Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 1

14 May 2026

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Session number

1 hour 30 minutes

Instructions to students

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

681

A005

10 pages

2226–5018

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Consider the function $f(x) = 2x + \sin x, x \in \mathbb{R}$.

(a) Find $f'(x)$. [2]

(b) Hence, find the equation of the tangent to the graph of f at the point where $x = \pi$. [4]

(a) $f'(x) = 2 + \cos x$

(b) When $x = \pi$, $f(\pi) = 2\pi + \sin\pi = 2\pi$

$\Rightarrow$ Common point: $(\pi, 2\pi)$.

$m_T = f'(\pi) = 2 + \cos\pi = 2 - 1 = 1$

Tangent: $y - 2\pi = 1 \times (x - \pi)$

$\Rightarrow y = x + \pi$

2. [Maximum mark: 5]

Consider the function $f(x) = 3(x-2)(x-q)$, where $x \in \mathbb{R}$ and q is a real constant.

The axis of symmetry of the graph of f has equation $x = 4$.

(a) Show that $q = 6$. [2]

(b) Find the coordinates of the vertex of the graph of f. [2]

(c) Hence, write down the range of f. [1]

(a) $f(x) = 3(x-2)(x-q) = 0 \Rightarrow x_1 = 2,\ x_2 = q.$

$\frac{x_1 + x_2}{2} = 4 \Rightarrow \frac{2+q}{2} = 4 \Rightarrow q = 2\times4 - 2 = 6.$

(b) When $x = 4$, $f(4) = 3\times(4-2)\times(4-6) = -12$

$\Rightarrow$ Vertex: $(4, -12)$

(c) Range of $f(x)$: $y \geqslant -12$

681

A005

3. [Maximum mark: 5]

A particle P is travelling along a straight line, with displacement measured relative to a point O on the line. The velocity, $v\ \mathrm{m\,s^{-1}}$, of the particle at time t seconds is given by

$$v = \frac{1}{t+1} + \mathrm{e}^{-t}\text{, where } t \geq 0.$$

(a) Find $\int \left(\frac{1}{t+1} + \mathrm{e}^{-t} \right) \mathrm{d}t$. [2]

Initially the particle is at O.

(b) Find an expression for the displacement, s metres, in terms of t. [3]

(a) $\int (\frac{1}{t+1} + e^{-t})\, dt = \ln|t+1| - e^{-t} + C$

(b) $S(t) = \int V(t)\, dt = \ln(t+1) - e^{-t} + C \quad (t \geq 0)$

$\because S(0) = 0$

$\therefore \ln 1 - e^{0} + C = 0 \Rightarrow 0 - 1 + C = 0 \Rightarrow C = 1.$

$\therefore S(t) = \ln(t+1) - e^{-t} + 1$

681

A005

4. [Maximum mark: 6]

Consider a geometric sequence, u_n, with common ratio r, where each term in the sequence is positive.

It is given that $u_6 = 1.6 \times 10^5$ and $u_8 = 6.4 \times 10^5$.

(a) Find the value of r. [3]

(b) Find the value of u_1, giving your answer in the form $a \times 10^k$, where $1 \le a < 10$ and $k \in \mathbb{Z}$. [3]

(a) Each term in the sequence is positive $\Rightarrow r > 0$

$u_8 = u_6 \cdot r^2 \Rightarrow r^2 = \frac{u_8}{u_6} = \frac{6.4 \times 10^5}{1.6 \times 10^5} = 4 \xrightarrow{r>0} r = 2$

(b) $u_6 = u_1 r^5$

$\Rightarrow u_1 = \frac{u_6}{r^5} = \frac{1.6 \times 10^5}{2^5} = \frac{160 \times 10^3}{32} = 5 \times 10^3$

681

A005

Turn over

5. [Maximum mark: 7]

(a) Show that $\log_{25}(81x^2) = \log_5 9 + \log_5 x$ for $x > 0$. [4]

(b) Hence, solve the equation $\log_5 \sqrt[3]{x} = \log_{25}(81x^2) - \log_5 4$, where $x > 0$. [3]

(a) $L.H.S = \dfrac{\log_5(81x^2)}{\log_5 25} = \dfrac{\log_5(9x)^2}{\log_5 5^2} = \dfrac{2\log_5(9x)}{2} = \log_5(9x)$

$= \log_5 9 + \log_5 x = R.H.S. \quad (x>0)$

(b) $\log_5 \sqrt[3]{x} = (\log_5 9 + \log_5 x) - \log_5 4 \Rightarrow \log_5 \sqrt[3]{x} = \log_5 \dfrac{9x}{4}$

$\Rightarrow \sqrt[3]{x} = \dfrac{9x}{4} \Rightarrow x = \left(\dfrac{9}{4}\right)^3 x^3 \xRightarrow{x>0} x^2 = \left(\dfrac{4}{9}\right)^3 = \left(\dfrac{2}{3}\right)^6$

$\xRightarrow{x>0} x = \left(\dfrac{2}{3}\right)^3 \Longrightarrow x = \dfrac{8}{27}$

6. [Maximum mark: 9]

Consider two events, A and B.

It is given that $\mathrm{P}(A \cap B') = \dfrac{5+x}{24}$, $\mathrm{P}(A \cap B) = \dfrac{7-x}{12}$ and $\mathrm{P}(A' \cap B) = \dfrac{1}{6}$, where $0 \le x \le 7$.

(a) Show that $\mathrm{P}(A) = \dfrac{19-x}{24}$. [3]

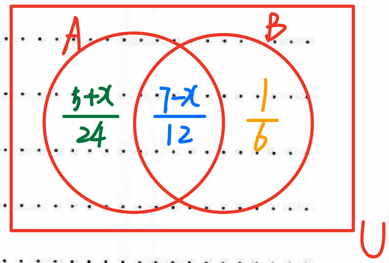
It is given that A and B are independent.

(b) Find the possible values of x. [6]

(a) $P(A) = P(A \cap B') + P(A \cap B) = \dfrac{5+x}{24} + \dfrac{7-x}{12}$

$= \dfrac{5+x+2(7-x)}{24} = \dfrac{5+x+14-2x}{24}$

$= \dfrac{19-x}{24}$



(b) $P(B) = P(A \cap B) + P(A' \cap B) = \dfrac{7-x}{12} + \dfrac{1}{6} = \dfrac{9-x}{12}$

A and B are independent $\Rightarrow P(A \cap B) = P(A) \times P(B)$

$\Rightarrow \dfrac{7-x}{12} = \dfrac{19-x}{24} \times \dfrac{9-x}{12} \Rightarrow 24(7-x) = (19-x)(9-x)$

$\Rightarrow 168 - 24x = 171 - 28x + x^2 \Rightarrow x^2 - 4x + 3 = 0$

$\Rightarrow (x-1)(x-3) = 0 \Rightarrow x = 1$ or $x = 3$

681

A005

Do **not** write solutions on this page.

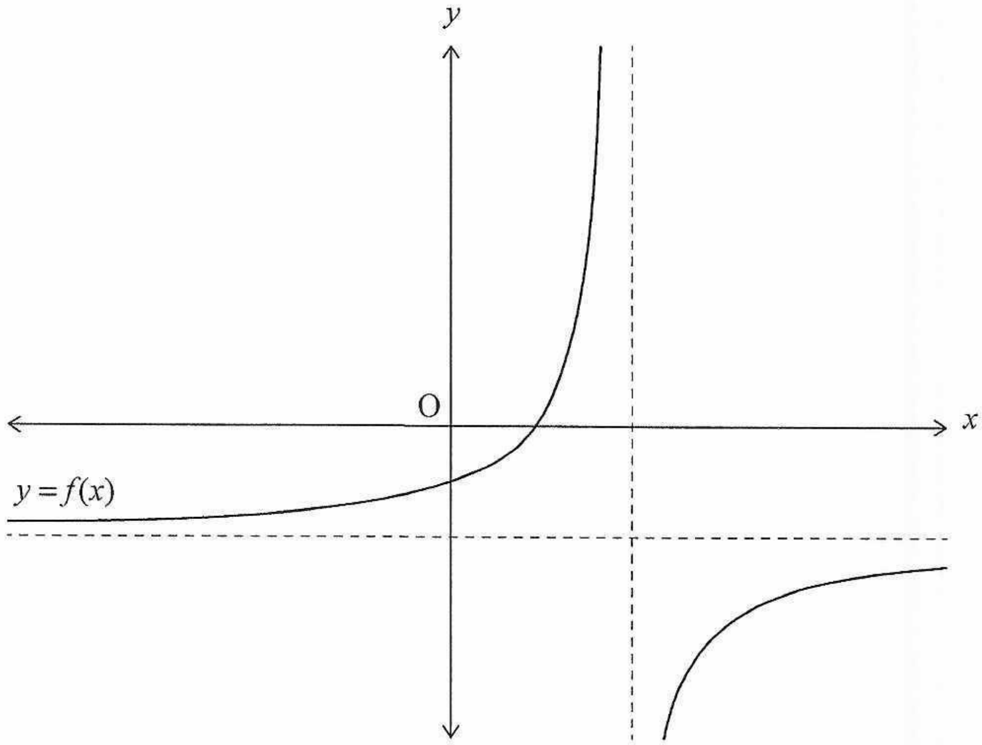
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 12]

Consider the function $f(x)=\dfrac{2x-3}{3-x}$, $x \in \mathbb{R}$, $x \neq 3$.

The graph of f and its asymptotes are shown in the following diagram.



(a) Find the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote. [3]

(b) Find the coordinates of the points where the graph of f intersects the coordinate axes. [4]

Consider the function $g(x)=x+4$, $x \in \mathbb{R}$.

(c) (i) Describe the transformation that transforms the graph of $y=f(x)$ to the graph of $y=(g \circ f)(x)$.

(ii) Hence, find the range of $g \circ f$. [5]

(a) $3 - x = 0 \implies$ V.A.: $x = 3$.

$$\lim_{x\to\infty} \frac{2x-3}{3-x} = \lim_{x\to\infty} \frac{2-\frac{3}{x}}{\frac{3}{x}-1} = \frac{2-0}{0-1} = -2 \implies \text{H.A.: } y = -2$$

(b) When $y=0 \implies 2x-3=0 \implies x = \frac{3}{2} \implies (\frac{3}{2}, 0)$

When $x=0 \implies y = \frac{0-3}{3-0} = -1 \implies (0, -1)$

(c) (i) $(g\circ f)(x) = g(f(x)) = f(x) + 4$.

Translate under the vector $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

(ii) Range of $(g\circ f)(x)$: $(g\circ f)(x) \neq 2$

Do **not** write solutions on this page.

8. [Maximum mark: 15]

Consider the function $f(x) = ax^3 + bx^2 + cx + d$, where $x \in \mathbb{R}$ and where a, b, c and d are real constants.

The graph of f has a point of inflexion at $(-2, -2c + d + 24)$.

(a) Show that $a = \frac{3}{2}$ and $b = 9$. [6]

It is given that the tangents to the graph of f at $x = -3$ and $x = k$ are horizontal.

(b) (i) Show that $c = \frac{27}{2}$.

(ii) Find the value of k.

(iii) State whether f has a local maximum or a local minimum at $x = k$, justifying your answer. [7]

The graph of f intersects the y-axis at the point P.

(c) Show that the tangent to the graph of f at $x = -3$ passes through P. [2]

681

A005

(a) $f'(x) = 3ax^2 + 2bx + C$

$f''(x) = 6ax + 2b$

$f''(-2) = 0 \Rightarrow -12a + 2b = 0 \Rightarrow b = 6a$ ①

$f(-2) = -2C + d + 24 \Rightarrow -8a + 4b - 2C + d = -2C + d + 24$

$\Rightarrow -8a + 4b = 24 \Rightarrow -2a + b = 6$ ②

Sub ① into ②: $-2a + 6a = 6 \Rightarrow 4a = 6 \Rightarrow \boxed{a = \frac{3}{2}}$

Sub $a = \frac{3}{2}$ into ①: $b = 6 \times \frac{3}{2} \Rightarrow \boxed{b = 9}$

(b) $f'(x) = \frac{9}{2}x^2 + 18x + C$

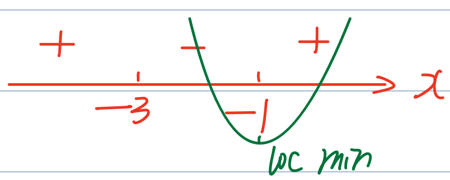
(i) $f'(-3) = 0 \Rightarrow \frac{9}{2} \times 9 + 18 \times (-3) + C = 0 \Rightarrow \boxed{C = \frac{27}{2}}$

(ii) $f'(k) = 0 \Rightarrow \frac{9}{2}k^2 + 18k + \frac{27}{2} = 0 \Rightarrow 9k^2 + 36k + 27 = 0$

$\Rightarrow k^2 + 4k + 3 = 0 \Rightarrow (k+1)(k+3) = 0 \xRightarrow{k \neq -3} \boxed{k = -1}$

(iii) $f'(x) = \frac{9}{2}x^2 + 18x + \frac{27}{2} = \frac{9}{2}(x^2 + 4x + 3) = \frac{9}{2}(x+1)(x+3)$

M1: S.D of $f'(x)$:

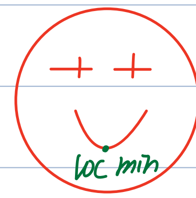


∴ The sign of $f'(x)$ changes from "negative" to "positive" around $x = -1$.

∴ There is a $\boxed{\text{loc min at } x = -1}$.

M2: $f''(x) = 9x + 18$

$f''(-1) = 9\times(-1) + 18 = 9 > 0$



$\Rightarrow$ There is a loc min at $x = -1$

(c) $f(x) = \frac{3}{2}x^3 + 9x^2 + \frac{27}{2}x + d$.

When $x = 0$, $f(0) = d$. $\Rightarrow P(0, d)$

When $x = -3$, $f(-3) = \frac{3}{2}\times(-27) + 9\times 9 + \frac{27}{2}\times(-3) + d$

$= -\frac{81}{2} + 81 - \frac{81}{2} + d$

$= d$

$\therefore$ Common point: $(-3, d)$

$f'(-3) = 0 \Rightarrow$ Tangent is horizontal $\Rightarrow$ Tangent equation: $y = d$.

$P(0, d)$ lies on $y = d$

$\therefore$ The tangent to the graph of f at $x = -3$ passes through P.

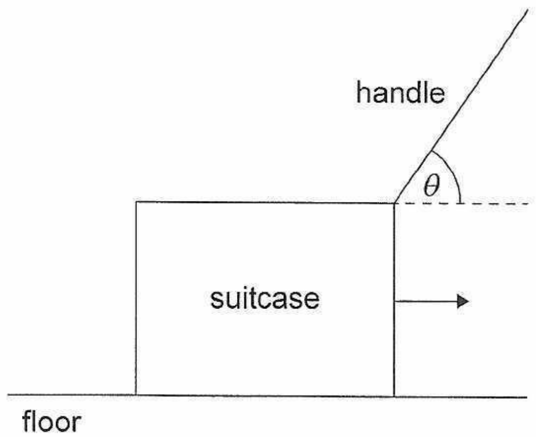
Do **not** write solutions on this page.

9. [Maximum mark: 15]

Eleanor uses the handle on a heavy suitcase to pull it so that it moves horizontally along the floor.

The handle makes an angle of θ radians with the horizontal, as shown in the following diagram.

diagram not to scale



The magnitude, F Newtons, of the minimum force that Eleanor needs to apply to make the suitcase move horizontally when the handle makes an angle of θ radians is given by

$$F(\theta) = \frac{A}{\cos\theta + \mu\sin\theta}, \text{ where } 0 \le \theta < \frac{\pi}{2} \text{ and } \mu \in \mathbb{R}.$$

When the handle is horizontal, $F = 300$.

(a) Show that $A = 300$. [2]

In the case when $\theta = \frac{\pi}{6}$, $F = 100\sqrt{3}$.

(b) Show that $\mu = \sqrt{3}$. [4]

Eleanor wishes to minimize the magnitude of the force she needs to apply to make the suitcase move horizontally. In order to achieve this, she needs to hold the handle at the angle α, where $0 < \alpha < \frac{\pi}{2}$.

(c) (i) Show that $\dfrac{\mathrm{d}F}{\mathrm{d}\theta} = \dfrac{300\left(\sin\theta - \sqrt{3}\cos\theta\right)}{\left(\cos\theta + \sqrt{3}\sin\theta\right)^2}$.

(ii) Hence, show that $\alpha = \frac{\pi}{3}$ and find the corresponding value of $F(\alpha)$. [9]

681

A005

(a) $F(0) = 300 \Rightarrow \dfrac{A}{\cos 0 + \mu \sin 0} = 300 \Rightarrow \dfrac{A}{1+0} = 300 \Rightarrow A = 300$

(b) $F(\theta) = \dfrac{300}{\cos\theta + \mu \sin\theta}$

$F(\frac{\pi}{6}) = 100\sqrt{3} \Rightarrow \dfrac{300}{\cos\frac{\pi}{6} + \mu \sin\frac{\pi}{6}} = 100\sqrt{3} \Rightarrow \dfrac{300}{\frac{\sqrt{3}}{2} + \frac{1}{2}\mu} = 100\sqrt{3}$

$\Rightarrow \dfrac{600}{\sqrt{3}+\mu} = 100\sqrt{3} \Rightarrow \sqrt{3} + \mu = \dfrac{600}{100\sqrt{3}}$

$\Rightarrow \sqrt{3} + \mu = 2\sqrt{3} \Rightarrow \mu = \sqrt{3}$

(c) (i) $F(\theta) = \dfrac{300}{\cos\theta + \sqrt{3}\sin\theta} = 300(\cos\theta + \sqrt{3}\sin\theta)^{-1}$

$\Rightarrow \dfrac{dF}{d\theta} = 300 \times \left[-(\cos\theta + \sqrt{3}\sin\theta)^{-2} \times (-\sin\theta + \sqrt{3}\cos\theta)\right] = \dfrac{300(\sin\theta - \sqrt{3}\cos\theta)}{(\cos\theta + \sqrt{3}\sin\theta)^2}$

(ii) $\dfrac{dF}{d\theta} = 0 \Rightarrow 300(\sin\theta - \sqrt{3}\cos\theta) = 0 \Rightarrow \sin\theta - \sqrt{3}\cos\theta = 0$

$\Rightarrow \sin\theta = \sqrt{3}\cos\theta \Rightarrow \tan\theta = \sqrt{3} \xRightarrow{0<\theta<\frac{\pi}{2}} \theta = \frac{\pi}{3}$

$\therefore \alpha = \frac{\pi}{3}$

$F(\frac{\pi}{3}) = \dfrac{300}{\cos\frac{\pi}{3} + \sqrt{3}\sin\frac{\pi}{3}} = \dfrac{300}{\frac{1}{2} + \sqrt{3} \times \frac{\sqrt{3}}{2}} = \dfrac{300}{\frac{1}{2} + \frac{3}{2}} = \dfrac{300}{2} = 150\,N$