Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 2

31 October 2023

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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Please **do not** write on this page.

Answers written on this page
will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

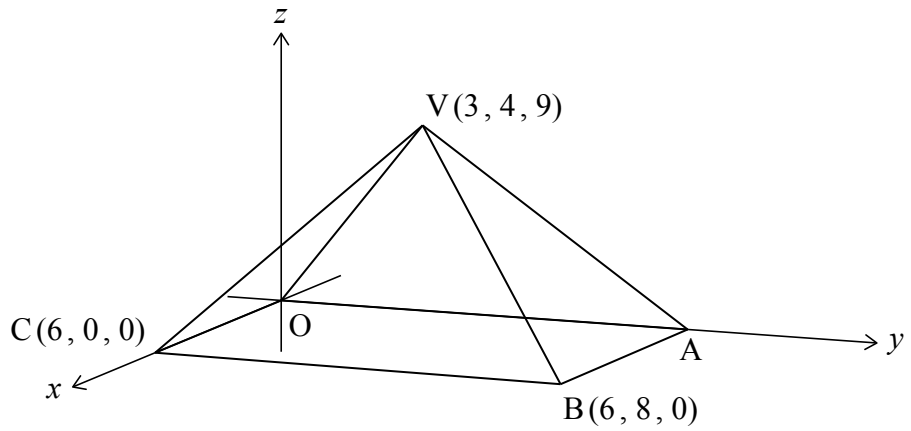
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following diagram shows a pyramid with vertex V and rectangular base OABC.

Point B has coordinates $(6, 8, 0)$, point C has coordinates $(6, 0, 0)$ and point V has coordinates $(3, 4, 9)$.

diagram not to scale



(a) Find BV. [2]

(b) Find the size of $\mathrm{B\hat{V}C}$. [4]

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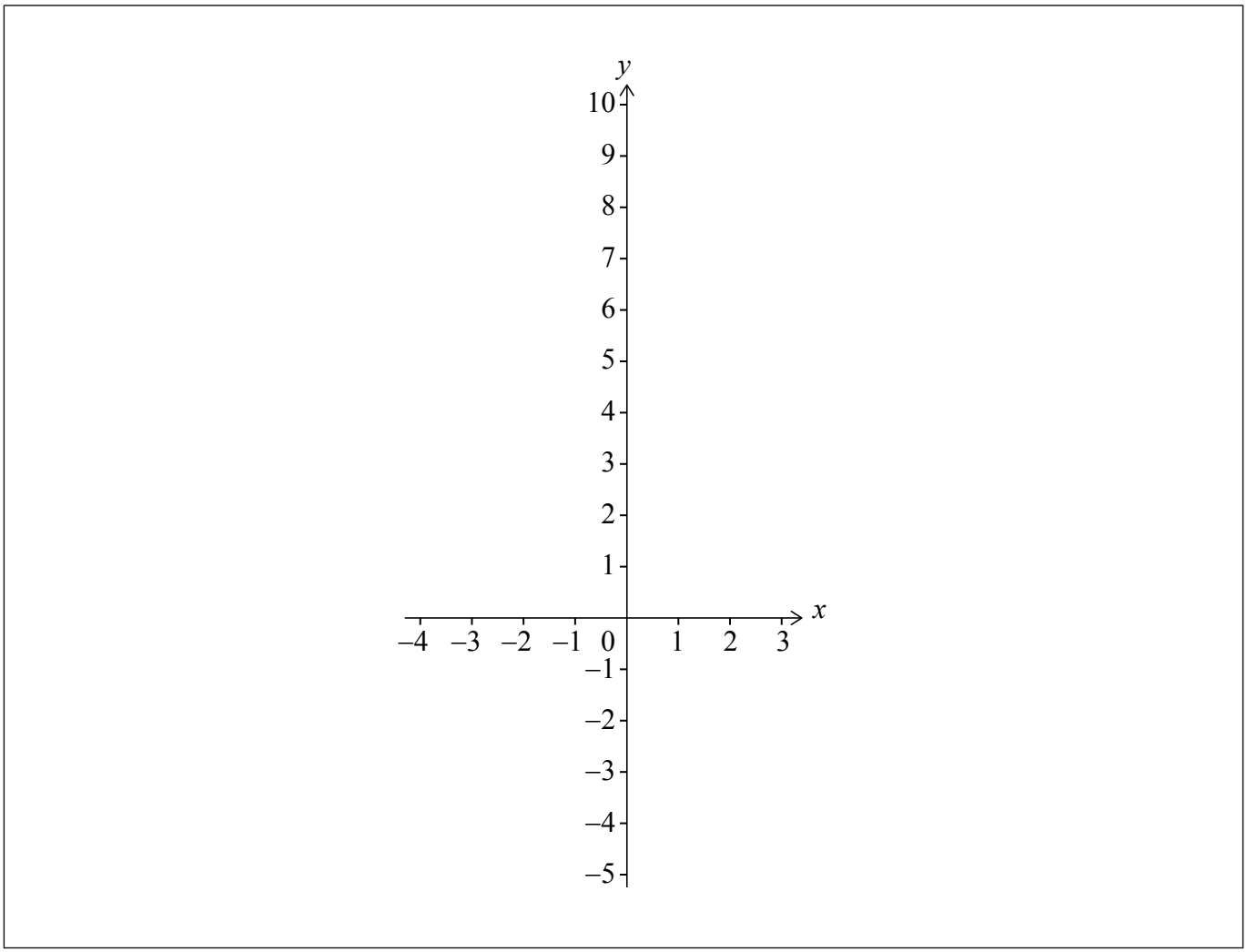
2. [Maximum mark: 5]

Consider the function $f(x) = e^x - 3x - 4$.

(a) On the following axes, sketch the graph of f for $-4 \leq x \leq 3$. [3]



The function g is defined by $g(x) = e^{2x} - 6x - 7$.

(b) The graph of g is obtained from the graph of f by a horizontal stretch with scale factor k, followed by a vertical translation of c units.

Find the value of k and the value of c. [2]

(This question continues on the following page)

(Question 2 continued)

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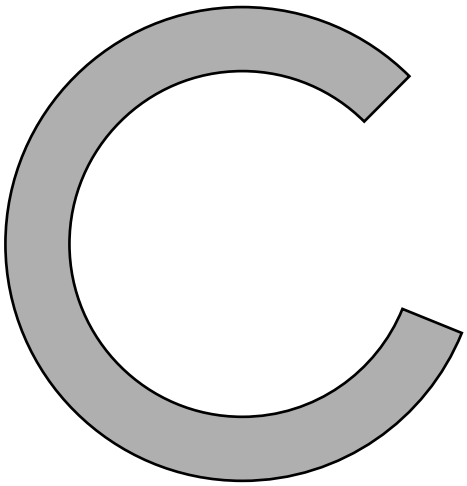
3. [Maximum mark: 7]

A company is designing a new logo in the shape of a letter “C”.



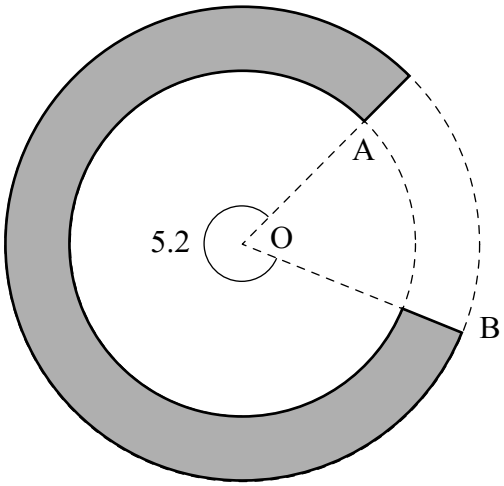
The letter “C” is formed between two circles with centre O.

The point A lies on the circumference of the inner circle with radius $r\,\mathrm{cm}$, where $r < 10$.

The point B lies on the circumference of the outer circle with radius $10\,\mathrm{cm}$.

The reflex angle $\mathrm{A\hat{O}B}$ is 5.2 radians. The letter “C” is shown by the shaded area in the following diagram.

diagram not to scale



(This question continues on the following page)

(Question 3 continued)

(a) Show that the area of the "C" is given by $260 - 2.6r^2$. [2]

The area of the "C" is $64\,\text{cm}^2$.

(b) (i) Find the value of r.

(ii) Find the perimeter of the "C". [5]

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4. [Maximum mark: 5]

A particle moves along a straight line. Its displacement, s metres, from a fixed point O after time t seconds is given by $s(t) = 4.3\sin\left(\sqrt{3t+5}\right)$, where $0 \le t \le 10$.

The particle first comes to rest after q seconds.

(a) Find the value of q. [2]

(b) Find the total distance that the particle travels in the first q seconds. [3]

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5. [Maximum mark: 5]

The following table shows the probability distribution of a discrete random variable X, where $a, k \in \mathbb{R}^+$.

x	1	2	3	4
$\mathrm{P}(X = x)$	k	k^2	a	k^3

Given that $\mathrm{E}(X) = 2.3$, find the value of a.

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6. [Maximum mark: 5]

The random variable X is such that $X \sim B(25, p)$ and $\mathrm{Var}(X) = 5.75$.

(a) Find the possible values of p. [3]

The random variable Y is such that $Y = 1 - 2X$.

(b) Find $\mathrm{Var}(Y)$. [2]

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7. [Maximum mark: 6]

A junior baseball team consists of six boys and three girls.

The team members are to be placed in a line to have their photograph taken.

(a) In how many ways can the team members be placed if

(i) there are no restrictions;

(ii) the girls must be placed next to each other. [3]

(b) Five members of the team are selected to attend a baseball summer camp. Find the number of possible selections that contain at least two girls. [3]

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8. [Maximum mark: 9]

Three points are given by $\mathrm{A}(0, p, 2)$, $\mathrm{B}(1, 1, 1)$ and $\mathrm{C}(p, 0, 4)$, where p is a positive constant.

(a) Show that $\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}} = \begin{pmatrix} 2-3p \\ -2-p \\ p^2-2p \end{pmatrix}$. [4]

(b) Hence, find the smallest possible value of $\left|\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}}\right|^2$. [3]

(c) Hence, find the smallest possible area of triangle ABC. [2]

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9. [Maximum mark: 9]

Consider the differential equation $\frac{dy}{dx} = \frac{4-y}{10}$, where $y = 2$ when $x = 0$.

(a) Use Euler's method with a step size of 0.1 to find an approximation for y when $x = 0.5$. Give your answer correct to four significant figures. [3]

(b) By solving the differential equation, show that $y = 4 - 2e^{-\frac{x}{10}}$. [5]

(c) Find the absolute value of the error in your approximation in part (a). [1]

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 16]

A farmer is growing a field of wheat plants. The height, $H\,\text{cm}$, of each plant can be modelled by a normal distribution with mean μ and standard deviation σ.

It is known that $\text{P}(H < 94.6) = 0.288$ and $\text{P}(H > 98.1) = 0.434$.

(a) Find the probability that the height of a randomly selected plant is between $94.6\,\text{cm}$ and $98.1\,\text{cm}$. [2]

(b) Find the value of μ and the value of σ. [5]

The farmer measures 100 randomly selected plants. Any plant with a height greater than $98.1\,\text{cm}$ is considered ready to harvest. Heights of plants are independent of each other.

(c) (i) Find the probability that exactly 34 plants are ready to harvest.

(ii) Given that fewer than 49 plants are ready to harvest, find the probability that exactly 34 plants are ready to harvest. [6]

In another field, the farmer is growing the same variety of wheat, but is using a different fertilizer. The heights of these plants, $F\,\text{cm}$, are normally distributed with mean 98.6 and standard deviation d. The farmer finds the interquartile range to be $4.82\,\text{cm}$.

(d) Find the value of d. [3]

Do **not** write solutions on this page.

11. [Maximum mark: 19]

Consider the function defined by $f(x) = \frac{x^2 - 14x + 24}{2x + 6}$, where $x \in \mathbb{R}$, $x \neq -3$.

(a) State the equation of the vertical asymptote on the graph of f. [1]

(b) Find the coordinates of the points where the graph of f crosses the x-axis. [2]

The graph of f also has an oblique asymptote of the form $y = ax + b$, where $a, b \in \mathbb{Q}$.

(c) Find the value of a and the value of b. [4]

(d) Sketch the graph of f for $-50 \leq x \leq 50$, showing clearly the asymptotes and any intersections with the axes. [4]

(e) Find the range of f. [4]

(f) Solve the inequality $f(x) > x$. [4]

Do **not** write solutions on this page.

12. [Maximum mark: 18]

Line L is given by the vector equation $\boldsymbol{r}_1 = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} + s \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$ where $s \in \mathbb{R}$.

Line M is given by the vector equation $\boldsymbol{r}_2 = \begin{pmatrix} 9 \\ 9 \\ 11 \end{pmatrix} + t \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$ where $t \in \mathbb{R}$.

(a) Show that lines L and M intersect at a point A and find the position vector of A. [5]

(b) Verify that the lines L and M both lie in the plane Π given by $\boldsymbol{r} \bullet \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 7$. [3]

Point B has position vector $\begin{pmatrix} -3 \\ 12 \\ 2 \end{pmatrix}$. A line through B perpendicular to Π intersects Π at point C.

(c) (i) Find the position vector of C.

(ii) Hence, find $\left|\vec{\mathrm{BC}}\right|$. [7]

(d) Find the reflection of the point B in the plane Π. [3]