Diploma Programme
Programme du diplôme
Programa del Diploma

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Mathematics: analysis and approaches
Higher level
Paper 2

2 May 2024

Zone A morning | **Zone B** morning | **Zone C** morning

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

15 pages

2224 – 7102

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Darren buys a car for $\$35\,000$. The value of the car decreases by 15% in the first year.

(a) Find the value of the car at the end of the first year. [2]

After the first year, the value of the car decreases by 11% in each subsequent year.

(b) Find the value of Darren's car 10 years after he buys it, giving your answer to the nearest dollar. [2]

When Darren has owned the car for n complete years, the value of the car is less than 10% of its original value.

(c) Find the least value of n. [3]

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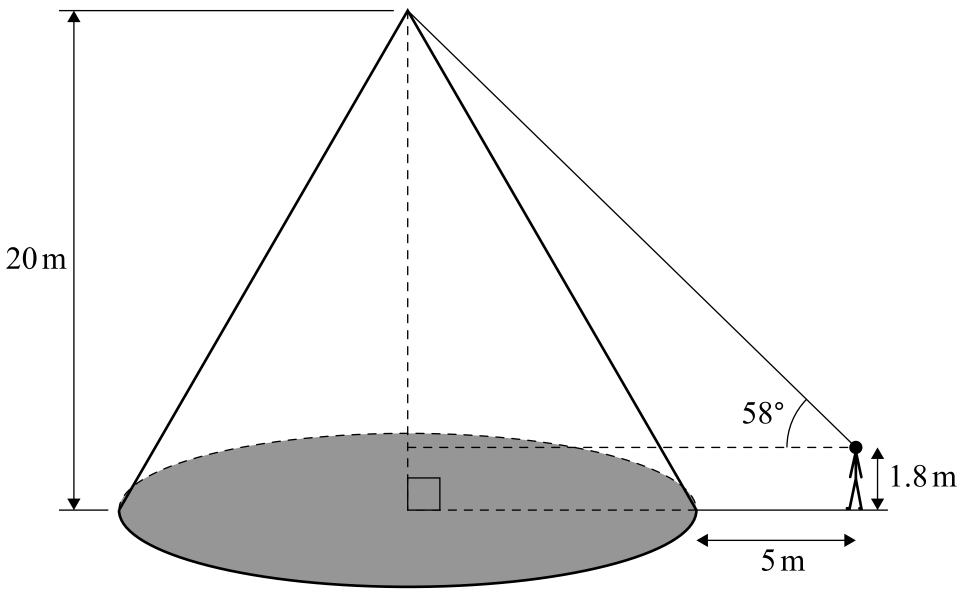
2. [Maximum mark: 5]

A monument is in the shape of a right cone with a vertical height of 20 metres. Oliver stands 5 metres from the base of the monument. His eye level is 1.8 metres above the ground and the angle of elevation from Oliver's eye level to the vertex of the cone is $58°$, as shown on the following diagram.

diagram not to scale



(a) Find the radius of the base of the cone. [3]

(b) Find the volume of the monument. [2]

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3. [Maximum mark: 4]

The random variable X is normally distributed with mean 10 and standard deviation 2.

(a) Find the probability that X is more than 1.5 standard deviations above the mean. [2]

The probability that X is more than k standard deviations above the mean is 0.1, where $k \in \mathbb{R}$.

(b) Find the value of k. [2]

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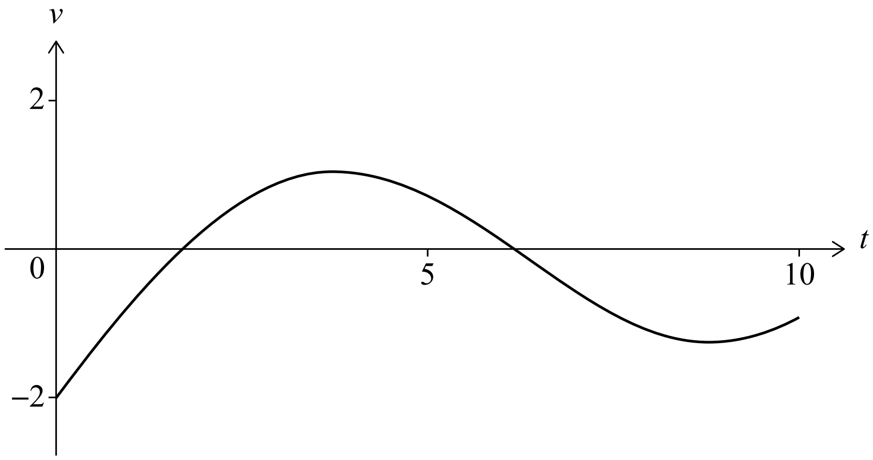
4. [Maximum mark: 6]

A particle moves in a straight line such that it passes through a fixed point O at time $t = 0$, where t represents time measured in seconds after passing O. For $0 \le t \le 10$ its velocity, v metres per second, is given by $v = 2\sin(0.5t) + 0.3t - 2$.

The graph of v is shown in the following diagram.



(a) Find the smallest value of t when the particle changes direction. [2]

The displacement of the particle is measured in metres from O.

(b) Find the range of values of t for which the displacement of the particle is increasing. [2]

(c) Find the displacement of the particle relative to O when $t = 10$. [2]

5. [Maximum mark: 7]

A class is given two tests, Test A and Test B. Each test is scored out of a total of 100 marks. The scores of the students are shown in the following table.

Student	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Test A	52	71	100	93	81	80	88	100	70	61
Test B	58	80	92	98	90	82	100	100	65	74

Let x be the score on Test A and y be the score on Test B.

The teacher finds that the equation of the regression line of y on x for these scores is $y = 0.822x + 18.4$.

(a) Find the value of Pearson's product-moment correlation coefficient, r. [2]

Giovanni was absent for Test A and Paulo was absent for Test B.

The teacher uses the regression line of y on x to estimate the missing scores.

Paulo scored 10 on Test A.

The teacher estimated his score on Test B to be 27 to the nearest integer using the following calculation:

$$y = 0.822(10) + 18.4 \approx 27$$

(b) Give a reason why this method is not appropriate for Paulo. [1]

Giovanni scored 90 on Test B.

The teacher estimated his score on Test A to be 87 to the nearest integer using the following calculation:

$$90 = 0.822x + 18.4\text{, so } x = \frac{90 - 18.4}{0.822} \approx 87$$

(c) (i) Give a reason why this method is not appropriate for Giovanni.

(ii) Use an appropriate method to show that the estimated Test A score for Giovanni is 86 to the nearest integer. [4]

(This question continues on the following page)

(Question 5 continued)

6. [Maximum mark: 6]

In Happyland, the weather on any given day is independent of the weather on any other day. On any day in May, the probability of rain is 0.2. May has 31 days.

Find the probability that

(a) it rains on exactly 10 days in May; [2]

(b) it rains on at least 10 days in May; [2]

(c) the first day that it rains in May is on the 10th day. [2]

7. [Maximum mark: 7]

Solve the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = x + y$, given that $y = 2$ when $x = 0$.

Give your answer in the form $y = f(x)$.

8. [Maximum mark: 6]

A continuous random variable X has a probability density function f given by

$$f(x) = \begin{cases} kx & 0 \le x \le k \\ 2kx - x^2 & k < x \le 2k \\ 0, & \text{otherwise} \end{cases}$$

where $k > 0$.

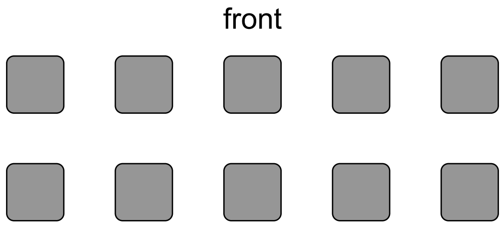
(a) Show that k satisfies the equation $7k^3 = 6$. [2]

(b) Find the median of X. [4]

9. [Maximum mark: 7]

A group of 10 children includes one pair of brothers, Alvin and Bobby, and one pair of sisters, Catalina and Daniela.

The children are to be seated at 10 desks which are arranged in two rows of five as shown in the following diagram.



Alvin and Bobby must be seated next to each other in the same row.

(a) Find the total number of ways the children can be seated. [3]

After an argument, Catalina and Daniela must not be seated next to each other. Alvin and Bobby must still be seated next to each other.

(b) Find the total number of ways the children can be seated. [4]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 16]

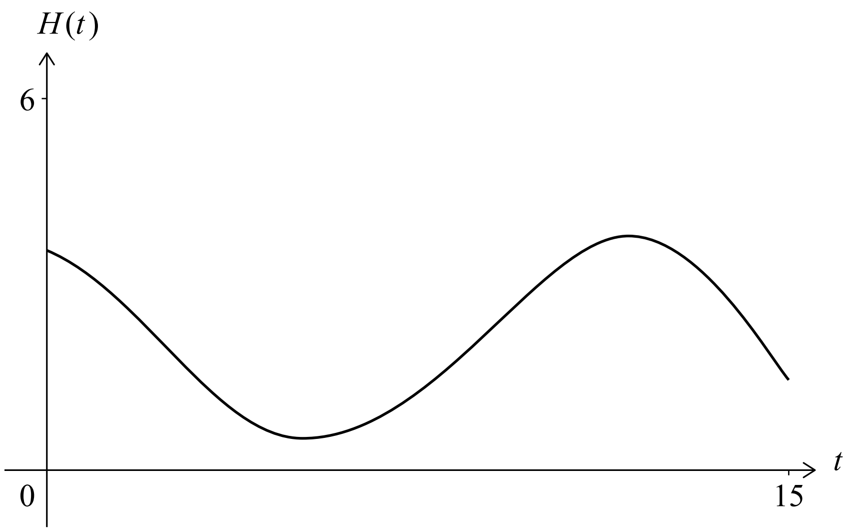
Sule Skerry and Rockall are small islands in the Atlantic Ocean, in the same time zone.

On a given day, the height of water in metres at Sule Skerry is modelled by the function $H(t) = 1.63\sin\left(0.513(t - 8.20)\right) + 2.13$, where t is the number of hours after midnight.

The following graph shows the height of the water for 15 hours, starting at midnight.

At low tide the height of the water is $0.50\,\text{m}$. At high tide the height of the water is $3.76\,\text{m}$.

All heights are given correct to two decimal places.



(a) The length of time between the first low tide and the first high tide is 6 hours and m minutes. Find the value of m to the nearest integer. [3]

(b) Between two consecutive high tides, determine the length of time, in hours, for which the height of the water is less than 1 metre. [2]

(c) Find the rate of change of the height of the water when $t = 13$, giving your answer in metres per hour. [2]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 10 continued)

On the same day, the height of water at the second island, Rockall, is modelled by the function $h(t) = a\sin\left(b(t-c)\right) + d$, where t is the number of hours after midnight, and a, b, c, $d > 0$.

The first low tide occurs at 02:41 when the height of the water is $0.40\,\text{m}$.

The first high tide occurs at 09:02 when the height of the water is $2.74\,\text{m}$.

(d) Find the values of a, b, c and d. [7]

When $t = T$, the height of the water at Sule Skerry is the same as the height of the water at Rockall for the first time.

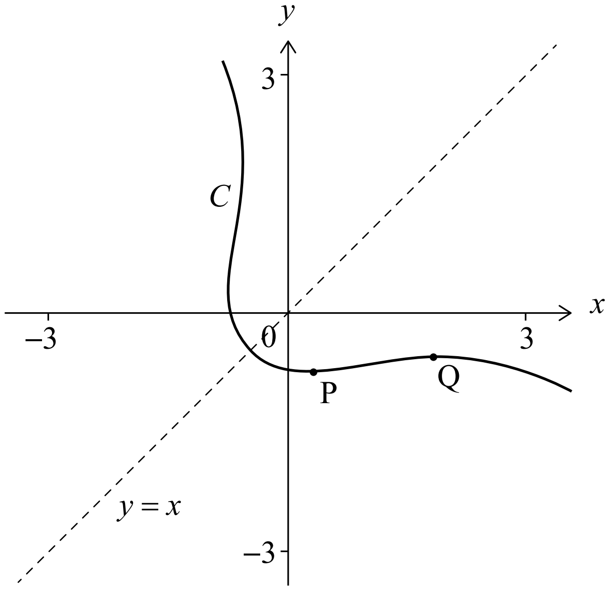
(e) Find the value of T. [2]

Do **not** write solutions on this page.

11. [Maximum mark: 19]

Consider the curve C defined by the equation $e^{x+y} = x^2 + y^2$, shown on the following diagram. The curve has a line of symmetry $y = x$.

There are two points on the curve C where the tangent is horizontal. These points are labelled P and Q.



(a) Show that $\frac{dy}{dx} = \frac{2x - e^{x+y}}{e^{x+y} - 2y}$. [5]

(b) (i) Show that the x-coordinates of points P and Q satisfy the equation $2x^2 + (\ln(2x))^2 - 2x\ln(2x) - 2x = 0$.

(ii) Hence, find the coordinates of P and the coordinates of Q. [9]

(c) Using the line of symmetry, write down the coordinates of the points on the curve C where the tangent is vertical. [1]

(d) Find the coordinates of the point on the curve C where the tangent has a gradient of -1. [4]

Do **not** write solutions on this page.

12. [Maximum mark: 20]

Consider the non-zero vectors $\boldsymbol{u}$ and $\boldsymbol{v}$. Let θ be the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$.

(a) Using the definitions of $\boldsymbol{u} \bullet \boldsymbol{v}$ and $\boldsymbol{u} \times \boldsymbol{v}$ in terms of $|\boldsymbol{u}|$, $|\boldsymbol{v}|$ and θ, show that $(\boldsymbol{u} \bullet \boldsymbol{v})^2 + |\boldsymbol{u} \times \boldsymbol{v}|^2 = |\boldsymbol{u}|^2 |\boldsymbol{v}|^2$. [2]

A triangle ABC has vertices $\text{A}(0, 1, 2)$, $\text{B}(p, q, 3)$ and $\text{C}(3, 2, 1)$, $p, q \in \mathbb{Q}$.

The vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are defined as $\boldsymbol{u} = \overrightarrow{\text{AB}}$ and $\boldsymbol{v} = \overrightarrow{\text{AC}}$.

It is given that $\boldsymbol{u} \bullet \boldsymbol{v} = 3$ and the area of triangle ABC is $\sqrt{6}$.

(b) (i) Find the value of $|\boldsymbol{u} \times \boldsymbol{v}|$.

(ii) Hence, or otherwise, find the value of $|\boldsymbol{u}|$.

(iii) Hence, or otherwise, find the possible values of p and the corresponding values of q. [13]

Consider a new point D, the vector $\boldsymbol{w}$ is defined as $\boldsymbol{w} = \overrightarrow{\text{CD}}$.

It is given that $\boldsymbol{u} \bullet \boldsymbol{w} = \boldsymbol{v} \bullet \boldsymbol{w} = 0$ and the area of triangle ACD is 5 square units.

(c) Assuming that $p = 1$, find the possible vectors for $\boldsymbol{w}$. [5]

Please **do not** write on this page.

Answers written on this page
will not be marked.