2225 – 7106M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2025

Mathematics: analysis and approaches

Higher level

Paper 1

25 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2.***

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.

- If the candidate's answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a "show that" question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was "Hence".

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.

- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.

- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

No calculator is allowed. The use of any calculator on this paper is malpractice and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1. $f'(x) = 4x^2 - 16$ ***A1***

sets their derivative equal to zero ***(M1)***

$4x^2 - 16 = 0$, ($x = \pm 2$)

$p = 2$ (accept $x = 2$) ***A1***

substitutes their **positive** *p* into $f(x)$ ***(M1)***

$$y = \frac{4\left(2^3\right)}{3} - 16(2)\left(= \frac{32}{3} - 32 = -\frac{64}{3}\right)$$

$q = -\frac{64}{3}$ (accept $y = \frac{-64}{3}$) ***A1***

Total [5 marks]

2. (a) $k = \frac{4}{400}\left(= \frac{1}{100} = 0.01\right)$ ***A1***

[1 mark]

(b) attempt to find binomial coefficients or multiply out brackets ***(M1)***

e.g. Pascal's triangle down to correct row OR $\left(1+2x+x^2\right)^2$ OR substitute into binomial expansion

$\left(1+x\right)^4$

$= 1+4x+6x^2+4x^3+x^4$ ***A1***

[2 marks]

(c) **METHOD 1**

recognition that the expansion can be used with x replaced with k ***(M1)***

$\left(1+\frac{1}{100}\right)^4$

$= 1+\frac{4}{100}+\frac{6}{100^2}+\ldots\left(=1+0.04+0.0006+\ldots\right)$ ***(A1)***

multiplies by 1000 (seen anywhere) ***(M1)***

$1000\left(1+\frac{1}{100}\right)^4$

$= 1000+40+0.6+\ldots\left(=1040.6\ldots\right)$

$= 1041$ (dinar) ***A1***

METHOD 2

attempt to find the value of $\left(1+k\right)^4$ by hand ***(M1)***

$\left(1.01\right)^4 = \left(1.0201\right)\left(1.01\right)^2 = \left(1.030301\right)\left(1.01\right)$

$= 1.0406\ldots$ ***(A1)***

multiplies by 1000 (seen anywhere) ***(M1)***

$1000\left(1.01\right)^4$

$= 1040.6\ldots$

$= 1041$ (dinar) ***A1***

[4 marks]

Total [7 marks]

3. **METHOD 1**

attempt to set up integral $e^x - \left(-e^x\right) = 2e^x$ or e^x and then double ***(M1)***

$\int\left(e^x - \left(-e^x\right)\right)dx$ OR $2\int e^x\,dx$

$= 2\int_{-1}^{1} e^x\,dx$

$= 2\left[e^x\right]_{-1}^{1}$ ***(A1)***

attempt to substitute correct limits into their integrated function and subtract ***(M1)***

$= 2\left(e - \frac{1}{e}\right)$, $2e - \frac{2}{e}$, $2e - 2e^{-1}$ ***A1***

METHOD 2

$\int_{-1}^{1} e^x\,dx = \left[e^x\right]_{-1}^{1}$ and $\int_{-1}^{1} -e^x\,dx = \left[-e^x\right]_{-1}^{1}$ ***(A1)***

attempt to substitute correct limits into both their integrated functions and subtract ***(M1)***

$e^1 - e^{-1}$ and $-e^1 - (-e^{-1})$

subtracts their two integrals in correct order ***(M1)***

$e^1 - e^{-1} - \left(-e^1 + e^{-1}\right)$

$= 2\left(e - \frac{1}{e}\right)$, $2e - \frac{2}{e}$, $2e - 2e^{-1}$ ***A1***

Total [4 marks]

4. (a) $\mathrm{P}(A)=\frac{1}{4}$ ***(A1)***

attempt to use $\mathrm{P}(B|A)=\frac{\mathrm{P}(B\cap A)}{\mathrm{P}(A)}$ ***(M1)***

$\frac{2}{3}=\frac{\mathrm{P}(B\cap A)}{\left(\frac{1}{4}\right)}$

$\mathrm{P}(A\cap B)=\frac{2}{3}\left(\frac{1}{4}\right)$

$=\frac{2}{12}\left(=\frac{1}{6}\right)$ ***A1***

[3 marks]

(b) attempt to use $\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$ OR a Venn diagram, with their values of $\mathrm{P}(A)$ and $\mathrm{P}(B\cap A)$ ***M1***

$\frac{3}{4}=\frac{1}{4}+\mathrm{P}(B)-\frac{1}{6}$

$\mathrm{P}(B)=\frac{1}{2}+\frac{1}{6}$

$=\frac{4}{6}\left(=\frac{2}{3}\right)$ ***A1***

$\mathrm{P}(B|A)=\mathrm{P}(B)$ OR $\mathrm{P}(A)\mathrm{P}(B)=\frac{1}{6}$ so $\mathrm{P}(A\cap B)=\mathrm{P}(A)\mathrm{P}(B)$ (hence A and B are independent) ***R1***

Note: The ***R1*** is dependent on all previous marks

[3 marks]

Total [6 marks]

5. (a) (i) for a sequence of areas, uses two consecutive terms to find a common ratio OR for sequences of both widths and heights uses two consecutive terms for both sequences to find both common ratios OR recognises that both widths and heights are geometric sequences with common ratio $\frac{3}{2}$ ***M1***

areas form a geometric sequence with first term 20 and common ratio $\frac{45}{20}$ ***A1***

OR area of picture frame F_n is $4\left(\frac{3}{2}\right)^{n-1}\times 5\left(\frac{3}{2}\right)^{n-1}$

area of F_n is $20\left(\frac{9}{4}\right)^{n-1}$ ***AG***

(ii) attempt to find the sum of the areas using $S_n=\frac{u_1\left(r^n-1\right)}{r-1}$ ***(M1)***

sum of areas $\frac{20\left(\left(\frac{9}{4}\right)^{10}-1\right)}{\frac{9}{4}-1}\left(=16\left(\left(\frac{9}{4}\right)^{10}-1\right)\right)$ ***(A1)***

mean area $=\frac{1}{10}\left(\frac{20\left(\left(\frac{9}{4}\right)^{10}-1\right)}{\frac{9}{4}-1}\right)\left(=\frac{1}{10}\left(16\left(\left(\frac{9}{4}\right)^{10}-1\right)\right)\right)$

$=\frac{16}{10}\left(\left(\frac{9}{4}\right)^{10}-1\right)\left(=\frac{8}{5}\left(\left(\frac{9}{4}\right)^{10}-1\right)\right)$ ***A1***

$p=\frac{8}{5},\ a=10$

[5 marks]

continued…

Question 5 continued.

(b) recognition that median is between 5th and 6th picture frame ***(M1)***

median area $=\dfrac{20\left(\frac{9}{4}\right)^4+20\left(\frac{9}{4}\right)^5}{2}$ ***(A1)***

$=\dfrac{20\left(\frac{9}{4}\right)^4\left(1+\frac{9}{4}\right)}{2}$

$=\dfrac{65}{2}\left(\dfrac{9}{4}\right)^4$ ***A1***

$q=\dfrac{65}{2}$

[3 marks]

Total [8 marks]

6. direction vectors are $a\boldsymbol{j}+\boldsymbol{k}\left(=\begin{pmatrix}0\\a\\1\end{pmatrix}\right)$ and $\boldsymbol{i}+2\boldsymbol{j}+3\boldsymbol{k}\left(=\begin{pmatrix}1\\2\\3\end{pmatrix}\right)$ ***(A1)***

recognition that the scalar product of the direction vectors is 0 ***(M1)***

$\begin{pmatrix}0\\a\\1\end{pmatrix}\bullet\begin{pmatrix}1\\2\\3\end{pmatrix}(=2a+3)=0$

$a=-\dfrac{3}{2}$ ***A1***

at point of intersection $4=1+\mu$, $-\dfrac{3}{2}\lambda=2\mu$ and $-1+\lambda=-b+3\mu$ ***(A1)***

attempt to solve 3 equations in μ, λ and b, derived from the point of intersection, to find μ, λ and b ***(M1)***

$\mu=3,\ \lambda=-4$

$b=14$ ***A1***

Total [6 marks]

7. (a) $(3=)e^{\ln 3}$ OR $a=\ln 3$ ***A1***

[1 mark]

(b) (i) $z=\frac{1}{3}e^{\mathrm{i}\ln 3}$ OR $(\mathrm{Re}(z))=e^{-\ln 3}\cos(\ln 3)$ ***(A1)***

$(\mathrm{Re}(z))=\frac{1}{3}\cos(\ln 3)$ ***A1***

(ii) $\frac{1}{z}=3e^{-\mathrm{i}\ln 3}\left(=3\left(\cos(-\ln 3)+\mathrm{i}\sin(-\ln 3)\right)\right)$ OR $\left(\mathrm{Re}\left(\frac{1}{z}\right)=\right)e^{\ln 3}\cos(-\ln 3)$ ***(A1)***

$\mathrm{Re}\left(\frac{1}{z}\right)=3\cos(-\ln 3)$

$=3\cos(\ln 3)$ OR $3\cos\left(\ln\frac{1}{3}\right)$ ***A1***

[4 marks]

Total [5 marks]

8. (a) $1+\log_2 n = \log_2 2 + \log_2 n$

$= \log_2(2n)$ ***A1***

$2n \geq n+1$ OR $\log_2(2n) \geq \log_2(n+1)$ ***R1***

(for $n \in \mathbb{Z}^+$) (since $\log_2 x$ is an increasing function)

$1+\log_2 n \geq \log_2(n+1)$ (for $n \in \mathbb{Z}^+$) ***AG***

Note: Do not award ***A0R1***.

[2 marks]

(b) for $n=1$

$\log_2 1 = 0$ and $1>0$ OR LHS $=1$, RHS = $\log_2 1 = 0$ OR $\log_2 2 > \log_2 1$ ***R1***

(so true for $n=1$)

assume true for $n=k$, ie $k > \log_2 k$ ***M1***

Note: Award ***M0*** for statements such as "let $n=k$", "assume $n=k$ is true". The assumption of truth must be clear. "Assume P_k true" is accepted.

The following two marks after this ***M1*** are independent of this mark and can be awarded.

hence

$1+k > 1+\log_2 k$ (using assumption) ***M1***

$\geq \log_2(k+1)$ (using result from part a)) ***A1***

hence if true for $n=k$ then true for $n=k+1$ ***R1***

and as true for $n=1$, therefore true for all $n \in \mathbb{Z}^+$.

Note: Only award the final **R1** if the first three marks have been awarded.

[5 marks]

Total [7 marks]

9. $y = vx,\ \dfrac{\mathrm{d}y}{\mathrm{d}x} = x\dfrac{\mathrm{d}v}{\mathrm{d}x} + v$

$x\dfrac{\mathrm{d}v}{\mathrm{d}x} + v = \dfrac{x - vx}{x + vx}\left(= \dfrac{1-v}{1+v}\right)$

$x\dfrac{\mathrm{d}v}{\mathrm{d}x} + v = \dfrac{1-v}{1+v}$ ***(A1)***

$x\dfrac{\mathrm{d}v}{\mathrm{d}x} = \dfrac{1-v}{1+v} - v\left(= \dfrac{1-2v-v^2}{1+v}\right)$

attempt to separate variables and form two integrals ***(M1)***

$\int \dfrac{1+v}{1-2v-v^2}\mathrm{d}v = \int \dfrac{1}{x}\mathrm{d}x$

$\int \dfrac{1}{x}\mathrm{d}x = \ln x + c$ ***A1***

use of substitution or inspection to integrate $\dfrac{1+v}{1-2v-v^2}$ or equivalent ***(M1)***

$u = 1-2v-v^2 \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}v} = -2-2v = -2(1+v)$

$\int \dfrac{1+v}{1-2v-v^2}\mathrm{d}v = -\dfrac{1}{2}\ln\left|1-2v-v^2\right|$ OR $-\dfrac{1}{2}\ln\left|v^2+2v-1\right| = \ln|x| + c$ ***A1***

$-\dfrac{1}{2}\ln\left|1-2\dfrac{y}{x}-\dfrac{y^2}{x^2}\right| - \ln|x| = c$ OR $-\dfrac{1}{2}\ln\left|\dfrac{y^2}{x^2}+2\dfrac{y}{x}-1\right| - \ln|x| = c$

EITHER

attempt to substitute $x = 2$ and either $y = 0$ or $v = 0$ to find a constant c ***(M1)***

$c = -\ln 2$

$-\dfrac{1}{2}\ln\left|1-2\dfrac{y}{x}-\dfrac{y^2}{x^2}\right| - \ln|x| = -\ln 2$ OR $-\dfrac{1}{2}\ln\left|\dfrac{y^2}{x^2}+2\dfrac{y}{x}-1\right| - \ln|x| = -\ln 2$ ***A1***

OR

$x^2\left|1-2\dfrac{y}{x}-\dfrac{y^2}{x^2}\right| = A$ OR $x^2\left|\dfrac{y^2}{x^2}+2\dfrac{y}{x}-1\right| = A$ ***A1***

attempt to substitute $x = 2$ either $y = 0$ or $v = 0$ to find a constant A ***(M1)***

THEN

$x^2\left|1-2\dfrac{y}{x}-\dfrac{y^2}{x^2}\right| = 4$ OR $x^2\left(1-2\dfrac{y}{x}-\dfrac{y^2}{x^2}\right) = 4$ ***A1***

checking boundary values confirms $x^2 - 2xy - y^2 = 4$ ***AG***

continued…

Question 9 continued.

Note: Condone absence of absolute value signs even if removed incorrectly until the final ***A1*** mark where they must be seen or have been removed to form a correct equation.

Total [8 marks]

Section B

10. (a) **METHOD 1**

$a = 5$ ***(A1)***

attempt to use roots and symmetry to find h ***(M1)***

$h = \frac{(-1)+(-3)}{2}$ OR half the distance between the roots $\frac{(-1)-(-3)}{2} = 1$ (may be seen on a diagram)

$h = -2$ (accept $x = -2$) ***(A1)***

$f(x) = 5(x-(-2))^2 - 5\left(= 5(x+2)^2 - 5\right)$ ***A1***

($a = 5,\ h = -2,\ k = -5$)

METHOD 2

$a = 5$ ***(A1)***

attempt to expand

$(x+1)(x+3) = x^2 + 4x + 3$ OR $5(x+1)(x+3) = 5x^2 + 20x + 15$

EITHER

uses their expansion to attempt to complete the square to the form ***(M1)***

$p(x+q)^2 + r$,where q is half the coefficient of their x term

$= (x+2)^2 - 2^2 + 3\left(= (x+2)^2 - 1\right)$ OR $5\left[(x+2)^2 - 2^2 + 3\right](= 5(x+2)^2 - 5)$ ***(A1)***

OR

uses their expansion to attempt to differentiate and sets equal to zero ***(M1)***

$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x + 4 = 0$ OR $\frac{\mathrm{d}y}{\mathrm{d}x} = 10x + 20 = 0$

$h = -2$ (accept $x = -2$) ***(A1)***

OR

uses their expansion to attempt to find axis of symmetry using $h = \frac{-b}{2a}$ ***(M1)***

$h = \frac{-4}{2}$ OR $h = \frac{-20}{10}$

$h = -2$ (accept $x = -2$) ***(A1)***

THEN

$f(x) = 5(x-(-2))^2 - 5\left(= 5(x+2)^2 - 5\right)$ ***A1***

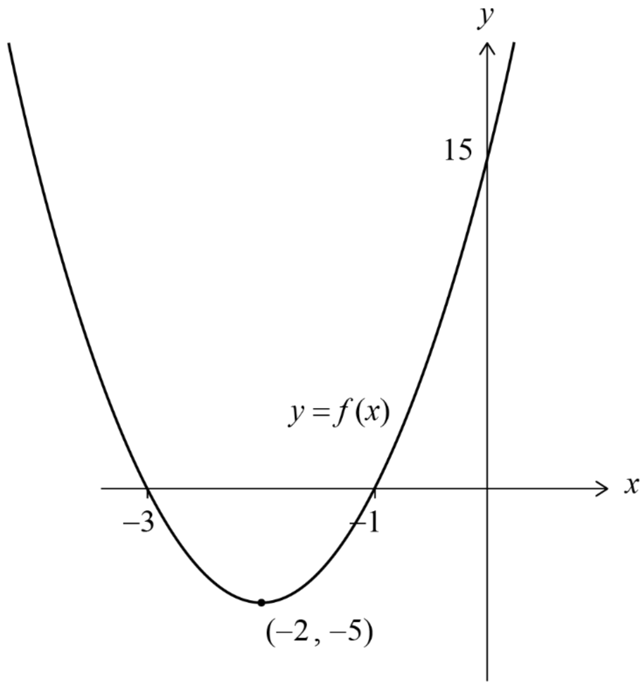
($a = 5,\ h = -2,\ k = -5$)

[4 marks]

continued…

Question 10 continued.

(b)



M1A1A1A1

award ***M1*** for a roughly symmetric curve which is concave up

award ***A1*** for x intercepts at -3 and -1

award ***A1*** for y intercept at 15

award ***A1*** for vertex at $(-2,-5)$

[4 marks]

(c) $5(x+2)^2-5\le 40$ OR $5\left(x+1\right)(x+3)\le 40$ OR $\left(x+1\right)(x+3)\le 8$ leading to

$(x+2)^2\le 9$ OR $5x^2+20x-25\le 0$ OR $x^2+4x-5\le 0$ ***(A1)***

valid attempt to find the critical values for their quadratic inequality ***(M1)***

$x+2=\pm 3$ OR $\left(x+5\right)\left(x-1\right)=0$

$x=-5,\ x=1$ ***(A1)***

$-5\le x\le 1$ ***A1***

Note: Accept $\left(x\in\right)[-5,1]$ or equivalent.

[4 marks]

continued…

Question 10 continued.

(d) (i) $(f \circ g)(x) = 5(\ln x + 1)(\ln x + 3)$ OR $5(\ln x + 2)^2 - 5$ OR
$5(\ln x)^2 + 20\ln x + 15$ ***A1***

(ii) attempt to replace x with $\ln x$ using their solution to part (c) ***(M1)***

$-5 \le \ln x \le 1$

$e^{-5} \le x \le e$ ***A1***

Note: Accept $(x \in)[e^{-5}, e]$ or equivalent.

[3 marks]

(e) $(g \circ f)(x) = \ln(f(x))$

recognition that the domain requires $f(x) > 0$ ***(M1)***

$(x+1)(x+3) > 0$

$x < -3, x > -1$ ***A1A1***

Note: award **A1** for critical values and **A1** for correct inequalities. accept $(x \in)(-\infty, -3) \cup (-1, \infty)$ or equivalent.

[3 marks]

Total [18 marks]

11. (a) normal of Π_1 and Π_2 are $\begin{pmatrix}1\\2\\1\end{pmatrix}$ and $\begin{pmatrix}1\\-1\\-2\end{pmatrix}$ ***(A1)***

attempt to use the scalar product for the angle between two vectors ***M1***

$$\begin{pmatrix}1\\2\\1\end{pmatrix}\bullet\begin{pmatrix}1\\-1\\-2\end{pmatrix}=\left|\begin{pmatrix}1\\2\\1\end{pmatrix}\right|\left|\begin{pmatrix}1\\-1\\-2\end{pmatrix}\right|\cos\theta$$

$-3=\sqrt{6}\sqrt{6}\cos\theta$ $\left(\text{so } -3=6\cos\theta\right)$ ***A1A1***

Note: Award ***A1*** for correct scalar product and ***A1*** for correct magnitudes of normals and $\cos\theta$.

$\cos\theta=-\frac{1}{2}$ ***A1***

$\theta=120^\circ$ OR $\cos\left(180^\circ-\theta\right)=\frac{1}{2}$ ***A1***

acute angle is 60° ***AG***

[6 marks]

(b) attempt to find vector product of their normal vectors ***M1***

$$\begin{pmatrix}1\\2\\1\end{pmatrix}\times\begin{pmatrix}1\\-1\\-2\end{pmatrix}=\begin{pmatrix}-4+1\\1+2\\-1-2\end{pmatrix}$$

$$=\begin{pmatrix}-3\\3\\-3\end{pmatrix}\left(=3\begin{pmatrix}-1\\1\\-1\end{pmatrix}\right)$$ ***(A1)***

equation of Π_3 is $-3x+3y-3z=d$ OR $-x+y-z=d$ or equivalent

attempt to substitute $x=5,\ y=-5,\ z=5$ into their equation ***(M1)***

equation of Π_3 is $-3x+3y-3z=-45$ $\left(\text{so } -x+y-z=-15\right)$ ***A1***

[4 marks]

continued…

Question 11 continued.

(c) (i) attempt to use the identity for $\sin\left(30^\circ + 45^\circ\right)$ ***(M1)***

$\sin\left(30^\circ + 45^\circ\right) = \sin\left(30^\circ\right)\cos\left(45^\circ\right) + \cos\left(30^\circ\right)\sin\left(45^\circ\right)$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$ ***A1A1***

$= \frac{\sqrt{2}+\sqrt{6}}{4}$ ***AG***

Note: ***A1*** *for each term.* Award ***A1 A0*** for correct answers where the denominator has not been rationalized such as $= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$.

(ii) let $x =$ QR

attempt to use the sine rule with the angles $75^\circ, 45^\circ$ ***(M1)***

$\frac{5}{\sin 75^\circ} = \frac{x}{\sin 45^\circ}$

$x = \frac{20}{\sqrt{2}\left(\sqrt{2}+\sqrt{6}\right)}\left(= \frac{20}{2\left(1+\sqrt{3}\right)}\right)$ or equivalent ***A1***

attempt to rationalise a denominator of the form $\left(\sqrt{a}+\sqrt{b}\right)$ by multiplying numerator and denominator by $\pm\left(\sqrt{a}-\sqrt{b}\right)$ or equivalent ***(M1)***

$x = \frac{20\left(\sqrt{2}-\sqrt{6}\right)}{\sqrt{2}\left(\sqrt{2}+\sqrt{6}\right)\left(\sqrt{2}-\sqrt{6}\right)}\left(= \frac{20\left(1-\sqrt{3}\right)}{2\left(1+\sqrt{3}\right)\left(1-\sqrt{3}\right)}\right)$

$x = 5\left(\sqrt{3}-1\right)$ (cm) ($p = 5,\ q = 3$) ***A1***

[7 marks]

Total [17 marks]

12. (a) attempt to use integration by parts on $f_n(x) = \cos^{n-1} x \cos x$ ***M1***

$u = \cos^{n-1} x,\ \dfrac{dv}{dx} = \cos x$

$\dfrac{du}{dx} = -(n-1)\cos^{n-2} x \sin x,\ v = \sin x$

$\int f_n(x)dx = \cos^{n-1} x \sin x + \int (n-1)\cos^{n-2} x \sin^2 x dx$ ***A1A1***

Note: ***A1*** for each term with correct signs

$= \cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x\left(1-\cos^2 x\right)dx$ ***A1***

$= \cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x dx - (n-1)\int \cos^n x dx$ ***AG***

[4 marks]

(b) attempt to rearrange the equation given in part (a) to collect terms in $\int f_n(x)dx$ or $\int \cos^n x dx$ ***M1***

$\int f_n(x)dx + (n-1)\int f_n(x)dx = \cos^{n-1} x \sin x + (n-1)\int f_{n-2}(x)dx$ OR
$\int \cos^n x dx + (n-1)\int \cos^n x dx = \cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x dx$

$n\int f_n(x)dx = \cos^{n-1} x \sin x + (n-1)\int f_{n-2}(x)dx$ OR
$n\int \cos^n x dx = \cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x dx$ ***A1***

$\int f_n(x)dx = \dfrac{1}{n}\cos^{n-1} x \sin x + \dfrac{n-1}{n}\int f_{n-2}(x)dx$ ***AG***

[2 marks]

continued…

Question 12 continued.

(c) attempt to use equation from part (a) to reduce the power of $\cos x$ ***(M1)***

$$\int \cos^4 x\mathrm{d}x = \frac{1}{4}\cos^3 x \sin x + \frac{3}{4}\int \cos^2 x\mathrm{d}x$$

EITHER

attempt to use equation from part (a) again to reduce the power of $\cos x$ ***(M1)***

$$\int \cos^4 x\mathrm{d}x = \frac{1}{4}\cos^3 x \sin x + \frac{3}{4}\left(\frac{1}{2}\cos x \sin x + \frac{1}{2}\int 1\mathrm{d}x\right)$$ ***(A1)***

OR

attempt to use double angle formula to rewrite $\cos^2 x$ in terms of $\cos 2x$ ***(M1)***

$$\int \cos^4 x\mathrm{d}x = \frac{1}{4}\cos^3 x \sin x + \frac{3}{4}\int\left(\frac{\cos 2x + 1}{2}\right)\mathrm{d}x$$ ***(A1)***

THEN

$$= \frac{1}{4}\cos^3 x \sin x + \frac{3}{8}\cos x \sin x + \frac{3}{8}x + c$$ ***A1***

[4 marks]

(d) attempt to use the formula for volume of revolution using π and $\left(\cos^2 x\right)^2$ ***(M1)***

volume $= \int \pi\left(\cos^2 x\right)^2 \mathrm{d}x$

volume $= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \pi\left(\cos^2 x\right)^2 \mathrm{d}x$ OR $2\int_{0}^{\frac{\pi}{2}} \pi\left(\cos^2 x\right)^2 \mathrm{d}x$ ***(A1)***

Note: Condone omission $\mathrm{d}x$ for the ***A1***.

attempt to substitute correct limits into their (c) and subtract ***(M1)***

$$\pi\left(\frac{1}{4}\cos^3\frac{\pi}{2}\sin\frac{\pi}{2} + \frac{3}{8}\cos\frac{\pi}{2}\sin\frac{\pi}{2} + \frac{3}{8}\left(\frac{\pi}{2}\right)\right) - \pi\left(\frac{1}{4}\cos^3\left(-\frac{\pi}{2}\right)\sin\left(-\frac{\pi}{2}\right) + \frac{3}{8}\cos\left(-\frac{\pi}{2}\right)\sin\left(-\frac{\pi}{2}\right) + \frac{3}{8}\left(-\frac{\pi}{2}\right)\right)$$

$$= \frac{3\pi^2}{8}$$ ***A1***

Note: ***FT*** marks may be awarded for a final answer of $r\pi^2$ based on non-zero values of p, q and r.

[4 marks]

continued…

Question 12 continued.

(e) (i) **METHOD 1**

attempt to raise Maclaurin expansion for $\cos x$ to the power of n ***(M1)***

$$f_n(x) = (\cos x)^n = \left(1 - \frac{x^2}{2} + \ldots\right)^n$$

$$= 1 - \frac{nx^2}{2} + \ldots$$ ***A2***

METHOD 2

attempt to differentiate $f_n(x)$ twice ***(M1)***

$$f_n\text{'}(x) = -n\cos^{n-1} x \sin x,\ f_n\text{''}(x) = -n\cos^n x + n(n-1)\cos^{n-2} x \sin^2 x$$ ***A1***

$$f_n(0) = 1,\ f_n\text{'}(0) = 0,\ f_n\text{''}(0) = -n$$

$$f_n(x) = 1 - \frac{nx^2}{2} + \ldots$$ ***A1***

(ii) **METHOD 1**

attempt to use Maclaurin expansion for $f_n(x)$ ***(M1)***

$$\lim_{x \to 0} \frac{f_n(x) - 1}{x^2} = \lim_{x \to 0} \frac{-\frac{nx^2}{2} + \ldots}{x^2} \left(= \lim_{x \to 0}\left(-\frac{n}{2} + \text{ powers of } x\right)\right)$$

$$= -\frac{n}{2}$$ ***A1***

METHOD 2

attempt to use l'Hôpital's rule twice on $\frac{\cos^n(x) - 1}{x^2}$ ***(M1)***

$$\lim_{x \to 0} \frac{\cos^n(x) - 1}{x^2} = \lim_{x \to 0} \frac{-n\cos^{n-1} x \sin x}{2x} = \lim_{x \to 0} \frac{n(n-1)\cos^{n-2} x \sin^2 x - n\cos^n x}{2}$$

$$= -\frac{n}{2}$$ ***A1***

Note: Do not award ***FT*** marks for an expression that does not involve n.

[5 marks]

Total [19 marks]