

# INTERNATIONAL BACCALAUREATE

Mathematics: Applications and Interpretation SL — Paper 1 Mark Scheme

## General Marking Instructions

- **M Marks:** Awarded for method. Explicitly show the substitution or technique used.
- **A Marks:** Awarded for accuracy (correct answers).
- **R Marks:** Awarded for reasoning or justification.
- **AG:** Answer Given in the question (cannot be awarded marks on its own).
- Full marks are awarded for a correct answer with appropriate working shown. If no working is shown and the answer is correct, full marks are typically given unless otherwise specified.

## Question 1

[Maximum mark: 5]

(a) Use this model to find the surface area of the Earth.

[2 marks]

- **Method:** Use the surface area of a sphere formula:  $Area = 4\pi r^2$   
 $Area = 4 \times \pi \times 6371^2$  (M1)
- **Answer:** =  $510064472 \text{ km}^2 \approx 5.10 \times 10^8 \text{ km}^2$  (or  $510000000 \text{ km}^2$ ) (A1)

(b) Use the model to find the percentage of the surface area of the Earth that is covered by water.

[3 marks]

- **Method 1:** Find the total water area first.  
 $Water \text{ Area} = 510064472 - 148900000 = 361164472 \text{ km}^2$  (A1)  
 $Percentage = (361164472 / 510064472) \times 100\%$  (M1)
- **Method 2:** Find the percentage of land and subtract from 100%.  
 $Land \% = (148.9 \times 10^6) / (510.06 \times 10^6) \times 100\% \approx 29.192\%$   
 $Water \% = 100\% - 29.192\%$  (M1)
- **Answer:** = 70.8% (accept 70.803%) (A1)

## Question 2

[Maximum mark: 6]

(a) Calculate the size of  $\hat{C}AB$ .

[3 marks]

- **Method:** Apply the Sine Rule:  $\sin A / a = \sin C / c$

$$\sin(\hat{C}AB) / 29 = \sin(40^\circ) / 35 \text{ (M1)}$$

$$\sin(\hat{C}AB) = (29 \times \sin(40^\circ)) / 35 \approx 0.5325 \text{ (A1)}$$

- **Answer:**  $\hat{C}AB = \arcsin(0.5325) \approx 32.2^\circ$  (accept  $32.173^\circ$  or  $0.562$  radians) (A1)

(b) Calculate the area of triangle ABC.

[3 marks]

- **Method:** Find the missing angle  $\hat{A}B\hat{C}$  first.

$$\hat{A}B\hat{C} = 180^\circ - 40^\circ - 32.173^\circ = 107.827^\circ \text{ (A1)}$$

Apply the Area of a Triangle formula:  $\text{Area} = \frac{1}{2}ab \sin C$

$$\text{Area} = \frac{1}{2} \times 35 \times 29 \times \sin(107.827^\circ) \text{ (M1)}$$

- **Answer:**  $\text{Area} \approx 483 \text{ m}^2$  (accept  $483.15 \text{ m}^2$ ) (A1)

## Question 3

[Maximum mark: 5]

(a) Write down the modal group.

[1 mark]

- **Answer:**  $120 < n \leq 140$  (A1)

(b) Calculate an estimate of the mean and standard deviation.

[3 marks]

- **Method:** Determine interval midpoints ( $x$ ): 70, 90, 110, 130, 150. Enter frequencies ( $f$ ) and midpoints into GDC 1-Variable Statistics. (M1)

- (i) **Mean ( $\mu$  or  $\bar{x}$ ):**  $\bar{x} = \sum fx / \sum f = 24220 / 200 = 121.1$  (A1)

- (ii) **Standard deviation ( $\sigma$ ):**  $\sigma \approx 17.5$  (accept  $17.498$ ) (A1)

(Note: if sample standard deviation  $s_{n-1} = 17.542$  is written, award A0 unless specified as sample).

(c) Explain why the values found in part (b) are said to be estimates.

[1 mark]

- **Answer:** The exact individual data points within each group interval are unknown; the calculation assumes all data points are located exactly at the midpoints of their respective intervals. (R1)

## Question 4

[Maximum mark: 6]

(a) (i) Calculate the product-moment correlation coefficient,  $r$ .

[3 marks total for (a)]

- **Answer:**  $r \approx 0.845$  (accept  $0.84547$ ) (A1)

(a) (ii) Find the equation of the regression line of  $n$  on  $T$ .

- **Method:** Calculate using GDC linear regression features ( $y = ax + b$  form).
- **Answer:**  $n = 2.37T - 21.0$  (A1)(A1)  
(Award A1 for 2.37, A1 for  $-21.0$ . Accept  $n = 2.3734T - 21.049$ ).

(b) (i) Use your equation to estimate the number of children... at  $30^\circ\text{C}$ .

[3 marks total for (b)]

- **Method:** Substitute  $T = 30$  into the regression line equation.  
 $n = 2.3734(30) - 21.049 \approx 50.153$  (M1)
- **Answer:**  $\approx 50$  children (accept answers rounding to 50) (A1)

(b) (ii) Explain why your answer may not be accurate.

- **Answer:** Extrapolation. The temperature  $30^\circ\text{C}$  lies well outside the recorded range of the data set ( $12^\circ\text{C}$  to  $25^\circ\text{C}$ ), meaning the linear trend may not hold true. (R1)

## Question 5

[Maximum mark: 5]

(a) Write down the value of  $a$ .

[1 mark]

- **Answer:** At  $t = 0$ ,  $f(0) = a \times b^0 = a$ .  
 $a = 104000$  (A1)

(b) Find the exact value of  $b$ .

[2 marks]

- **Method:** Substitute  $t = 1$  and  $f(1) = 105144$ .  
 $105144 = 104000 \times b^1$  (M1)
- **Answer:**  $b = 105144 / 104000 = 1.011$  (A1)

(c) Use the model to estimate the population... at the beginning of 2034.

[2 marks]

- **Method:** Find  $t$ :  $2034 - 2024 = 10$  years. Substitute  $t = 10$ .  
 $f(10) = 104000 \times (1.011)^{10} \approx 116016.31$  (M1)
- **Answer:**  $116016$  (A1) (Must be correct to the nearest integer).

## Question 6

[Maximum mark: 5]

(a) Find the coordinates of F.

[2 marks]

- **Method:** Use the 3D midpoint formula between A(0, 60, 240) and B(40, 0, 240).

$$F = ((0+40)/2, (60+0)/2, (240+240)/2) \text{ (M1)}$$

- **Answer:**  $F(20, 30, 240)$  (A1)

(b) Calculate the distance between the spider and the fly.

[3 marks]

- **Method:** Use the 3D distance formula between F(20, 30, 240) and S(40, 0, 160).

$$d = \sqrt{[(40-20)^2 + (0-30)^2 + (160-240)^2]} \text{ (M1)}$$

$$d = \sqrt{[20^2 + (-30)^2 + (-80)^2]} = \sqrt{[400 + 900 + 6400]} = \sqrt{[7700]} \text{ (A1)}$$

- **Answer:**  $d \approx 87.7 \text{ cm}$  (accept  $87.749 \text{ cm}$  or  $10\sqrt{77}$ ) (A1)

## Question 7

[Maximum mark: 9]

(a) Write down the probability that this person is less than 30 years...

[1 mark]

- **Answer:**  $18 / 180 = 1/10 = 0.1$  (A1)

(b) Given that this person is more than 50 years... find the probability...

[2 marks]

- **Method:** Total people > 50 is  $19 + 34 = 53$ .

$$P(\text{Hot} | n > 50) = 19 / 53 \text{ (M1)}$$

- **Answer:**  $\approx 0.358$  (accept  $0.35849$ ) (A1)

(c) Write down the null hypothesis.

[1 mark]

- **Answer:** Preferred type of drink and age are independent. (A1)

(d) Calculate the test statistic and p-value.

[3 marks]

- **Method:** Enter the contingency matrix into GDC  $\chi^2$  two-way test function. (M1)

- (i)  $\chi^2$  test statistic:  $\chi^2 \approx 9.77$  (accept  $9.7692$ ) (A1)

- (ii) p-value:  $p \approx 0.00756$  (accept  $0.007562$ ) (A1)

(e) Write down the conclusion for the test, giving a reason.

[2 marks]

- **Reasoning:** Compare either  $\chi^2$  statistic ( $9.77$ ) > critical value ( $5.991$ ) OR p-value ( $0.00756$ ) < significance level ( $0.01$ ). (R1)

- **Conclusion:** Reject the null hypothesis ( $H_0$ ). There is significant evidence to suggest that age and preferred drink are dependent. (A1)

## Question 8

[Maximum mark: 6]

(a) Find the height of the bean plant at the end of day 10.

[3 marks]

- **Method:** This models a geometric sequence where  $u_1 = 3$  and  $r = 1.14$ .

$$u_{10} = 3 \times (1.14)^{10-1} \text{ (M1)(A1)}$$

(Award M1 for geometric setup, A1 for exponent index of 9).

- **Answer:**  $= 3 \times 3.2519 \approx 9.76 \text{ cm}$  (accept 9.7557 cm) (A1)

(b) Calculate the number of days it takes... to be higher than 1.9 metres.

[3 marks]

- **Method:** Convert units:  $1.9 \text{ m} = 190 \text{ cm}$ . Set up inequality:

$$3 \times (1.14)^{n-1} > 190 \text{ (M1)}$$

$$(1.14)^{n-1} > 190 / 3$$

$$(n-1) \ln(1.14) > \ln(190 / 3) \Rightarrow n - 1 > 31.67 \text{ (A1)}$$

$$n > 32.67$$

- **Answer:** 33 days (A1) (Must be a whole integer value).

## Question 9

[Maximum mark: 8]

(a) Calculate the value of Samira's investment at the end of 5 years...

[3 marks]

- **Method:** Compound interest formula or Financial TVM solver profile:

$$N = 60, I\% = 1.8, PV = -2000, P/Y = 12, C/Y = 12 \text{ (M1)}$$

$$A = 2000 \times (1 + 0.018/12)^{60} \text{ (A1)}$$

- **Answer:**  $= \$2188.13$  (A1) (Must be exactly 2 decimal places).

(b) Calculate the number of years it would take... to reach \$5000.

[2 marks]

- **Method:** Set up equation or use TVM solver:  $5000 = 2000(1.0015)^{12t}$

$$12t = \ln(2.5) / \ln(1.0015) \approx 611.45 \text{ months (M1)}$$

$$t = 50.95 \text{ years}$$

- **Answer:** 51 years (A1) (Must be correct to the nearest year).

(c) Calculate her monthly payments, correct to two decimal places.

[3 marks]

- **Method:** Find loan balance value:  $4000 - 1400 = \$2600$ .

Use Financial TVM app parameters:

$$N = 24, I\% = 3.8, PV = 2600, FV = 0, P/Y = 12, C/Y = 12 \text{ (M1)(A1)}$$

(Award M1 for correct loan computation, A1 for correct inputs).

- **Answer:**  $PMT = \$112.67$  (A1)

## Question 10

[Maximum mark: 6]

(a) Write down the null and alternative hypotheses.

[1 mark]

• **Answer:**

$H_0$ : The spinner is unbiased (all outcomes have a probability of 0.25).

$H_1$ : The spinner is biased. (A1)

(b) Write down the expected frequency of the spinner landing on 1.

[1 mark]

• **Answer:**  $Expected = \frac{1}{4} \times 60 = 15$  (A1)

(c) (i) Determine the p-value.

[4 marks total for (c)]

• **Method:** Perform  $\chi^2$  goodness-of-fit test via GDC with degrees of freedom  $df = 4 - 1 = 3$ . Expected counts are (15, 15, 15, 15).

$$\chi^2 = (12-15)^2/15 + (18-15)^2/15 + (9-15)^2/15 + (21-15)^2/15 = 6.0 \text{ (M1)}$$

• **Answer:**  $p\text{-value} \approx 0.112$  (accept 0.11161) (A1)

(c) (ii) State what the result of the test indicates about Kalia's spinner. Justify your answer.

• **Reasoning:** Since  $p\text{-value} (0.112) > \text{significance level} (0.10)$ . (R1)

• **Conclusion:** Fail to reject  $H_0$ . There is insufficient evidence to conclude that the spinner is biased; Kalia's belief stands. (A1)

## Question 11

[Maximum mark: 7]

(a) Find the 42<sup>nd</sup> term of the sequence.

[2 marks]

- **Method:** Use arithmetic sequence formula:  $u_n = u_1 + (n-1)d$

$$u_{42} = 18 + (42-1)7 \text{ (M1)}$$

- **Answer:**  $= 18 + 287 = 305$  (A1)

(b) Find a quadratic equation in terms of  $n$  that represents this information...

[3 marks]

- **Method:** Use arithmetic sum formula:  $S_n = n/2 [2u_1 + (n-1)d]$

$$9475 = n/2 [2(18) + (n-1)7] \text{ (M1)}$$

$$18950 = n[36 + 7n - 7] \Rightarrow 18950 = 29n + 7n^2 \text{ (A1)}$$

- **Answer:**  $7n^2 + 29n - 18950 = 0$  (A1) (Must be in the explicit form requested).

(c) Find the value of  $n$ .

[2 marks]

- **Method:** Solve via factoring, formula, or polynomial solver:

$$(7n + 395)(n - 50) = 0 \text{ (M1)}$$

- **Answer:**  $n = 50$  (A1) (Reject negative root since  $n$  must be a positive integer).

## Question 12

[Maximum mark: 5]

Find  $f(x)$ .

[5 marks]

- **Method:** Find original expression by integrating the gradient function.

$$f(x) = \int (6x^2 - 2x + 3) dx \text{ (M1)}$$

$$f(x) = (6x^3/3) - (2x^2/2) + 3x + C = 2x^3 - x^2 + 3x + C \text{ (A1)(A1)(A1)}$$

(Award A1 for each integrated term respectively).

- **Method:** Use the initial boundary point condition (2, 12) to calculate  $C$ .

$$12 = 2(2)^3 - (2)^2 + 3(2) + C$$

$$12 = 16 - 4 + 6 + C \Rightarrow 12 = 18 + C \Rightarrow C = -6 \text{ (M1)}$$

- **Answer:**  $f(x) = 2x^3 - x^2 + 3x - 6$  (A1)

## Question 13

[Maximum mark: 7]

Calculate the coordinates of the point D...

[7 marks]

- **Method:** Find gradient of road passing through A(1, 10) and B(5, 2).

$$m_{\text{road}} = (2 - 10) / (5 - 1) = -8 / 4 = -2 \text{ (A1)}$$

- **Method:** Formulate line equation representing the road layout.

$$y - 2 = -2(x - 5) \Rightarrow y = -2x + 12 \text{ (A1)}$$

- **Method:** Shortest distance from C to the line layout follows a perpendicular path. Determine inverse reciprocal gradient.

$$m_{\perp} = -1 / (-2) = \frac{1}{2} \text{ (A1)}$$

- **Method:** Formulate linear profile equation crossing factory position C(4, 5).

$$y - 5 = \frac{1}{2}(x - 4) \Rightarrow y = \frac{1}{2}x + 3 \text{ (A1)}$$

- **Method:** Calculate intersecting junction point D via synchronous evaluation.

$$-2x + 12 = \frac{1}{2}x + 3 \Rightarrow 9 = 2.5x \Rightarrow x = 3.6 \text{ (M1)(A1)}$$

$$y = -2(3.6) + 12 = 4.8$$

- **Answer:** D(3.6, 4.8) or (18/5, 24/5) (A1)