Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 2

15 May 2026

Zone A morning | **Zone B** morning | **Zone C** morning

Session number

1 hour 30 minutes

Instructions to students

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

438

A002

13 pages

2226–5019

438

A002

Please **do not** write on this page.

Answers written on this page will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Jane makes a cup of tea and forgets to drink it so that it cools towards the temperature of the room.

The temperature $T°C$ of the cup of tea after t minutes is given by $T(t) = 18 + 72e^{-\frac{1}{20}t}$.

(a) Find the initial temperature of the tea. [2]

(b) Find the temperature of the tea after 15 minutes. [2]

(c) Write down the temperature of the room. [1]

(a) $T(0) = 18 + 72e^{0} = 18 + 72 = 90\,°C$

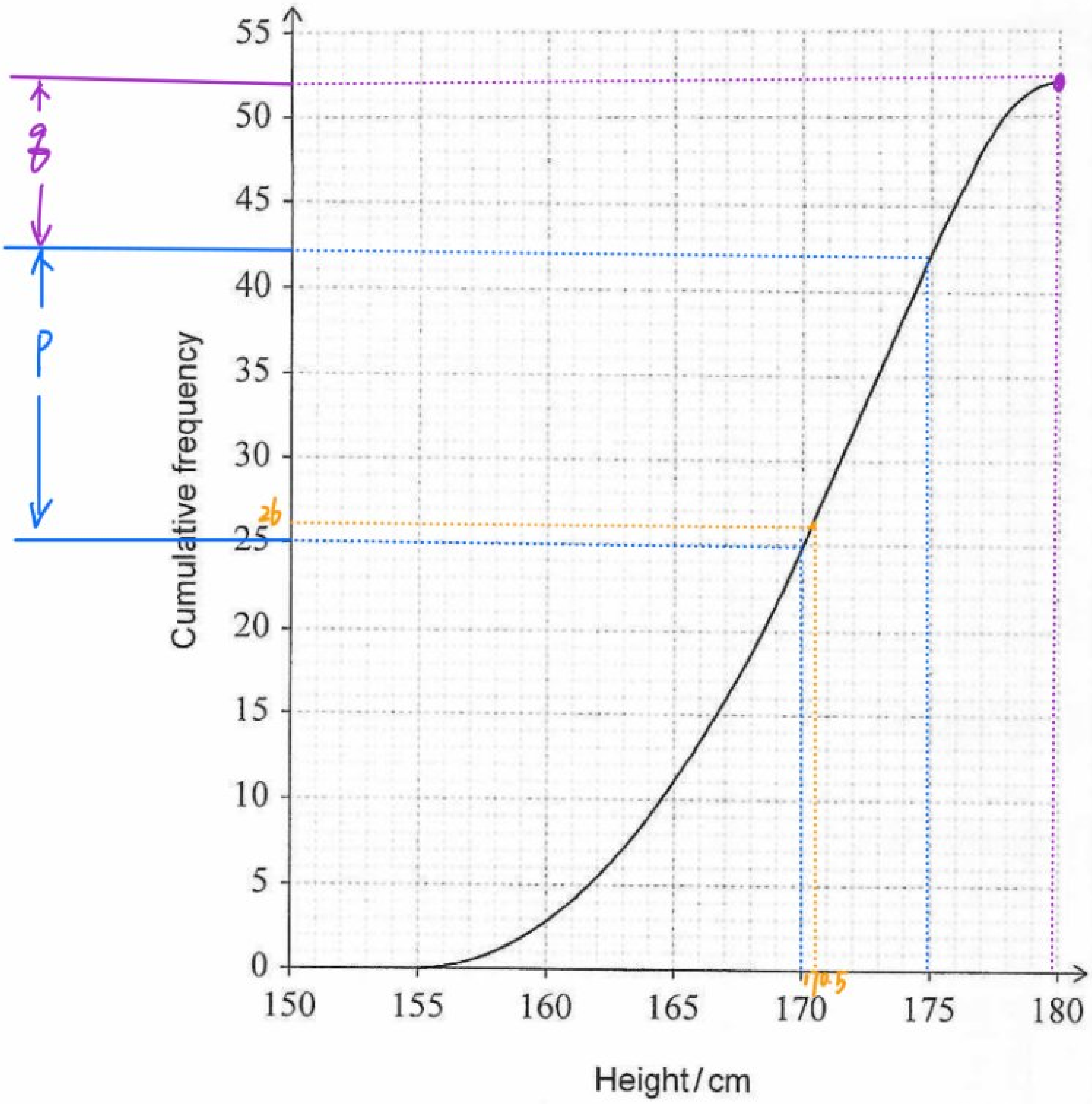
(b) $T(15) = 18 + 72e^{-\frac{1}{20}\times 15} \doteq 52.0\,°C$

(c) $t \to \infty$, $e^{-\frac{1}{20}t} \to 0$, $T(t) \to 18$.

The temperature of the room is $18\,°C$

2. [Maximum mark: 7]

A group of 52 students have their heights measured in cm. The cumulative frequency graph for the heights of the students is shown in the following diagram.



(a) Use the graph to find an estimate of the median height. [2]

438

A002

(This question continues on the following page)

(Question 2 continued)

The heights of the students are shown in the following frequency table where $p, q \in \mathbb{N}$.

Midpoint	Height (h cm)	Frequency
157.5	$155 \le h < 160$	3
162.5	$160 \le h < 165$	8
167.5	$165 \le h < 170$	14
172.5	$170 \le h < 175$	p
177.5	$175 \le h < 180$	q

(b) (i) Find the value of p and the value of q.

(ii) Use the frequency table to find an estimate of the mean height, giving your answer to four significant figures. [5]

(a) $\frac{52}{2} = 26^{th} \Rightarrow$ Median height $= 170.5$ cm

(b) (i) $p = 42 - 25 = 17$

$q = 52 - 42 = 10$

(ii)

Midpoint	Height (h cm)	Frequency
157.5	$155 \le h < 160$	3
162.5	$160 \le h < 165$	8
167.5	$165 \le h < 170$	14
172.5	$170 \le h < 175$	17
177.5	$175 \le h < 180$	10

By GDC: Mean height $= 169.7$ cm

or Mean height $= \dfrac{157.3 \times 3 + 162.5 \times 8 + 167.5 \times 14 + 172.5 \times 17 + 177.5 \times 10}{52} = 169.7$ cm

438

A002

3. [Maximum mark: 4]

Consider a function f which has a first derivative given by $f'(x) = x^2 - x - 1$, where $x \in \mathbb{R}$.

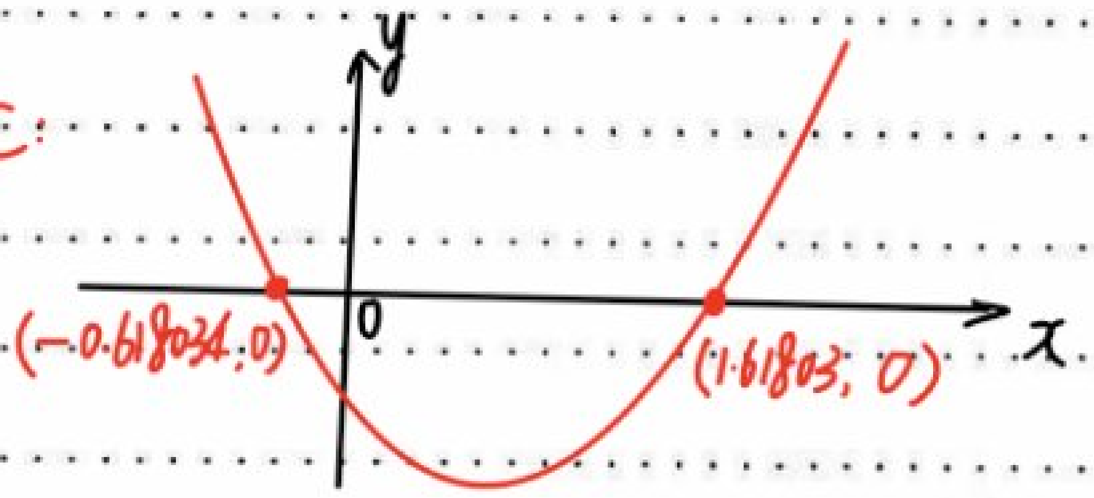
Find the range of values of x for which f is decreasing. [4]

f is decreasing $\Rightarrow f'(x) < 0$

$\Rightarrow x^2 - x - 1 < 0$

M1:

By GDC:



$\therefore -0.618 < x < 1.62$

M2: $x^2 - x - 1 = 0$

$\Rightarrow x = \dfrac{1 \pm \sqrt{5}}{2}$

S.D. of $x^2 - x - 1$:

$+ \quad \dfrac{1-\sqrt{5}}{2} \quad - \quad \dfrac{1+\sqrt{5}}{2} \quad +$ (x)

$\therefore \dfrac{1-\sqrt{5}}{2} < x < \dfrac{1+\sqrt{5}}{2}$

438

A002

4. [Maximum mark: 6]

A Ferris wheel with radius 20 metres rotates at a constant speed. The centre of the wheel is 23 metres above the ground, as shown in the following diagram. P is a point on the wheel. The wheel starts moving with P at the lowest point and completes one revolution in 4 minutes.

$T = 4$

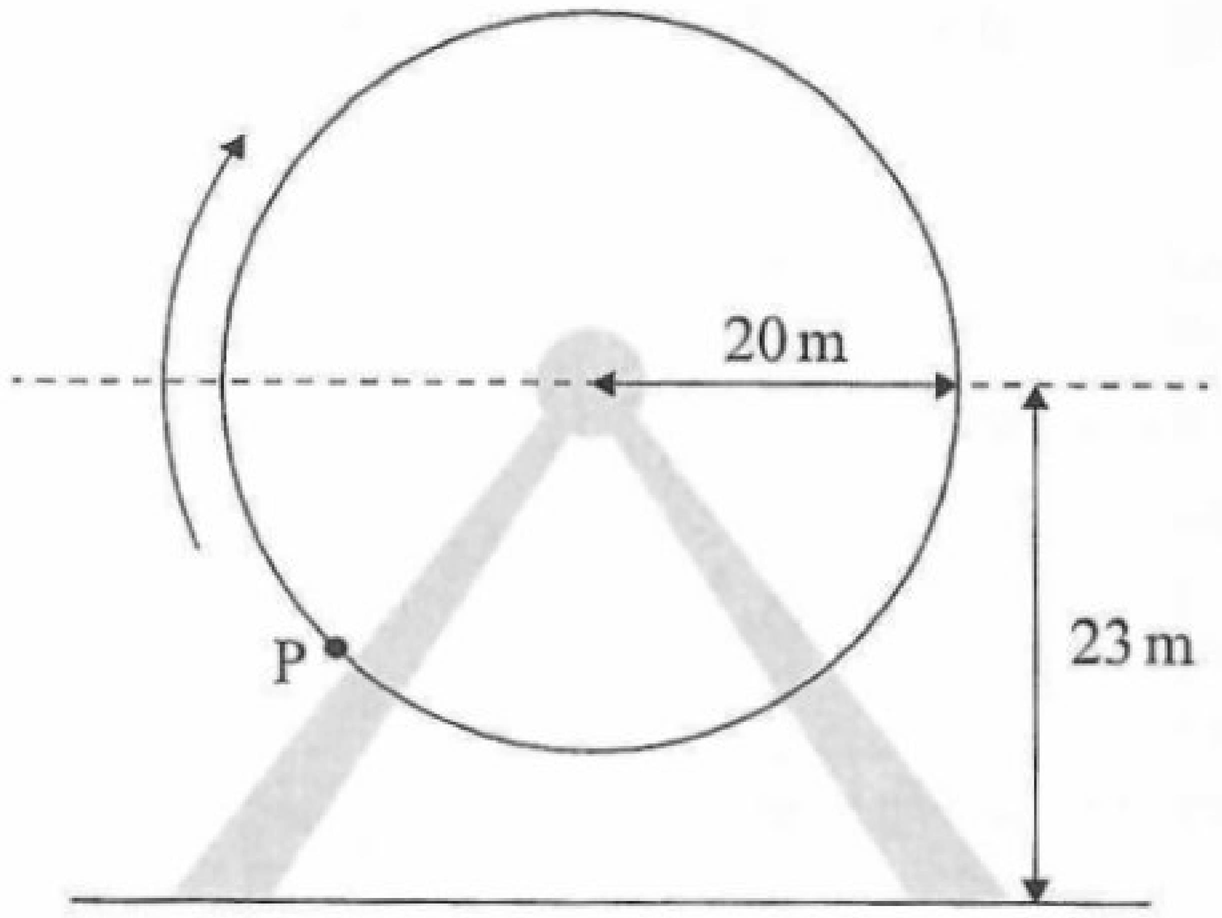
diagram not to scale

$h_{max} = 43$

$h_{min} = 3$

$\frac{43+3}{2} = 23$

$\frac{43-3}{2} = 20.$



Mean line : $h = 23.$

The height, h metres, of P above the ground after t minutes is given by

$$h = a - b\cos(ct)°, \text{ where } a, b, c \in \mathbb{Z}^{+}.$$

Note : GDC Mode : Degree.

(a) Find the value of a, the value of b and the value of c. [4]

The height can also be given as $h(t) = a - b\sin(c(t+d))°$, where $d \in \mathbb{Z}^{+}$.

(b) Find the smallest positive value of d. [2]

(a) Mean line : $h = 23 \Rightarrow a = 23.$

Amplitude $= 20$

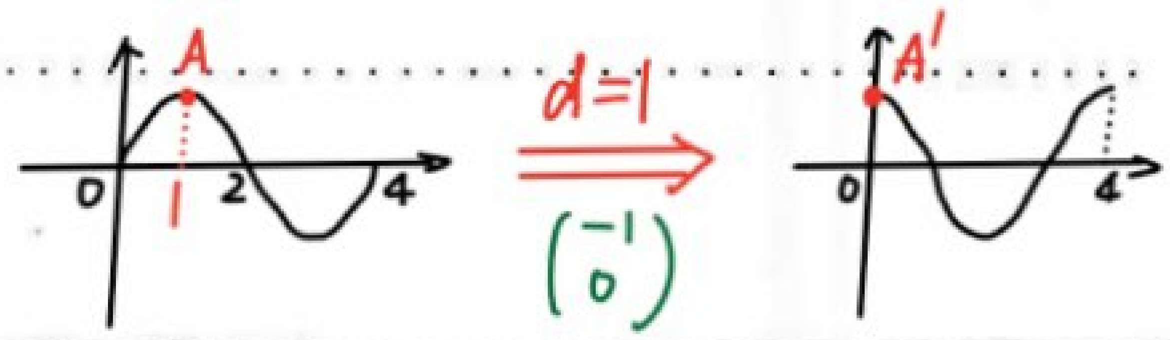
The wheel starts moving with P at the lowest point

$\Rightarrow -b = -20 \Rightarrow b = 20$

$T = 4 \Rightarrow \frac{360}{c} = 4 \Rightarrow c = 90.$

(b) $\sin(90(t+d))° = \cos(90t)°$

$d = 1$



438

A002

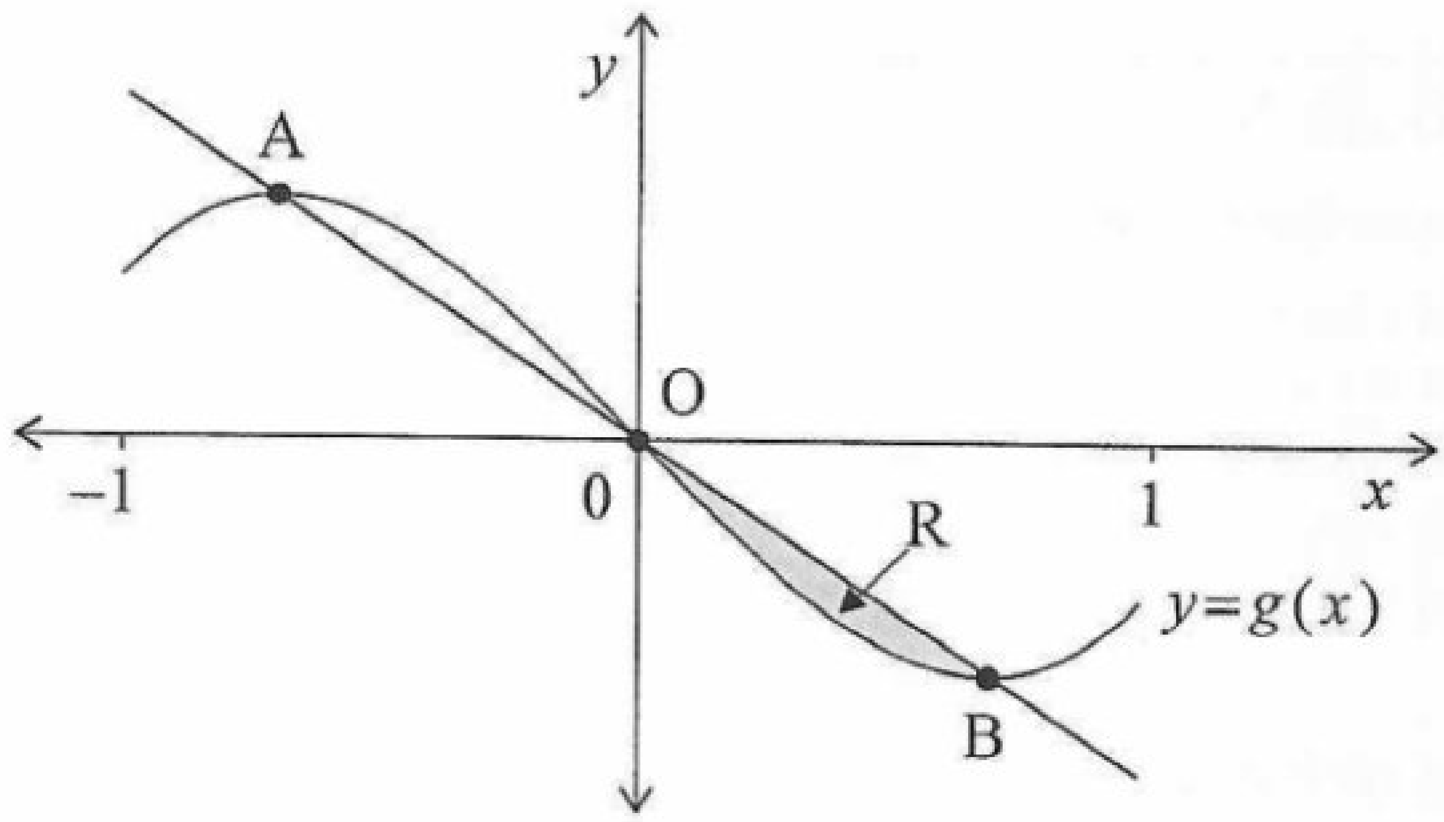
5. [Maximum mark: 7]

Note:
GDC Mode: Radian

Consider the function g defined as $g(x) = x^3 - 3x + 2\sin x$, where $x \in \mathbb{R}$.

The graph of g has a local maximum at point A and a local minimum at point B. The line (AB) passes through the origin O at $(0, 0)$.

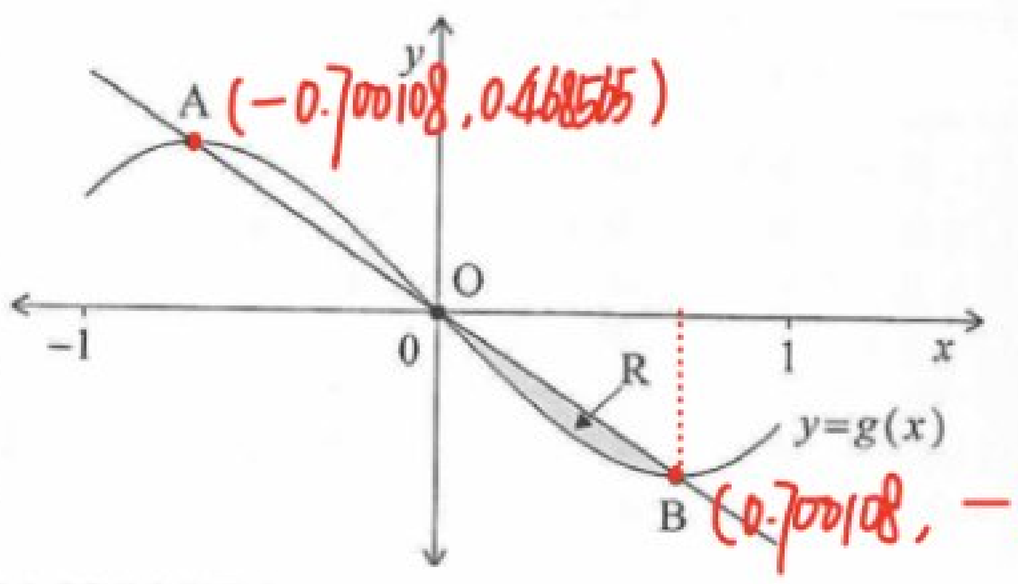
The region R is enclosed by the graph of g and the line segment [OB], as shown on the following diagram.



(a) Find the coordinates of point A and the coordinates of point B. [2]

(b) Find the area of R. [5]

(a) By GDC:



A(−0.700; 0.469) B(0.700; −0.469)

(b) $m_{AB} = m_{OB} = \dfrac{-0.468565 - 0}{0.700108 - 0} = -0.669275 \Rightarrow l_{AB}: y = -0.669275x.$

$A = \int_0^{0.700108} [-0.669275x - (x^3 - 2x + 2\sin x)]\, dx \overset{\text{By GDC}}{=\!=} 0.0407$

438

A002

6. [Maximum mark: 7]

A large group of adults have their heights measured. The heights of the adults are normally distributed with mean μ cm and standard deviation σ cm.

The probability that one of the adults chosen at random has a height greater than 171 cm is 0.4.

If one of the adults with a height greater than 171 cm is chosen at random, the probability that their height is greater than 175 cm is 0.35.

Find the value of μ and the value of σ.

X: The height of the adults.

$X \sim N(\mu, \sigma^2)$

$P(X > 171) = 0.4 \Rightarrow P(X \le 171) = 0.6 \Rightarrow P(Z \le \frac{171-\mu}{\sigma}) = 0.6$

$\xRightarrow{\text{By GDC}} \frac{171-\mu}{\sigma} = 0.253347$ ①

$P(X > 175 \mid X > 171) = 0.35 \Rightarrow \frac{P(X>175)}{P(X>171)} = 0.35$

$\Rightarrow \frac{P(X>175)}{0.4} = 0.35 \Rightarrow P(X>175) = 0.14 \Rightarrow P(X \le 175) = 0.86$

$\Rightarrow P(Z \le \frac{175-\mu}{\sigma}) = 0.86 \xRightarrow{\text{By GDC}} \frac{175-\mu}{\sigma} = 1.08032$ ②

From ①: $\mu + 0.253347\sigma = 171$

From ②: $\mu + 1.08032\sigma = 175$.

By GDC: $\begin{cases} \mu = 169.775 \doteq 170 \\ \sigma = 4.83692 \doteq 4.84 \end{cases}$

438

A002

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

The points $A(-3, 6, 4)$ and $B(-1, 2, -2)$ lie on the surface of a sphere with centre C such that the line segment $[AB]$ is a diameter.

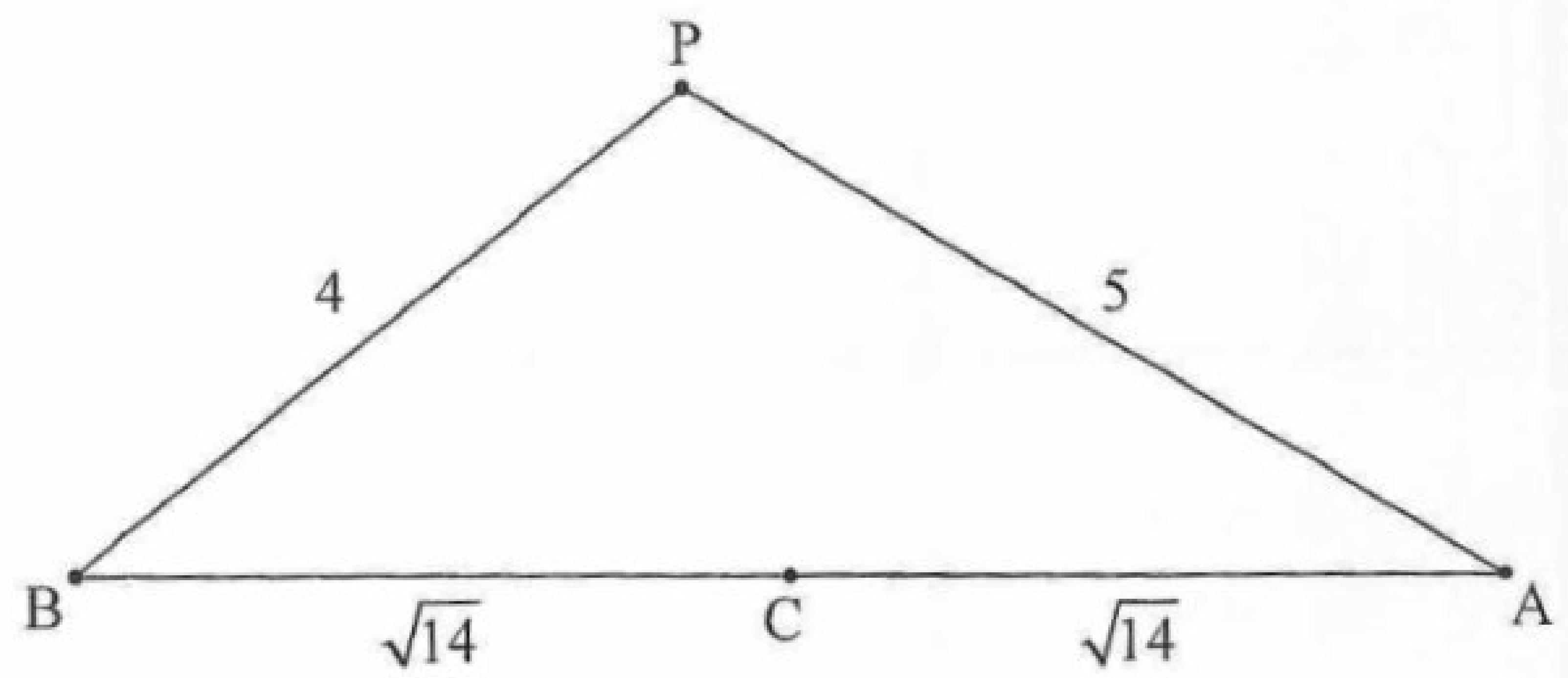
(a) Find the coordinates of C. [2]

(b) Show that the radius of the sphere is $\sqrt{14}$. [2]

(c) Find the volume of the sphere. [2]

Consider a point P where $AP = 5$ and $BP = 4$, as shown on the following diagram.

diagram not to scale



(d) Find

(i) the angle $B\hat{A}P$;

(ii) the area of triangle BAP. [7]

(e) (i) Find CP.

(ii) Hence, show that P lies inside the sphere. [3]

438

A002

(a) $C\left(\frac{(-3)+(-1)}{2}, \frac{6+2}{2}, \frac{4+(-2)}{2}\right) \Rightarrow C(-2,4,1)$

(b) M1: $d = AB = \sqrt{(-1-(-3))^2+(2-6)^2+(-2-4)^2} = \sqrt{4+16+36} = \sqrt{56} = 2\sqrt{14}$

$\Rightarrow r = \frac{d}{2} = \sqrt{14}$

M2: $r = AC = \sqrt{(-2-(-3))^2+(4-6)^2+(1-4)^2} = \sqrt{1+4+9} = \sqrt{14}$

or $r = BC = \sqrt{(-2-(-1))^2+(4-2)^2+(1-(-2))^2} = \sqrt{1+4+9} = \sqrt{14}$.

(c) $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(\sqrt{14})^3 = \frac{56\sqrt{14}}{3}\pi$ or 219

(d) (i) $\cos B\hat{A}P = \frac{AB^2+AP^2-BP^2}{2\times AB\times AP} = \frac{(2\sqrt{14})^2+5^2-4^2}{2\times 2\sqrt{14}\times 5} = \frac{65}{20\sqrt{14}} = \frac{13}{4\sqrt{14}}$

$\Rightarrow B\hat{A}P = \cos^{-1}\frac{13}{4\sqrt{14}} = 29.703756 \doteq 29.7°$

(or $B\hat{A}P = \cos^{-1}\frac{13}{4\sqrt{14}} = 0.51842834 \doteq 0.518$)

(ii) $A_{\triangle BAP} = \frac{1}{2}\times AB\times AP\sin B\hat{A}P = \frac{1}{2}\times 2\sqrt{14}\times 5\sin 29.703756° = 9.27$

(or $A_{\triangle BAP} = \frac{1}{2}\times AB\times AP\sin B\hat{A}P = \frac{1}{2}\times 2\sqrt{14}\times 5\sin 0.51842834 = 9.27$)

(e) (i) $CP^2 = AP^2+AC^2-2\times AP\times AC\cos C\hat{A}P = 5^2+(\sqrt{14})^2-2\times 5\times\sqrt{14}\times\frac{13}{4\sqrt{14}} = \frac{13}{2}$

$\xRightarrow{CP>0} CP = \sqrt{\frac{13}{2}} = \frac{\sqrt{26}}{2}$ or 2.55

(ii) $r = \sqrt{14} \doteq 3.74$. $\because CP = 2.55 < r = 3.74$ $\therefore P$ lies inside the sphere.

Do **not** write solutions on this page.

8. [Maximum mark: 13]

At the start of 2026, Lavinia receives a gift of $\$15\,000$. She wants to buy a boat which costs $\$28\,000$ so she decides to invest her money.

On 1 January 2026 Lavinia invests her money in a bank account which pays interest at a nominal annual rate of 4.8%, compounded **quarterly.** The interest is paid into her account on the last day of each quarter. She makes no further deposits to, or withdrawals from, the account.

(a) Find the amount of money Lavinia will have in her bank account on 1 January 2031. Give your answer correct to the nearest dollar. [3]

(b) Show that Lavinia will first have more than $\$28\,000$ in her bank account during the year 2039. [3]

The cost of the boat at the start of 2026 was $\$28\,000$, and it depreciates at a constant annual rate of $r\%$. After one year the cost of the boat is $\$26\,222$.

(c) Find the value of r. [2]

Due to this depreciation, Lavinia will be able to buy the boat before 2039. She will buy the boat as soon as she has enough money in her bank account to pay for it.

(d) Determine the year during which Lavinia will buy the boat. [5]

438

A002

(a) $15000\left(1+\frac{4.8\%}{4}\right)^{5\times4} = 19041.5 = \$1.90\times10^4$.

(b) $15000\left(1+\frac{4.8\%}{4}\right)^{4n} > 28000 \Rightarrow (1.012)^{4n} > \frac{28}{15}$

$\Rightarrow 4n > \log_{1.012}\frac{28}{15} \Rightarrow n > \frac{1}{4}\log_{1.012}\frac{28}{15} \Rightarrow n > 13.0811$

$\Rightarrow 2026 + n > 2039.0811$

$\Rightarrow$ Lavinia will first have more than \$28000 in her bank during the year 2039.

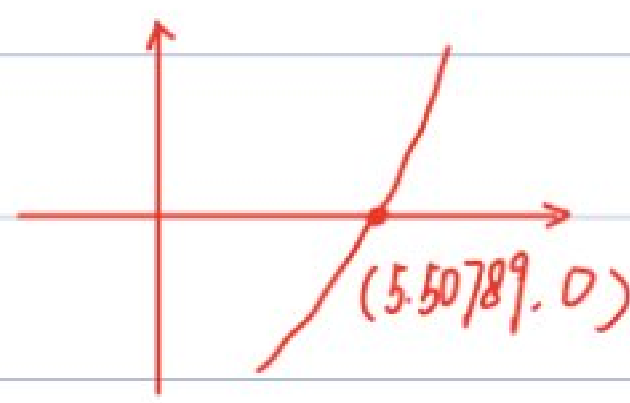
(c) $28000(1-r\%) = 26222 \Rightarrow 1-r\% = \frac{26222}{28000} \Rightarrow r\% = 1-\frac{26222}{28000}$

$\Rightarrow r = 100\times\left(1-\frac{26222}{28000}\right) \Rightarrow r = 6.35$

(d) $15000\left(1+\frac{4.8\%}{4}\right)^{4n} \geq 28000(1-6.35\%)^n$

$\Rightarrow 15000\left(1+\frac{4.8\%}{4}\right)^{4n} - 28000(1-6.35\%)^n \geq 0$

By GDC:



$n \geq 5.50789 \Rightarrow 2026 + n \geq 2031.50789 \Rightarrow$ Year ~~2032~~ 2031

$\Rightarrow$ Lavinia will buy the boat during the year ~~2032~~ 2031

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9. [Maximum mark: 15]

Jacinta has a bag of counters which are coloured either red, blue or yellow.

The bag contains exactly 12 yellow counters. The number of red counters is the same as the number of blue counters.

Jacinta plays a game where she takes 10 counters out of the bag, one at a time. She notes the colour of each counter and returns it to the bag before taking the next counter out of the bag.

The probability that the first counter is yellow is 0.4.

(a) Show that there are 9 red counters in the bag. [2]

(b) Find the probability that at least 6 of the 10 counters that Jacinta takes are yellow. **Give your answer correct to five significant figures.** [3]

438

Jacinta has to pay $\$5$ to take part in the game. If she takes at least 6 counters of the same colour, she wins a prize. If she does not take at least 6 counters of the same colour, then she does not win a prize. There is a different prize for each colour, as shown in the following table, where $B \in \mathbb{N}$.

Outcome	At least 6 red counters	At least 6 blue counters	At least 6 yellow counters
Prize	$\$40$	$\$B$	$\$10$

Let the random variable X represent Jacinta's net gain in dollars when she plays the game once. For example, if she takes at least 6 red counters, her net gain is $40 - 5 = 35$ since she gains $\$35$.

The probability distribution of X is shown in the following table, with probabilities given correct to four significant figures, where $A \in \mathbb{Z}, p \in \mathbb{R}$.

A002

x	35	$B-5$	A	-5
$\mathrm{P}(X=x)$	0.0473	0.0473	0.1662	p

(c) Write down the value of A. [1]

(d) (i) Use the probabilities in the table to find the value of p.

(ii) Determine the smallest integer value of B for which Jacinta could expect to make a positive net gain. [5]

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(Question 9 continued)

Jacinta wants to play the game until she wins a prize.

(e) Find the minimum number of times Jacinta needs to play the game in order that the probability of winning at least one prize is greater than 0.99. [4]

438

A002

(a) Let red counters = blue counters = x.

$P(Y) = 0.4 \Rightarrow \frac{12}{x+x+12} = 0.4 \Rightarrow \frac{12}{2x+12} = \frac{2}{5}$

$\Rightarrow 4x + 24 = 60 \Rightarrow 4x = 36 \Rightarrow x = 9$

$\therefore$ There are 9 red counters in the bag.

(b) T: The number of yellow counters of the 10 counters.

$T \sim B(10, 0.4)$

$P(T \geq 6) = 0.16624$

(C) $A = 5$

(d) (i) $P = 1 - (0.0473 + 0.0473 + 0.1662) = 0.7392$

(ii) $E(X) = 35 \times 0.0473 + (B-5) \times 0.0473 + 5 \times 0.1662 + (-5) \times 0.7392$

$= 0.0473B - 1.446$

Positive net gain $\Rightarrow E(X) > 0$

$\Rightarrow 0.0473B - 1.446 > 0$

$\Rightarrow B > \frac{1.446}{0.0473} = 30.570825$

$\because B \in \mathbb{N}$. $\therefore B_{min} = 31$

(e) Probability of success $= 0.0473 + 0.0473 + 0.1662 = 0.2608$

W: Number of success in n times.

$W \sim B(n, 0.2608)$

$P(W \geq 1) > 0.99$

$\Rightarrow 1 - P(W=0) > 0.99$

$\Rightarrow 1 - (1 - 0.2608)^n > 0.99$

$\Rightarrow (0.7392)^n < 0.01$

$\Rightarrow n > \log_{0.7392} 0.01 = 15.239484.$

$\because n \in \mathbb{Z}^+.$

$\therefore n_{min} = 16$