IB Mathematics: Analysis and Approaches SL — Paper 1 TZ A — Mark Scheme

May 2026 | Maximum mark: 80 | No calculator

Question	Answer & Explanation	Marks
Question 1 [6 marks] — $f(x) = 2x + \sin x,\ x \in \mathbb{R}$		
1 (a)	$f'(x) = 2 + \cos x$ Differentiate term by term: derivative of $2x$ is 2, derivative of $\sin x$ is $\cos x$.	2
1 (b)	Point: $f(\pi) = 2\pi + \sin\pi = 2\pi \rightarrow (\pi, 2\pi)$ Gradient: $f'(\pi) = 2 + \cos\pi = 2 - 1 = 1$ Tangent: $y - 2\pi = 1\cdot(x - \pi) \rightarrow y = x + \pi$	4
Question 2 [5 marks] — $f(x) = 3(x - 2)(x - q)$, axis of symmetry $x = 4$		
2 (a)	Zeros at $x = 2$ and $x = q$. Axis of symmetry: $(2 + q)/2 = 4 \rightarrow q = 6$ ✓	2
2 (b)	$f(4) = 3(4 - 2)(4 - 6) = 3 \times 2 \times (-2) = -12$ Vertex: $(4, -12)$	2
2 (c)	Parabola opens upward (leading coefficient $3 > 0$), so vertex is a minimum. Range: $f(x) \geq -12$	1
Question 3 [5 marks] — $v = 1/(t + 1) + e^{-t},\ t \geq 0$		
3 (a)	$\int [1/(t+1) + e^{-t}]\,dt = \ln\lvert t + 1\rvert - e^{-t} + C$ Integrate $1/(t+1) \rightarrow \ln\lvert t+1\rvert$; integrate $e^{-t} \rightarrow -e^{-t}$.	2
3 (b)	$s(t) = \ln(t + 1) - e^{-t} + C$ Apply $s(0) = 0$: $\ln(1) - e^{0} + C = 0 \rightarrow 0 - 1 + C = 0 \rightarrow C = 1$ $s(t) = \ln(t + 1) - e^{-t} + 1$	3
Question 4 [6 marks] — Geometric sequence: $u_6 = 1.6 \times 10^5$, $u_8 = 6.4 \times 10^5$		
4 (a)	$r^2 = u_8 / u_6 = (6.4 \times 10^5) / (1.6 \times 10^5) = 4$ All terms positive $\rightarrow r > 0 \rightarrow r = 2$	3
4 (b)	$u_1 = u_6 / r^5 = (1.6 \times 10^5) / 2^5 = (1.6 \times 10^5) / 32 = 5000$ $u_1 = 5 \times 10^3$ ($a = 5, k = 3$)	3
Question 5 [7 marks] — Logarithms		
5 (a)	Write $81x^2 = (9x)^2$ and $25 = 5^2$. $\log_{25}(81x^2) = \log_{5^2}((9x)^2) = [2\cdot\log_5(9x)] / 2 = \log_5(9x) = \log_5 9 + \log_5 x$ ✓	4
5 (b)	Apply (a): $\log_5 \sqrt[3]{x} = \log_5 9 + \log_5 x - \log_5 4 = \log_5(9x/4)$ So $x^{1/3} = 9x/4$. Let $u = x^{1/3}$: $u = 9u^3/4 \rightarrow u^2 = 4/9 \rightarrow u = 2/3$ $x = (2/3)^3 = 8/27$	3
Question 6 [9 marks] — Events A and B		
6 (a)	$P(A) = P(A\cap B') + P(A\cap B) = (5+x)/24 + (7-x)/12 = (5+x)/24 + 2(7-x)/24$ $= (5 + x + 14 - 2x) / 24 = (19 - x) / 24$ ✓	3
6 (b)	$P(B) = P(A\cap B) + P(A'\cap B) = (7-x)/12 + 1/6 = (9-x)/12$ Independence: $P(A\cap B) = P(A)\cdot P(B) \rightarrow (7-x)/12 = [(19-x)/24]\cdot[(9-x)/12]$ $24(7-x) = (19-x)(9-x) \rightarrow 168 - 24x = 171 - 28x + x^2$ $x^2 - 4x + 3 = 0 \rightarrow (x-1)(x-3) = 0 \rightarrow x = 1$ or $x = 3$	6

Question	Answer & Explanation	Marks
Question 7 [12 marks] — $f(x) = (2x - 3)/(3 - x),\ x \neq 3$		
7 (a)	(i) Vertical asymptote: $3 - x = 0 \rightarrow x = 3$ (ii) Horizontal asymptote: $\lim (2x-3)/(3-x)$ as $x\to\infty = -2 \rightarrow y = -2$	3
7 (b)	x-intercept: $2x - 3 = 0 \rightarrow x = 3/2 \rightarrow (3/2, 0)$ y-intercept: $f(0) = (0-3)/(3-0) = -1 \rightarrow (0, -1)$	4
7 (c)	(i) $(g\circ f)(x) = f(x) + 4$. Translation by vector (0, 4) — 4 units upward. (ii) Range of f is $\mathbb{R} \setminus \{-2\}$. Shifting up 4 moves the excluded value to 2. Range of $g\circ f$: $y \in \mathbb{R},\ y \neq 2$	5
Question 8 [15 marks] — $f(x) = ax^3 + bx^2 + cx + d$, inflexion at $(-2, -2c+d+24)$		
8 (a)	$f''(x) = 6ax + 2b$. Inflexion at $x = -2$: $f''(-2) = 0 \rightarrow b = 6a$ ...(1) $f(-2) = -2c+d+24 \rightarrow -8a+4b = 24 \rightarrow -2a+b = 6$ ...(2) Sub (1) into (2): $4a = 6 \rightarrow a = 3/2,\ b = 9$ ✓	6
8 (b)	$f'(x) = (9/2)x^2 + 18x + c$ (i) $f'(-3) = 0$: $81/2 - 54 + c = 0 \rightarrow c = 27/2$ ✓ (ii) $f'(k) = 0$: $k^2 + 4k + 3 = 0 \rightarrow (k+1)(k+3) = 0$. $k \neq -3 \rightarrow k = -1$ (iii) $f''(-1) = 9(-1) + 18 = 9 > 0 \rightarrow$ local minimum at $x = -1$	7
8 (c)	$P = (0, d)$. $f(-3) = -81/2 + 81 - 81/2 + d = d \rightarrow$ point $(-3, d)$ is on the curve. $f'(-3) = 0 \rightarrow$ tangent at $x = -3$ is horizontal: $y = d$. $P = (0, d)$ lies on $y = d \rightarrow$ tangent passes through P. ✓	2
Question 9 [15 marks] — $F(\theta) = A / (\cos\theta + \mu \sin\theta)$		
9 (a)	$\theta = 0$: $F(0) = A / (1 + 0) = A = 300$ ✓	2
9 (b)	$\theta = \pi/6$, $F = 100\sqrt{3}$: $100\sqrt{3} = 300 / (\sqrt{3}/2 + \mu/2) = 600 / (\sqrt{3} + \mu)$ $\sqrt{3} + \mu = 600/(100\sqrt{3}) = 2\sqrt{3} \rightarrow \mu = \sqrt{3}$ ✓	4
9 (c)(i)	$F(\theta) = 300(\cos\theta + \sqrt{3}\sin\theta)^{(-1)}$. Chain rule: $dF/d\theta = 300\times(-1)(\cos\theta + \sqrt{3}\sin\theta)^{(-2)}\times(-\sin\theta + \sqrt{3}\cos\theta)$ $= 300(\sin\theta - \sqrt{3}\cos\theta) / (\cos\theta + \sqrt{3}\sin\theta)^2$ ✓	5
9 (c)(ii)	$dF/d\theta = 0$: $\sin\theta = \sqrt{3}\cos\theta \rightarrow \tan\theta = \sqrt{3} \rightarrow \theta = \pi/3 \rightarrow \alpha = \pi/3$ ✓ $F(\pi/3) = 300 / (\cos(\pi/3) + \sqrt{3}\sin(\pi/3)) = 300 / (1/2 + 3/2) = 300/2 = 150$ N	4