

#### © International Baccalaureate Organization 2024

All rights reserved. No part of this product may be reproduced in any form or by any electronic or mechanical means, including information storage and retrieval systems, without the prior written permission from the IB. Additionally, the license tied with this product prohibits use of any selected files or extracts from this product. Use by third parties, including but not limited to publishers, private teachers, tutoring or study services, preparatory schools, vendors operating curriculum mapping services or teacher resource digital platforms and app developers, whether fee-covered or not, is prohibited and is a criminal offense.

More information on how to request written permission in the form of a license can be obtained from https://ibo.org/become-an-ib-school/ib-publishing/licensing/applying-for-a-license/.

#### © Organisation du Baccalauréat International 2024

Tous droits réservés. Aucune partie de ce produit ne peut être reproduite sous quelque forme ni par quelque moyen que ce soit, électronique ou mécanique, y compris des systèmes de stockage et de récupération d'informations, sans l'autorisation écrite préalable de l'IB. De plus, la licence associée à ce produit interdit toute utilisation de tout fichier ou extrait sélectionné dans ce produit. L'utilisation par des tiers, y compris, sans toutefois s'y limiter, des éditeurs, des professeurs particuliers, des services de tutorat ou d'aide aux études, des établissements de préparation à l'enseignement supérieur, des fournisseurs de services de planification des programmes d'études, des gestionnaires de plateformes pédagogiques en ligne, et des développeurs d'applications, moyennant paiement ou non, est interdite et constitue une infraction pénale.

Pour plus d'informations sur la procédure à suivre pour obtenir une autorisation écrite sous la forme d'une licence, rendez-vous à l'adresse https://ibo.org/become-an-ib-school/ib-publishing/licensing/applying-for-a-license/.

#### © Organización del Bachillerato Internacional, 2024

Todos los derechos reservados. No se podrá reproducir ninguna parte de este producto de ninguna forma ni por ningún medio electrónico o mecánico, incluidos los sistemas de almacenamiento y recuperación de información, sin la previa autorización por escrito del IB. Además, la licencia vinculada a este producto prohíbe el uso de todo archivo o fragmento seleccionado de este producto. El uso por parte de terceros —lo que incluye, a título enunciativo, editoriales, profesores particulares, servicios de apoyo académico o ayuda para el estudio, colegios preparatorios, desarrolladores de aplicaciones y entidades que presten servicios de planificación curricular u ofrezcan recursos para docentes mediante plataformas digitales—, ya sea incluido en tasas o no, está prohibido y constituye un delito.

En este enlace encontrará más información sobre cómo solicitar una autorización por escrito en forma de licencia: https://ibo.org/become-an-ib-school/ib-publishing/licensing/applying-for-a-license/.





# Mathematics: analysis and approaches Standard level Paper 2

2 May 2024

Zone A	morning	Zone B	morning	Zone C	morning
--------	---------	--------	---------	--------	---------

 (	Cano	didat	e se	ssio	n nu	mbe	r	

## 1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the mathematics: analysis and approaches SL formula booklet is required for this paper.
- The maximum mark for this examination paper is [80 marks].





2224-7110

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

#### **Section A**

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

con	inued	below	the line	s, if neces	ssary.							
1.	[Max	kimum	n mark: 6	]								
		study cular		oile phone	usage o	f a randoı	m sample	of ten stu	udents wa	ıs examin	ed on a	
	The	lengtl	n of time,	t hours,	that the t	en studer	nts used t	heir phon	es for are	listed be	low.	
	0	.7	1.2	1.9	4.0	4.4	4.5	4.9	5.7	6.5	11.7	
	(a)	For	these da	ta, find th	e							
		(i)	mediar	ı;								
		(ii)	interqu	artile ranç	ge.							[3]
	An c	utlier	is a valu	e that is le	ess than	$Q_1 - 1.5$	× IQR or	greater th	$an Q_3 + 1$	1.5 × IQR		
	(b)			1.7 is an o								[3]



2. [Maximum mark: 6]

The functions f and g are both defined for  $-1 \le x \le 0$  by

$$f(x) = 1 - x^2$$

$$g(x) = e^{2x}.$$

The graphs of f and g intersect at x = a and x = b, where a < b.

(a) Find the value of a and the value of b.

[3]

(b) Find the area of the region enclosed by the graphs of f and g.

[3]




[1]

### 3. [Maximum mark: 5]

Consider the following bivariate data set where p,  $q \in \mathbb{Z}^+$ .

x	5	6	6	8	10
y	9	13	p	q	21

The regression line of y on x has equation y = 2.1875x + 0.6875.

The regression line passes through the mean point  $(\overline{x}, \overline{y})$ .

(a) Given that 
$$\overline{x} = 7$$
, verify that  $\overline{y} = 16$ .

(b) Given that q - p = 3, find the value of p and the value of q. [4]

	 			 						-																				
				 																•					•				•	



4. jiviaxiiilulli illaik. o	4.	[Maximum	mark: 6	31
-----------------------------	----	----------	---------	----

The loudness of a sound, L, measured in decibels, is related to its intensity, I units, by  $L=10\log_{10}(I\times 10^{12})$ .

Consider two sounds,  $\,S_1^{}\,$  and  $\,S_2^{}.\,$ 

- $\boldsymbol{S}_{\scriptscriptstyle 1}$  has an intensity of  $\,10^{\text{--}6}$  units and a loudness of 60 decibels.
- $S_2$  has an intensity that is twice that of  $S_1$ .
- (a) State the intensity of  $S_2$ .

[1]

(b) Determine the loudness of  $S_2$ .

[2]

The maximum loudness of thunder in a thunderstorm was measured to be 115 decibels.

(c) Find the corresponding intensity, I, of the thunder.

[3]


<ol><li>[Maximum mark: 6]</li></ol>
-------------------------------------

A particle moves in a straight line such that its velocity,  $v \, \mathrm{m} \, \mathrm{s}^{-1}$ , at time t seconds is given by  $v(t) = 1 + \mathrm{e}^{-t} - \mathrm{e}^{-\sin 2t}$  for  $0 \le t \le 2$ .

- (a) Find the velocity of the particle at t = 2. [1]
- (b) Find the maximum velocity of the particle. [2]
- (c) Find the acceleration of the particle at the instant it changes direction. [3]




6.	[Maximum	mark:	5
٠.	liviaxiiiiaiii	man.	v

Consider a random variable X such that  $X \sim B(n, 0.25)$ .

Determine the least value of n such that  $P(X \ge 1) > 0.99$ .

 	 	 • •	 ٠.	 	 ٠.	٠.	٠.	•	 			 	٠.	٠.	٠.	٠.		٠.	٠.	٠.	٠.		٠.	• •
 	 	 	 	 	 				 			 	٠.	٠.			٠.	٠.						
 	 	 	 	 	 		٠.		 			 												
 	 • • •	 • •	 • •	 	 • •	•	• •	•	 • •	• •	• •	 	•	• •	•	• •	• •	•	•		•	•	• •	• •
 	 	 	 ٠.	 	 ٠.	٠.	٠.		 			 	٠.	٠.	٠.	٠.	٠.	٠.		٠.	٠.	٠.	٠.	
 	 	 	 	 	 		٠.		 			 	٠.								٠.			



-8-

[2]

Do **not** write solutions on this page.

#### **Section B**

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

#### **7.** [Maximum mark: 14]

A lake contains a type of fish called carp. The lengths,  $L\,\mathrm{cm}$ , of the carp can be modelled by a normal distribution with mean  $45.6\,\mathrm{cm}$  and standard deviation  $4.2\,\mathrm{cm}$ .

According to this model, carp with a length between  $41.4\,\mathrm{cm}$  and  $k\,\mathrm{cm}$  lie within one standard deviation of the mean.

- (a) Write down the value of k.
- (b) Find the probability that a randomly selected carp is greater than 48 cm in length. [2]
- (c) It is known that 99% of carp in the lake have a length greater than x cm. Find the value of x. [2]
- (d) Consider a random sample of 100 carp from the lake.
  - (i) Find the expected number of carp with lengths between  $40 \, \mathrm{cm}$  and  $56 \, \mathrm{cm}$ .
  - (ii) Find the probability that in this sample, exactly 95 carp have a length between 40 cm and 56 cm. [5]

A large sample of carp from the lake is studied. The length of each fish is measured and recorded correct to the nearest  $0.1\,\mathrm{cm}$ .

(e) Find the probability that a randomly selected carp has a length recorded as 45.6 cm. [3]



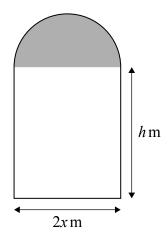
Do **not** write solutions on this page.

#### **8.** [Maximum mark: 15]

A window is designed in the shape of a semicircle attached to a rectangle.

The rectangular section of the window has dimensions 2x metres by h metres.

The window consisting of its two sections is shown in the following diagram.



Let the area of the window be A square metres.

(a) Write down an expression for A in terms of x and h. [2]

Let the perimeter of the window be P metres.

(b) Given that 
$$P = 10$$
, show that  $h = \frac{1}{2}(10 - 2x - \pi x)$ . [2]

The window is designed to let in the maximum amount of light.

The rectangular section of the window consists of clear glass and lets in three units of light per square metre.

The semicircular section of the window consists of tinted glass and lets in one unit of light per square metre.

- (c) Show that the amount of light, L units, let in by the window is given by  $L = 30x 6x^2 \frac{5}{2}\pi x^2.$  [4]
- (d) (i) Find an expression for  $\frac{dL}{dx}$ .
  - (ii) Find the value of x so that the window lets in the maximum amount of light. Justify that this value of x gives a maximum.
  - (iii) Find the value of h so that the window lets in the maximum amount of light. [7]



[3]

Do **not** write solutions on this page.

#### 9. [Maximum mark: 17]

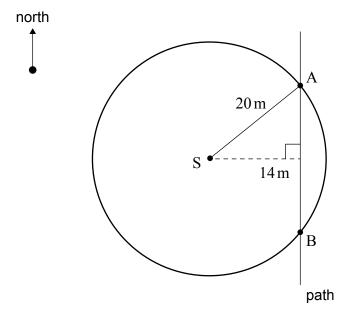
A rotating sprinkler is at a fixed point S.

It waters all points inside and on a circle of radius 20 metres.

Point S is 14 metres from the edge of a path which runs in a north-south direction.

The edge of the path intersects the circle at points A and B.

This information is shown in the following diagram.



(a) Show that AB = 28.57, correct to four significant figures.

The sprinkler rotates at a constant rate of one revolution every 16 seconds.

(b) Show that the sprinkler rotates through an angle of  $\frac{\pi}{8}$  radians in one second. [1] Let T seconds be the time that [AB] is watered in each revolution.

(c) Find the value of T. [4]

(This question continues on the following page)



**– 11 –** 2224–7110

Do **not** write solutions on this page.

#### (Question 9 continued)

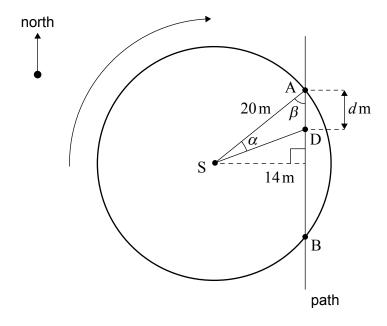
Consider one clockwise revolution of the sprinkler.

At t = 0, the water crosses the edge of the path at A.

At time t seconds, the water crosses the edge of the path at a movable point D which is a distance d metres south of point A.

Let  $\alpha = A\hat{S}D$  and  $\beta = S\hat{A}B$ , where  $\alpha$ ,  $\beta$  are measured in radians.

This information is shown in the following diagram.



(d) Write down an expression for  $\alpha$  in terms of t.

[1]

It is known that  $\beta = 0.7754$  radians, correct to four significant figures.

(e) By using the sine rule in  $\triangle ASD$ , show that the distance, d, at time t, can be modelled by

$$d(t) = \frac{20\sin\left(\frac{\pi t}{8}\right)}{\sin\left(2.37 - \frac{\pi t}{8}\right)}.$$
 [3]

(This question continues on the following page)

**– 12 –** 2224–711

Do **not** write solutions on this page.

#### (Question 9 continued)

A turtle walks south along the edge of the path.

At time t seconds, the turtle's distance, g metres south of A, can be modelled by

$$g(t) = 0.05t^2 + 1.1t + 18$$
, where  $t \ge 0$ .

(f) At 
$$t = 0$$
, state how far south the turtle is from A. [1]

Let w represent the distance between the turtle and point D at time t seconds.

- (g) (i) Use the expressions for g(t) and d(t) to write down an expression for w in terms of t.
  - (ii) Hence find when and where on the path the water first reaches the turtle. [4]



12FP12