Diploma Programme
Programme du diplôme
Programa del Diploma

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Mathematics: analysis and approaches
Higher level
Paper 1

1 May 2024

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

16 pages

2224 – 7106

Please **do not** write on this page.

Answers written on this page
will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

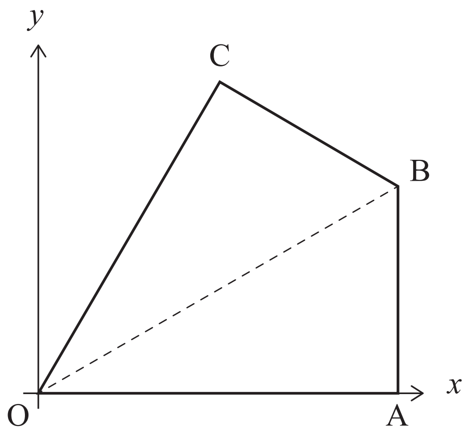
Solve $\tan(2x - 5°) = 1$ for $0° \le x \le 180°$.

2. [Maximum mark: 5]

Solve $3 \times 9^x + 5 \times 3^x - 2 = 0$.

3. [Maximum mark: 7]

Quadrilateral OABC is shown on the following set of axes.



OABC is symmetrical about [OB].

A has coordinates $(6, 0)$ and C has coordinates $\left(3, 3\sqrt{3}\right)$.

(a) (i) Write down the coordinates of the midpoint of [AC].

(ii) Hence or otherwise, find the equation of the line passing through the points O and B. [4]

(b) Given that [OA] is perpendicular to [AB], find the area of the quadrilateral OABC. [3]

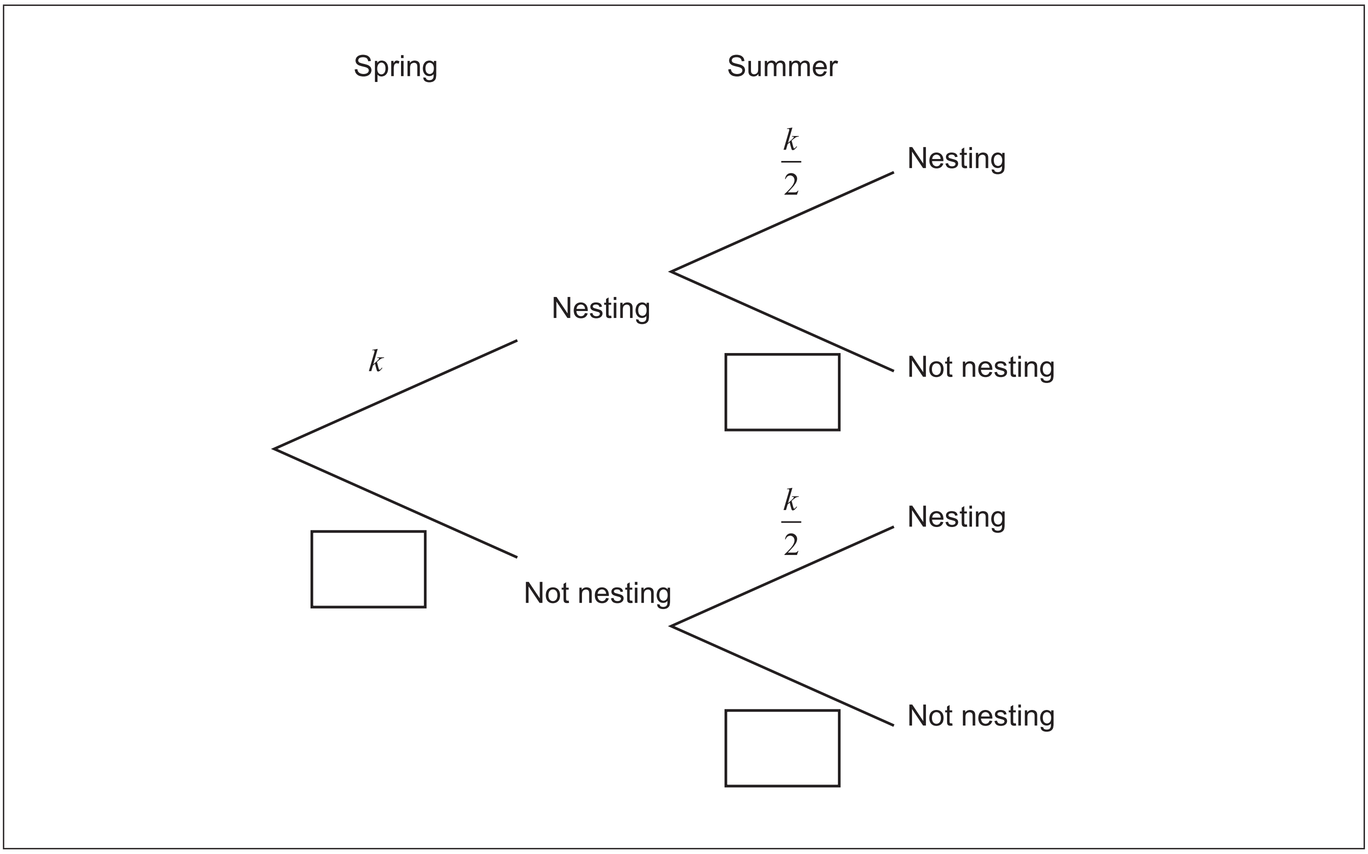
4. [Maximum mark: 6]

A species of bird can nest in two seasons: Spring and Summer.

The probability of nesting in Spring is k.

The probability of nesting in Summer is $\frac{k}{2}$.

This is shown in the following tree diagram.



(a) Complete the tree diagram to show the probabilities of not nesting in each season. Write your answers in terms of k. [2]

It is known that the probability of not nesting in Spring and not nesting in Summer is $\frac{5}{9}$.

(b) (i) Show that $9k^2 - 27k + 8 = 0$.

(ii) Both $k = \frac{1}{3}$ and $k = \frac{8}{3}$ satisfy $9k^2 - 27k + 8 = 0$.

State why $k = \frac{1}{3}$ is the only valid solution. [4]

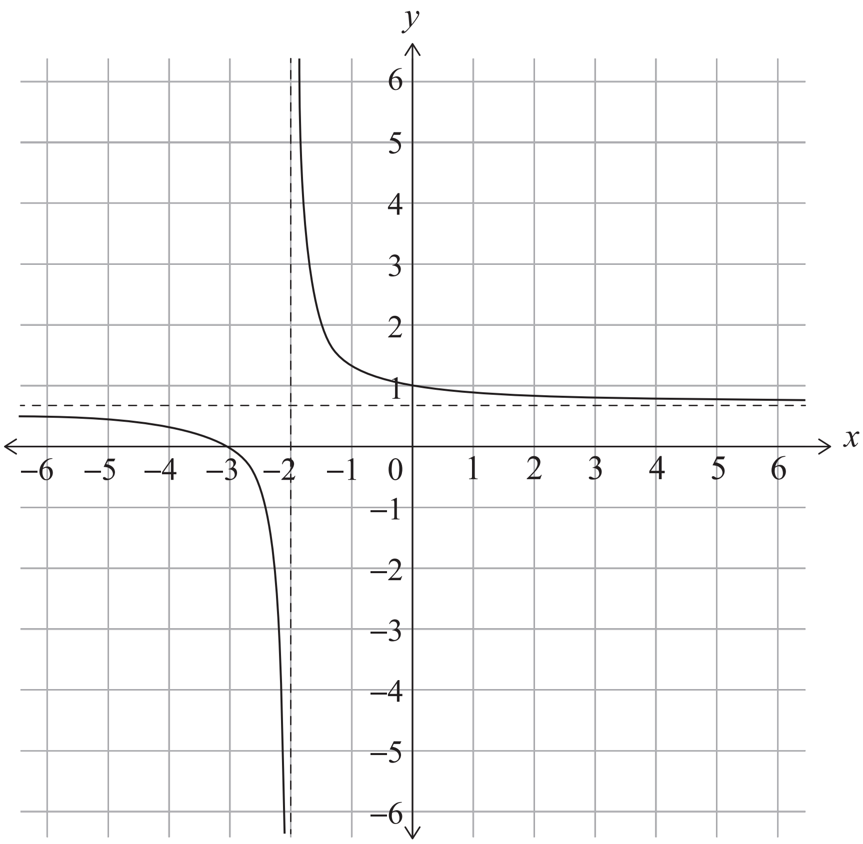
(This question continues on the following page)

(Question 4 continued)

5. [Maximum mark: 8]

A function f is defined by $f(x) = \dfrac{2(x+3)}{3(x+2)}$, where $x \in \mathbb{R}, x \neq -2$.

The graph $y = f(x)$ is shown below.



(a) Write down the equation of the horizontal asymptote. [1]

Consider $g(x) = mx + 1$, where $m \in \mathbb{R}, m \neq 0$.

(b) (i) Write down the number of solutions to $f(x) = g(x)$ for $m > 0$.

(ii) Determine the value of m such that $f(x) = g(x)$ has only one solution for x.

(iii) Determine the range of values for m, where $f(x) = g(x)$ has two solutions for $x \geq 0$. [7]

(This question continues on the following page)

(Question 5 continued)

6. [Maximum mark: 5]

A farmer grows two types of apples—cooking apples and eating apples. The weights of the apples, in grams, can be modelled as normal distributions with the following parameters.

Apple type	**Mean μ**	**Standard deviation σ**
Eating	$100\,\text{g}$	$20\,\text{g}$
Cooking	$140\,\text{g}$	$40\,\text{g}$

For each type of apple you can assume that 95% of the weights are within two standard deviations of the mean.

(a) Find the percentage of eating apples that have a weight greater than $140\,\text{g}$. [1]

The farmer grows a large number of apples of which 80% are eating apples.

Both types of apples are picked and randomly mixed together in a cleaning machine.

After cleaning, the machine separates out those that have a weight greater than $140\,\text{g}$ into a container.

(b) An apple is randomly selected from this container. Find the probability that it is an eating apple. Give your answer in the form $\frac{c}{d}$, where $c, d \in \mathbb{Z}^+$. [4]

7. [Maximum mark: 7]

A function $g(x)$ is defined by $g(x) = 2x^3 - 7x^2 + dx - e$, where $d, e \in \mathbb{R}$.

α, β and γ are the three roots of the equation $g(x) = 0$ where $\alpha, \beta, \gamma \in \mathbb{R}$.

(a) Write down the value of $\alpha + \beta + \gamma$. [1]

A function $h(z)$ is defined by $h(z) = 2z^5 - 11z^4 + rz^3 + sz^2 + tz - 20$, where $r, s, t \in \mathbb{R}$.

α, β and γ are also roots of the equation $h(z) = 0$.

It is given that $h(z) = 0$ is satisfied by the complex number $z = p + 3\mathrm{i}$.

(b) Show that $p = 1$. [3]

It is now given that $h\left(\frac{1}{2}\right) = 0$, and $\alpha, \beta \in \mathbb{Z}^+$, $\alpha < \beta$ and $\gamma \in \mathbb{Q}$.

(c) (i) Find the value of the product $\alpha\beta$.

(ii) Write down the value of α and the value of β. [3]

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8. [Maximum mark: 6]

Use l'Hôpital's rule to find $\lim_{x\to 0}\dfrac{\sec^4 x - \cos^2 x}{x^4 - x^2}$.

9. [Maximum mark: 7]

A teacher takes n students on a field trip. The students are assigned randomly into two groups.

For safety reasons there must be exactly three students in the first group and at least three students in the second group.

The teacher will randomly assign three students to the first group and the other students to the second group.

(a) Write down an expression for the number of ways that the students could be assigned. [1]

Two of the students ask the teacher not to work in the same group.

The teacher agrees and now finds that the number of ways to assign the students is halved.

(b) Determine the value of n. [6]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 16]

Consider the arithmetic sequence a, p, $q \ldots$, where a, p, $q \neq 0$.

(a) Show that $2p - q = a$. [2]

Consider the geometric sequence a, s, $t \ldots$, where a, s, $t \neq 0$.

(b) Show that $s^2 = at$. [2]

The first term of both sequences is a.

It is given that $q = t = 1$.

(c) Show that $p > \frac{1}{2}$. [2]

Consider the case where $a = 9$, $s > 0$ and $q = t = 1$.

(d) Write down the first four terms of the

(i) arithmetic sequence;

(ii) geometric sequence. [4]

The arithmetic and the geometric sequence are used to form a new arithmetic sequence u_n.

The first three terms of u_n are $u_1 = 9 + \ln 9$, $u_2 = 5 + \ln 3$, and $u_3 = 1 + \ln 1$.

(e) (i) Find the common difference of the new sequence in terms of $\ln 3$.

(ii) Show that $\sum_{i=1}^{10} u_i = -90 - 25\ln 3$. [6]

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11. [Maximum mark: 19]

The plane Π_1 has equation $2x + 6y - 2z = 5$.

(a) Verify that the point $\mathrm{A}\left(2, \frac{1}{2}, 1\right)$ lies on the plane Π_1. [1]

The plane Π_2 is given by $(k^2 - 6)x + (2k + 3)y + pz = q$, where $p, q, k \in \mathbb{R}$ and $p \neq 0$.

(b) In the case where $p = -6$, Π_2 is perpendicular to Π_1 and A lies on Π_2.
Find the value of k and the value of q. [5]

For parts (c), (d) and (e) it is now given that Π_2 is parallel to Π_1 with $k = 3$.

(c) Determine the value of p. [2]

It is also given that $q = -\frac{51}{2}$.

The line through A that is perpendicular to Π_1 meets Π_2 at the point B.

(d) (i) Find the coordinates of B.

(ii) Hence, show that the perpendicular distance between Π_1 and Π_2 is $\sqrt{11}$. [7]

(e) Find the equation of a third parallel plane Π_3 which is also a perpendicular distance of $\sqrt{11}$ from Π_1. [4]

Do **not** write solutions on this page.

12. [Maximum mark: 20]

(a) Let $f(x)=(1-ax)^{-\frac{1}{2}}$, where $ax<1$, $a\neq 0$.

The n^{th} derivative of $f(x)$ is denoted by $f^{(n)}(x)$, $n\in\mathbb{Z}^{+}$.

Prove by induction that $f^{(n)}(x)=\dfrac{a^{n}(2n-1)!(1-ax)^{-\frac{2n+1}{2}}}{2^{2n-1}(n-1)!}$, $n\in\mathbb{Z}^{+}$. [8]

(b) By using part (a) or otherwise, show that the Maclaurin series for $f(x)=(1-ax)^{-\frac{1}{2}}$ up to and including the x^2 term is $1+\frac{1}{2}ax+\frac{3}{8}a^{2}x^{2}$. [2]

(c) Hence, show that $(1-2x)^{-\frac{1}{2}}(1-4x)^{-\frac{1}{2}}\approx\dfrac{2+6x+19x^{2}}{2}$. [4]

(d) Given that the series expansion for $(1-ax)^{-\frac{1}{2}}$ is convergent for $|ax|<1$, state the restriction which must be placed on x for the approximation $(1-2x)^{-\frac{1}{2}}(1-4x)^{-\frac{1}{2}}\approx\dfrac{2+6x+19x^{2}}{2}$ to be valid. [1]

(e) Use $x=\dfrac{1}{10}$ to determine an approximate value for $\sqrt{3}$.

Give your answer in the form $\dfrac{c}{d}$, where $c, d\in\mathbb{Z}^{+}$. [5]