle, dividing each circle into 5 equal sectors, as shown in the following diagram.

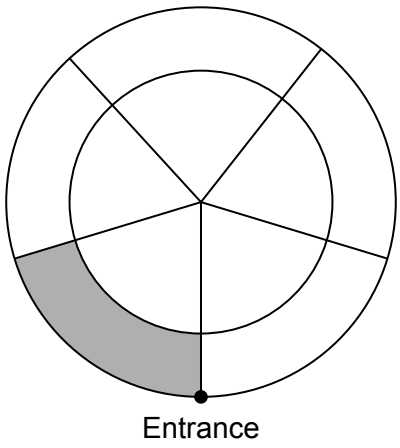
For your calculations, ignore the widths of the paths.

diagram not to scale



Each region **between** paths will be coloured with environmentally friendly dye. The shaded region below will be coloured orange.

diagram not to scale



Maureen has enough orange dye to cover an area of $6\,\mathrm{m}^2$.

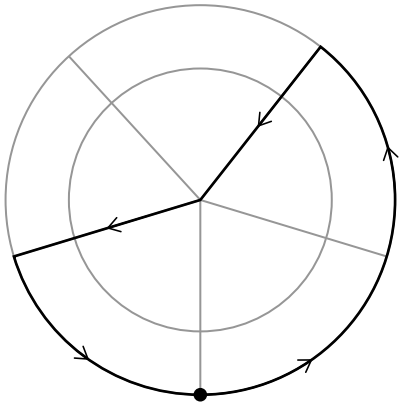
(This question continues on the following page)

(Question 1 continued)

(a) Show that Maureen has enough orange dye to cover the shaded region. [3]

During the game, the players start at the entrance and must travel only along the paths made in the snow. Maureen travels from the entrance along the path shown in the following diagram.

diagram not to scale



Entrance

(b) Calculate the distance Maureen travels along this path, starting from the entrance and returning to the entrance. [3]

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2. [Maximum mark: 7]

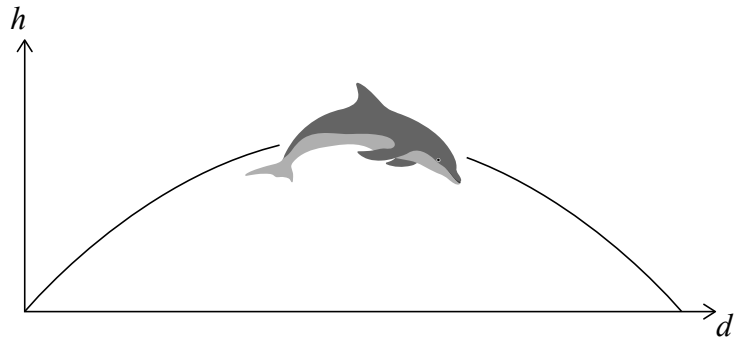
A dolphin jumps out of water. The path of its jump can be modelled by a quadratic function using the following variables:

d is the horizontal distance from the point where the dolphin leaves the water

h is the height of the dolphin above the surface of the water.

All distances are measured in metres, from the point where the dolphin leaves the water.

diagram not to scale



On a particular jump, the dolphin first reaches a height of $5.1\,\text{m}$ above the surface of the water when the horizontal distance is $3\,\text{m}$ and again when the horizontal distance is $8.5\,\text{m}$.

(a) Write down the equation of the axis of symmetry of the quadratic function. [2]

The quadratic function that models the dolphin's jump is $h(d) = -0.2d^2 + bd$, where $\text{d} > 0$ and b is a constant.

(b) Calculate the value of b. [2]

(c) Find the horizontal distance when the dolphin re-enters the water. [2]

(d) Explain what a negative h-value would mean in this context. [1]

(This question continues on the following page)

(Question 2 continued)

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3. [Maximum mark: 8]

Raul wants to buy a new truck for $42\,695$ US dollars (USD). Raul is offered two different loan options to pay for the truck. For both options, Raul will pay back the loan over a period of 6 years.

The two options are shown in the following table.

Loan option	**Size of loan (% purchase price)**	**Nominal annual interest rate (compounded monthly)**
A	100% of purchase price (after no initial payment)	7.15%
B	80% of purchase price (after a 20% initial payment from Raul)	6.40%

For both loan options, Raul will make monthly payments, at the end of each month, until the loan is completely paid off.

(a) Calculate the amount of Raul's monthly payment, to two decimal places, if he chooses

(i) option A.

(ii) option B. [6]

(b) State a reason why Raul might choose

(i) option A.

(ii) option B. [2]

(This question continues on the following page)

(Question 3 continued)

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4. [Maximum mark: 7]

A manufacturer designs hats that businesses can order.

For an order of 15 hats or fewer, the price per hat is p euros (EUR). For larger orders, the price of **each** hat after the first 15 ordered is reduced by $5\,\text{EUR}$.

$C(n)$ represents the total cost of purchasing n hats from the manufacturer.

(a) Write down an expression in terms of p for the total cost of ordering 15 hats. [1]

(b) Write down an equation for $C(n)$, in terms of p and n, for the total cost of ordering n hats when $n > 15$. [3]

A company decides to order 100 hats. The mean price per hat for this order is $25.73\,\text{EUR}$.

(c) (i) Calculate the exact total cost of the order.

(ii) Hence, determine the value of p, correct to 2 decimal places. [3]

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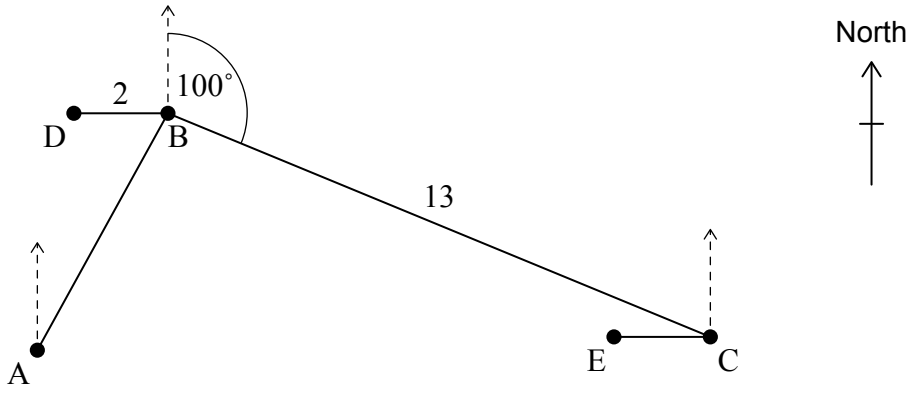
5. [Maximum mark: 9]

Ticky is sailing a boat in the ocean. For the first part of her trip, she plans to start at point A and travel to point B. She then plans to turn and travel $13\,\text{km}$ on a bearing of $100°$ to point C as shown in the following diagram.

diagram not to scale



Due to navigational error in the first part of the trip, Ticky arrives at point D instead of point B. Point D is $2\,\text{km}$ due west of point B.

(a) Determine

(i) the distance from point D to point C.

(ii) the bearing Ticky must use to travel directly from point D to point C. [7]

Due to navigational error in the second part of the trip, Ticky arrives at point E instead of point C. Point E is $2\,\text{km}$ due west of point C.

(b) Write down the distance between point D and point E. Justify your answer. [2]

(This question continues on the following page)

(Question 5 continued)

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6. [Maximum mark: 6]

Walkea Tree is a furniture store. They offer a 30% discount on all chairs priced over 250 Canadian dollars (CAD).

A different furniture store, LuxeCraft, offer customers a reduction of $150\,\text{CAD}$ on all chairs priced over $250\,\text{CAD}$.

(a) Given that x is the original price of a chair and $x > 250$,

(i) write down a function $f(x)$ that models the final price of a chair at Walkea Tree, after the discount is applied.

(ii) write down a function $g(x)$ that models the final price of a chair at LuxeCraft, after the reduction is applied. [2]

A third store, Opulent Furnishings, decides to offer both a 30% discount **and** a reduction of $150\,\text{CAD}$ on all their chairs priced over $250\,\text{CAD}$.

(b) Describe the meaning of $(f \circ g)(x)$, in context. [2]

(c) State, with justification, whether $(f \circ g)(x)$ or $(g \circ f)(x)$ is better for the **customer** when they buy a chair with an original price of $450\,\text{CAD}$. [2]

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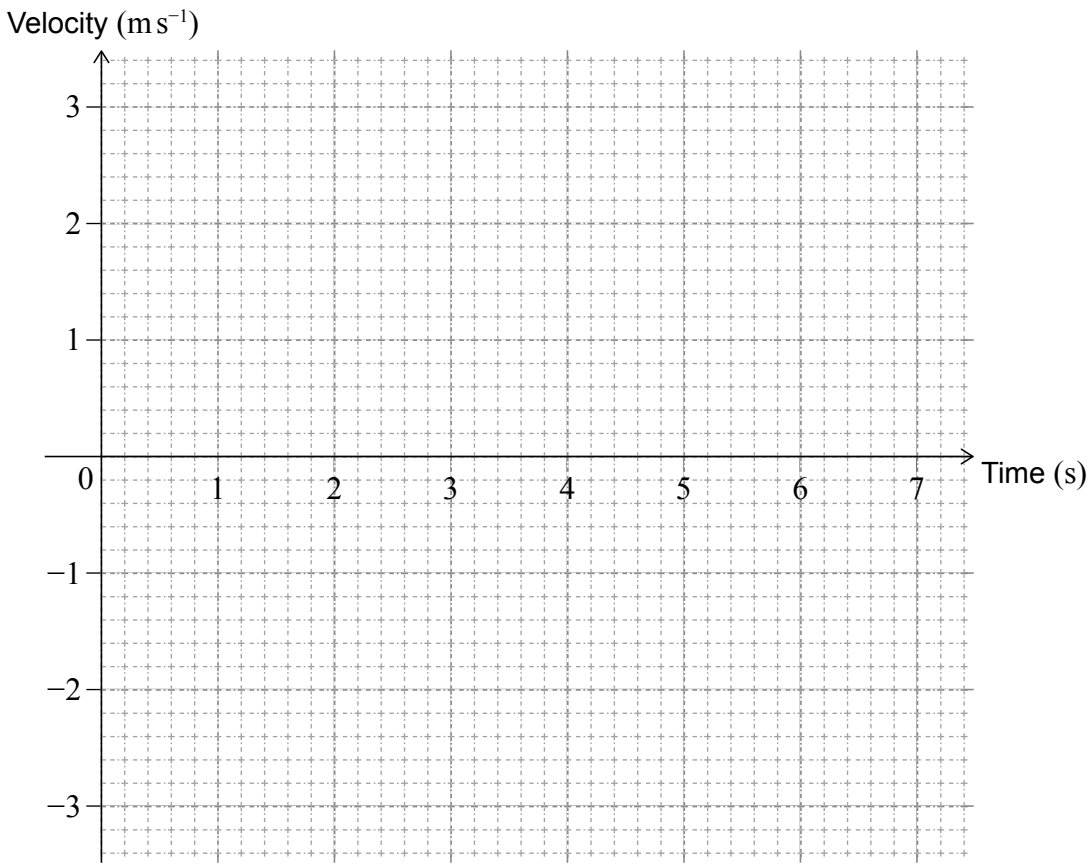
7. [Maximum mark: 8]

Conrad is investigating the motion of a particle. The velocity of the particle, in $\mathrm{m\,s^{-1}}$, is given by $v(t) = 2\cos t + \sin 2t - 0.2$ where t is the time, in seconds, after the investigation begins.



(a) On the axes,

(i) sketch the graph of $v(t)$ for $0 \le t \le 6$.

(ii) label, with coordinates, the point(s) where the particle has zero acceleration. [5]

(b) (i) Write down an integral expression for the distance the particle travels during the first 6 seconds of the investigation.

(ii) Hence, find the distance the particle travels during the first 6 seconds of the investigation. [3]

(This question continues on the following page)

(Question 7 continued)

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8. [Maximum mark: 7]

Consider the function $f(x) = x^3 - 9x^2 + 23x - 15$ for $0 \le x \le 5$.

The graph of $f(x)$ crosses the x-axis when $x = 1$, $x = 5$ and $x = a$.

(a) Find the value of a. [2]

(b) Use your graphic display calculator to find the area enclosed by the curve $y = f(x)$ and the x-axis, when $1 \le x \le a$. [2]

Another function is defined as $g(x) = -x^2 + 6x - 5$.

The area enclosed by the curve $y = f(x)$ and the x-axis, when $1 \le x \le b$, is equal to the area enclosed by the curve $y = g(x)$ and the x-axis, when $1 \le x \le b$.

(c) Find the value of b, where $1 < b < 3$. [3]

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9. [Maximum mark: 7]

In a year, Louis attends school for a total of 180 school days. Louis leaves his house at exactly $7{:}45\,\text{am}$ every day and takes public transportation from his house to school.

His travel times to school are normally distributed with a mean of 35.5 minutes and a standard deviation of 5.2 minutes. The travel times are independent of each other.

He is late if he arrives at school after $8{:}30\,\text{am}$.

(a) Find the probability that Louis is late on at least 10 days in the year. [4]

Mrs. Beale is responsible for recording students who arrive late. However, Mrs. Beale does not always accurately mark lateness.

On days when Louis is late, there is a $35\,\%$ chance that Mrs. Beale will still mark him as being on time. Mrs. Beale's accuracy for one school day is independent of her accuracy for any other school day.

Louis will receive a detention if he is marked late to school at least 4 times during the year.

(b) Find the probability of Louis receiving a detention in the year. [3]

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10. [Maximum mark: 9]

Consider the function $f(x) = \ln(2 - x) + \ln(-2 - x) - 2\ln(-x)$, where $x < -2$.

The function $f(x)$ can be rewritten in the form $\ln(g(x))$.

(a) Find an expression for $g(x)$. [2]

(b) Write down the range of $f^{-1}(x)$. [2]

The function $f^{-1}(x)$ can be written in the form $\frac{a}{\sqrt{1 - e^x}}$.

(c) Find the value of a. [5]

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11. [Maximum mark: 8]

Nanthana is completing an exploration on multiplication using complex numbers. She considers a geometric sequence where $u_1 = 9$ and $r = \frac{2}{3} + \frac{2}{3}\mathrm{i}$.

(a) Write down the value of

(i) u_2.

(ii) u_3. [2]

Nanthana claims that the sequence $|u_1|, |u_2|, |u_3|, \ldots$ also forms a geometric sequence.

(b) Show that Nanthana's claim is correct, stating the exact value of the common ratio for this sequence. [4]

(c) Hence, find the sum of the infinite sequence $|u_1|, |u_2|, |u_3|, \ldots$. [2]

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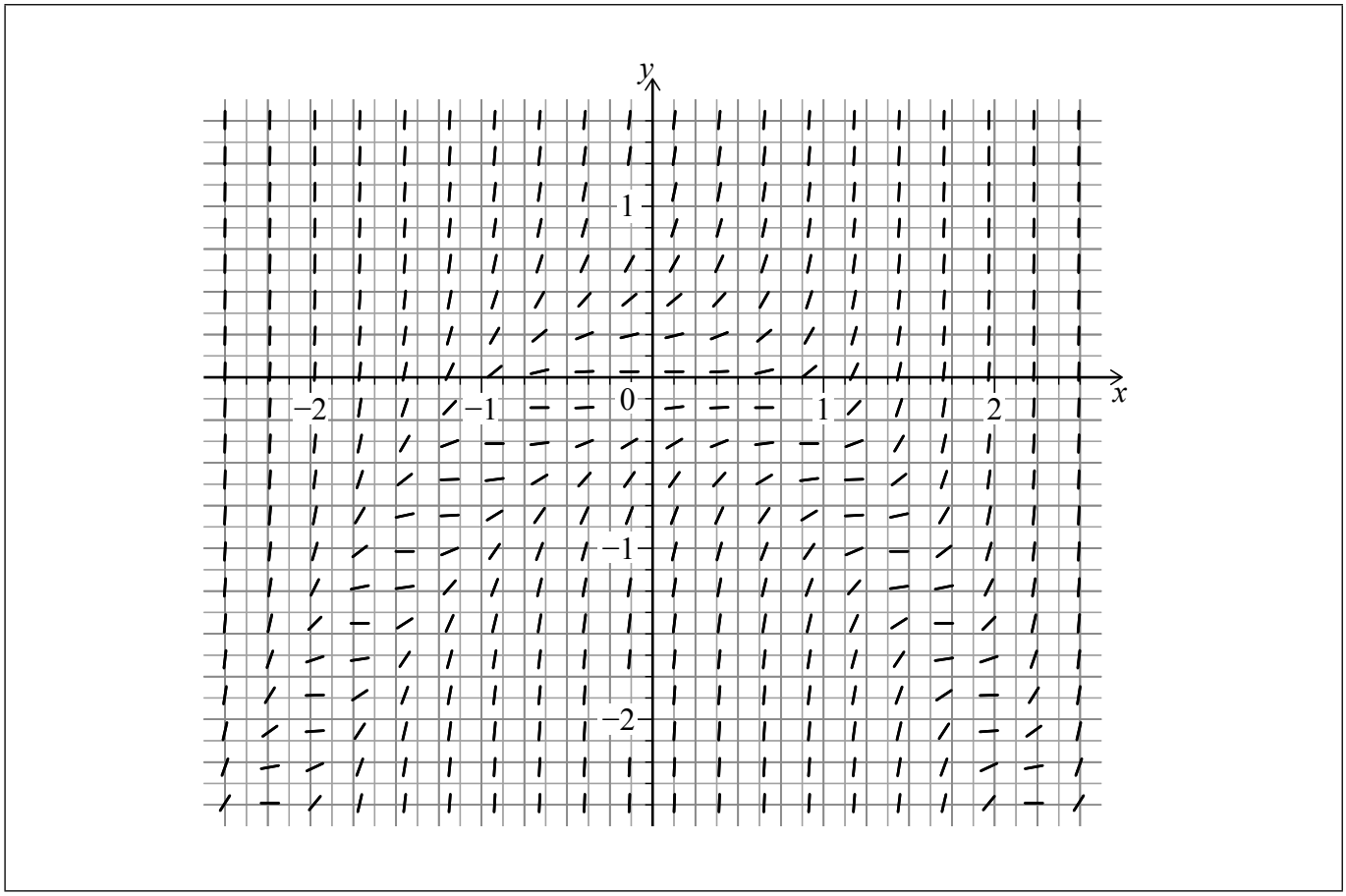
12. [Maximum mark: 7]

The following graph shows the slope field for the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^4 + 4x^2y + 4y^2.$$



(a) On the slope field, sketch the solution to the differential equation that passes through the point $(0, 1)$. [2]

Some of the solutions to the differential equation have a point at which the gradient is zero. The curve on which all these points lie is given by the equation $y = ax^2$, where a is a constant to be found.

(b) Sketch this curve on the slope field. [2]

(c) Use the differential equation to find the value of a. [3]

(This question continues on the following page)

(Question 12 continued)

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13. [Maximum mark: 7]

The points $\mathrm{R}(-2, 2)$, $\mathrm{S}(3, -5)$, and $\mathrm{T}(5, 1)$ are the vertices of a triangle. The triangle goes through a sequence of transformations in the following order:

I. a transformation using the matrix $\boldsymbol{A} = \begin{bmatrix} \cos\frac{\pi}{3} & -\sin\frac{\pi}{3} \\ \sin\frac{\pi}{3} & \cos\frac{\pi}{3} \end{bmatrix}$

II. a transformation using the matrix $\boldsymbol{B} = \begin{bmatrix} -0.8 & 0.6 \\ 0.6 & 0.8 \end{bmatrix}$.

(a) Describe the nature of the transformation represented by $\boldsymbol{A}$. [2]

Instead of using this sequence of transformations, the image of triangle RST can be found using only a reflection of triangle RST in line $y = ax$.

(b) Given that $a > 0$, find the value of a. [5]

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14. [Maximum mark: 6]

Consider the following functions:

$$L_1(x) = 14.8\sin(0.35x + 1.27)$$

$$L_2(x) = 18.5\sin(0.35x - 2.36)$$

(a) Write down both functions in the form $\mathrm{Im}(a\mathrm{e}^{bx\mathrm{i}}\,\mathrm{e}^{c\mathrm{i}})$. [2]

The sum of $L_1(x)$ and $L_2(x)$ can be expressed in the form $L_3(x) = q\sin(0.35x + r)$, where $0 \le r < 2\pi$.

(b) Use your answer to part (a) to find the value of q and of r. [4]

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15. [Maximum mark: 8]

Quentin is investigating the motion of a mass hanging on a large spring.

From research, Quentin thinks the displacement, x, in metres, of the mass at time t, in seconds, will satisfy the differential equation

$$9.8 = 2\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2.4\frac{\mathrm{d}x}{\mathrm{d}t} + 8x\,.$$

Quentin begins his investigation when the mass is at a displacement of 0.5 metres and is moving such that $\frac{\mathrm{d}x}{\mathrm{d}t} = 9.8$.

(a) By using Euler's method with $h = 0.1$ to estimate the motion of the mass after being released, find:

(i) the displacement of the mass after 1 second,

(ii) the first estimated time the mass reaches $x = 0$. [5]

Using technology, Quentin correctly measures the displacement of the mass, after 1 second. His software gives him a value of 385 centimetres, which is accurate to the nearest centimetre.

(b) Find the **maximum possible** percentage error between your estimate from part (a)(i) and the exact displacement after 1 second. [3]

(This question continues on the following page)

(Question 15 continued)

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References:

2. fitie, 2017. *Vector illustration of dolphins* [image online] Available at: https://www.gettyimages.co.uk/detail/illustration/vector-illustration-of-dolphins-royalty-free-illustration/638431854 [Accessed 19 April 2024] Source adapted.

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