



Oxford Cambridge and RSA

**GCE**

**Further Mathematics A**

**Y545/01: Additional Pure Mathematics**

A Level

**Mark Scheme for June 2025**

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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## MARKING INSTRUCTIONS

### PREPARATION FOR MARKING

#### RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training: OCR Essential Guide to Marking*.
2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are available in RM Assessor
3. Log-in to RM Assessor and mark the **required number** of practice responses (“scripts”) and the **required number** of standardisation responses.

### MARKING

1. Mark strictly to the mark scheme.
2. Marks awarded must relate directly to the marking criteria.
3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.
4. If you are in any doubt about applying the mark scheme, consult your Team Leader by telephone, email or via the RM Assessor messaging system.
5. **Crossed-Out Responses**  
Where a candidate has crossed out a response and provided a clear alternative then the crossed-out response is not marked. Where no alternative response has been provided, examiners may give candidates the benefit of the doubt and mark the crossed-out response where legible.

**Rubric Error Responses – Optional Questions**

Where candidates have a choice of question across a whole paper or a whole section and have provided more answers than required, then all responses are marked and the highest mark allowable within the rubric is given. Enter a mark for each question answered into RM Assessor, which will select the highest mark from those awarded. *(The underlying assumption is that the candidate has penalised themselves by attempting more questions than necessary in the time allowed.)*

**Multiple-Choice Question Responses**

When a multiple-choice question has only a single, correct response and a candidate provides two responses (even if one of these responses is correct), then no mark should be awarded (as it is not possible to determine which was the first response selected by the candidate).

*When a question requires candidates to select more than one option/multiple options, then local marking arrangements need to ensure consistency of approach.*

**Contradictory Responses**

When a candidate provides contradictory responses, then no mark should be awarded, even if one of the answers is correct.

**Short Answer Questions (requiring only a list by way of a response, usually worth only one mark per response)**

Where candidates are required to provide a set number of short answer responses then only the set number of responses should be marked. The response space should be marked from left to right on each line and then line by line until the required number of responses have been considered. The remaining responses should not then be marked. Examiners will have to apply judgement as to whether a 'second response' on a line is a development of the 'first response', rather than a separate, discrete response. *(The underlying assumption is that the candidate is attempting to hedge their bets and therefore getting undue benefit rather than engaging with the question and giving the most relevant/correct responses.)*

**Short Answer Questions (requiring a more developed response, worth two or more marks)**

If the candidates are required to provide a description of, say, three items or factors and four items or factors are provided, then mark on a similar basis – that is downwards (as it is unlikely in this situation that a candidate will provide more than one response in each section of the response space).

**Longer Answer Questions (requiring a developed response)**

Where candidates have provided two (or more) responses to a medium or high tariff question which only required a single (developed) response and not crossed out the first response, then only the first response should be marked. Examiners will need to apply professional judgement as to whether the second (or a subsequent) response is a 'new start' or simply a poorly expressed continuation of the first response.

6. Always check the pages (and additional objects if present) at the end of the response in case any answers have been continued there. If the candidate has continued an answer there, then add the annotation 'SEEN' to confirm that the work has been seen and mark any responses using the annotations in section 11.

7. There is a NR (**No Response**) option. Award NR (No Response):

- if there is nothing written at all in the answer space
- OR if there is a comment which does not in any way relate to the question (e.g., 'can't do', 'don't know')
- OR if there is a mark (e.g., a dash, a question mark) which is not an attempt at the question.

Note: Award 0 marks – for an attempt that earns no credit (including copying out the question).

8. The RM Assessor **comments box** is used by your Team Leader to explain the marking of the practice responses. Please refer to these comments when checking your practice responses. **Do not use the comments box for any other reason.**

9. Assistant Examiners will send a brief report on the performance of candidates to their Team Leader (Supervisor) via email by the end of the marking period. The report should contain notes on particular strengths displayed as well as common errors or weaknesses. Constructive criticism of the question paper/mark scheme is also appreciated.

10. For answers marked by levels of response: Not applicable in F501

**To determine the level** – start at the highest level and work down until you reach the level that matches the answer

**To determine the mark within the level**, consider the following

<b>Descriptor</b>	<b>Award mark</b>
On the borderline of this level and the one below	At bottom of level
Just enough achievement on balance for this level	Above bottom and either below middle or at middle of level (depending on number of marks available)
Meets the criteria but with some slight inconsistency	Above middle and either below top of level or at middle of level (depending on number of marks available)
Consistently meets the criteria for this level	At top of level

**11. Annotations**

<b>Annotation</b>	<b>Meaning</b>
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

<b>Other abbreviations in mark scheme</b>	<b>Meaning</b>
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

**Subject Specific Marking Instructions**

- a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

**Award NR (No Response)**

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

- b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

- c. The following types of marks are available.

**M**

A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

**A**

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

**B**

Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep\*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f. We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
  - When a value is **not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
- NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.

Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

- g. Rules for replaced work and multiple attempts:
- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
  - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
  - if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.
- h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
- If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
- i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Qn.		Answer	Marks	AO	Guidance
1	a	The order of 5 mod 37 must be a factor of $37 - 1 = 36$ or $5 \nmid 36$	<b>B1</b> [1]	<b>2.3</b>	(and 5 is not)
	b	$5^2 \equiv 25, 5^3 \equiv 14$ $5^4 \equiv 33, 5^6 \equiv 11, 5^9 \equiv 6, 5^{12} \equiv 10$ and $5^{18} \equiv 36$ oe  Showing that there is no $n$ , with $1 < n < 36$ , for which $5^n \equiv 1$ (mod 37) (from part (a), only the factors of 36 need be checked) At least four values of such $n$ considered.  $5^2 \equiv 25, 5^3 \equiv 14$ $5^4 \equiv 33, 5^6 \equiv 11, 5^9 \equiv 6, 5^{12} \equiv 10$ and $5^{18} \equiv 36$ or $-1$	<b>B1</b>  <b>M1</b>  <b>A1</b> [3]	<b>1.1</b>  <b>2.1</b>  <b>2.2a</b>	Evaluating at least two of these powers of 5 correctly  Evidence of a valid strategy for checking only factors of 36 (or other valid methods/arguments). <b>Or</b> powers of 5 with ALL values of $n$ from 2 to 35 checked <b>Or</b> listing factors of 36 and evaluating 5 to the power of one of them (not 1 or 36)  All factors checked <b>with</b> correct numerical working. Condone appearance of irrelevant powers

Question	Answer	Marks	AO	Guidance
2	<p>(a) <math>(\text{Area } \triangle ABC) = \frac{1}{2} \left  \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 3 \\ 6 \\ -6 \end{pmatrix} \right  \quad \text{oe}</math></p> <p><math>\pm \begin{pmatrix} -6 \\ 18 \\ 15 \end{pmatrix}</math></p> <p><math>(\text{Area } \triangle ABC) = \frac{3}{2} \sqrt{65} \text{ oe}</math></p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>1.2</p> <p>1.1</p> <p>1.1</p>	<p>Correct formula with attempt at vector product of two suitable vectors. Condone poor notation.</p> <p>FT A correct vector product of two suitable vectors. Allow sign errors and one incorrect component. Condone poor notation.</p> <p>Correct answer gains full marks.</p>
	<p>(b)</p> <p><math>(\text{Vol. } ABCD) = \frac{1}{6} \left  \text{their} \begin{pmatrix} -6 \\ 18 \\ 15 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} \right  \quad \text{or} \quad \frac{1}{6} \text{their} \begin{vmatrix} 2 &amp; 3 &amp; 4 \\ -1 &amp; 6 &amp; 3 \\ 2 &amp; -6 &amp; 1 \end{vmatrix}</math></p> <p><math>(\text{Vol. } ABCD) = \frac{1}{6} \left  \begin{pmatrix} -6 \\ 18 \\ 15 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} \right  \quad \text{or} \quad \frac{1}{6} \begin{vmatrix} 2 &amp; 3 &amp; 4 \\ -1 &amp; 6 &amp; 3 \\ 2 &amp; -6 &amp; 1 \end{vmatrix}</math></p> <p><math>= \frac{15}{2} \text{ oe}</math></p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>[3]</p>	<p>1.2</p> <p>1.1</p> <p>1.1</p>	<p>Correct formula with attempt at scalar product of suitable third vector with their vector product from (a) or scalar triple product of three suitable vectors Condone poor notation throughout this question.</p> <p>A correct scalar product of their (a) with third correct vector or A correct scalar triple product of three suitable vectors, unsimplified</p> <p>Correct answer gains full marks.</p>

2	(c)	<p>their Vol. <math>ABCD = \frac{1}{3} \times (\text{their Area of base } ABC) \times \text{height}</math></p> <p><b>i.e.</b> <math>\frac{15}{2} = \frac{1}{3} \times \left(\frac{3}{2}\sqrt{65}\right) \times h</math></p> <p><math>\Rightarrow h = \frac{15}{\sqrt{65}}</math> or <math>\frac{15\sqrt{65}}{65} = \frac{3\sqrt{65}}{13}</math></p>	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>[2]</b></p>	<p><b>3.1a</b></p> <p><b>1.1</b></p>	<p>With all their components used</p> <p>Any correct exact (surd) form</p> <p><b>SCB1</b> for using the formula for the distance between a point and a plane</p>
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Qn.	Answer	Marks	AO	Guidance
3	<p data-bbox="203 196 465 229"><b>(a)</b> <math>3N + M = 31a</math></p> <p data-bbox="275 288 1077 395"><math>N = 31k \Rightarrow M = 31(a - 3k) \Rightarrow 31 M</math> <b>or</b> if <math>31 N \Rightarrow 31 3N</math> (since <math>\text{hcf}(3, 31) = 1 \Rightarrow 31 M</math> as <math>M = 31a - 3N</math>)</p> <p data-bbox="275 459 1099 566"><math>M = 31l \Rightarrow 3N = 31(a - l)</math> (and <math>\text{hcf}(3, 31) = 1 \Rightarrow 31 N</math> <b>or</b> if <math>31 M \Rightarrow 31 3N</math> as <math>3N = 31a - M \Rightarrow 31 N</math> (since <math>\text{hcf}(3, 31) = 1</math>)</p> <p data-bbox="275 587 1099 655">Both correctly shown multiples of 31 including explanation that <math>\text{hcf}(3, 31) = 1</math></p>	<p data-bbox="1160 196 1218 229"><b>M1</b></p> <p data-bbox="1160 325 1218 359"><b>M1</b></p> <p data-bbox="1160 496 1218 529"><b>M1</b></p> <p data-bbox="1160 603 1218 636"><b>A1</b></p> <p data-bbox="1160 676 1218 710"><b>[4]</b></p>	<p data-bbox="1283 196 1341 229"><b>1.1</b></p> <p data-bbox="1283 325 1341 359"><b>2.1</b></p> <p data-bbox="1283 496 1344 529"><b>2.2a</b></p> <p data-bbox="1283 603 1341 636"><b>2.4</b></p>	<p data-bbox="1384 309 1962 378">Proof attempted in one direction. Could prove that <math>31 \nmid M</math> implies that <math>31 \nmid N</math>.</p> <p data-bbox="1384 477 1962 545">Proof attempted in other direction. Could prove that <math>31 \nmid N</math> implies that <math>31 \nmid M</math>.</p>
	<p data-bbox="203 735 819 804"><b>(b)</b> <math>7\ 167\ 758 \rightarrow (716\ 775 - 24 \Rightarrow) \mathbf{716\ 751} \rightarrow</math> <math>(71\ 675 - 3 \Rightarrow) \mathbf{71\ 672}</math></p> <p data-bbox="275 842 555 876"><math>(7167 - 6 \Rightarrow) 7\ 161</math></p> <p data-bbox="275 884 533 917"><math>(716 - 3 \Rightarrow) 713</math></p> <p data-bbox="275 925 1077 959"><math>(71 - 9 \Rightarrow) 62 = 2 \times 31</math> (or <math>31 62</math>) <b>or</b> <math>62 \rightarrow 6 - 6 = 0 (= 0 \times 31)</math></p> <p data-bbox="275 967 566 1000"><math>(\Rightarrow \text{divisibility by } 31)</math></p>	<p data-bbox="1160 751 1218 785"><b>M1</b></p> <p data-bbox="1160 938 1218 971"><b>A1</b></p> <p data-bbox="1160 1011 1218 1045"><b>[2]</b></p>	<p data-bbox="1283 751 1344 785"><b>3.1a</b></p> <p data-bbox="1283 938 1341 971"><b>1.1</b></p>	<p data-bbox="1384 751 1798 785">At least two iterations attempted</p> <p data-bbox="1384 938 1525 971">All correct</p>

Question		Answer	Marks	AO	Guidance	
4	(a)	<b>DR</b> $\frac{\partial z}{\partial x} = 8xy - 3y^2 - 5$ $\frac{\partial z}{\partial y} = 4x^2 - 6xy$  $z = -4 + '0'(x-1) + '-2'(y-1)$ $2y + z = -2$ oe simplified	<b>M1*</b>  <b>A1</b>  <b>M1dep*</b> <b>A1</b>	<b>1.1</b>  <b>1.1</b>  <b>1.1</b> <b>1.1</b>	Attempt at first partial derivative(s). Allow errors in coefficients  Both correct (could be awarded in (b) if both partial derivatives not attempted here)  Attempt at tangent plane equation in this form  <b>Or</b> $\mathbf{r} \cdot (2\mathbf{j} + \mathbf{k}) = -2$ (any correct vector form) etc.	
		<b>ALT for the last two marks</b> $\mathbf{n} = \begin{pmatrix} 1 \\ '0' \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ '-2' \end{pmatrix} = \begin{pmatrix} 0 \\ '2' \\ '1' \end{pmatrix}$ and $\mathbf{n} \cdot \begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix} = -2$  $2y + z = -2$ oe simplified	<b>M1dep*</b>  <b>A1</b>	Finding the normal vector to the plane and $k$ in $\mathbf{n} \cdot \mathbf{r} = k$ with $\mathbf{n} = a\mathbf{j} + b\mathbf{k}$  <b>or</b> $\mathbf{r} \cdot (2\mathbf{j} + \mathbf{k}) = -2$ (any correct vector form) etc.		
			<b>[4]</b>			
	(b)	i	<b>DR</b> $\frac{\partial z}{\partial y} = 0$ and $x \neq 0$ or $x > 0$  $y = \frac{2}{3}x$ <b>or</b> $x = \frac{3}{2}y$ and substitute for $y$ ( <b>or</b> $x$ ) in $\frac{\partial z}{\partial x} = 0$ $\Rightarrow \frac{16}{3}x^2 - \frac{4}{3}x^2 - 5 = 0$ <b>or</b> $12y^2 - 3y^2 - 5 = 0$ oe  $\Rightarrow x = \frac{1}{2}\sqrt{5}$ oe <b>or</b> $y = \frac{1}{3}\sqrt{5}$ oe  $x = \frac{1}{2}\sqrt{5}, y = \frac{1}{3}\sqrt{5}, z = -\frac{5}{3}\sqrt{5}$ oe	<b>B1</b>  <b>M1</b>  <b>M1</b>  <b>A1</b>  <b>A1</b> <b>[5]</b>	<b>3.1a</b>  <b>2.1</b>  <b>1.1</b>  <b>1.1</b> <b>1.1</b>	$x \neq 0$ or $x > 0$ can be seen at a later stage  Eliminating one of the variables  Quadratic eqn. in $x$ ( <b>OR</b> $y$ )  First coordinate correct. (Ignore negative solutions)  All three coordinates and no extras

Question			Answer	Marks	AO	Guidance
4	b	ii	<b>DR</b> $\frac{\partial^2 z}{\partial x^2} = 8y$ and $\frac{\partial^2 z}{\partial y^2} = -6x$	<b>B1</b>	<b>1.1</b>	<b>FT</b> correct from part (a) Can be implied by terms in the main diagonal of the Hessian matrix or in its determinant if explicitly seen
			$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = 8x - 6y$	<b>B1</b>	<b>1.1</b>	<b>FT</b> correct from part (a) Can be implied by equal terms in the subleading diagonal of the Hessian matrix or in its determinant if explicitly seen
			$(\det(\mathbf{H}) =) \left(\frac{8}{3}\sqrt{5}\right)(-3\sqrt{5}) - (2\sqrt{5})^2$ <b>or</b> $8y(-6x) - (8x - 6y)^2$ <b>oe</b>	<b>M1</b>	<b>1.1</b>	Determinant attempted with their second-order partial derivatives. If either <b>B1</b> is <b>FT</b> or if any <b>B0</b> , explicit substitution of second partial derivatives in the determinant formula must be seen.
			-60	<b>A1</b>	<b>1.1</b>	<b>cao</b> Partial derivatives must be numerical (or values of $x$ and $y$ must be seen to be substituted) at some point to gain A1.
			<b>[4]</b>			
		iii	Since $\det(\mathbf{H}) < 0$ , ( $Q$ is a) saddle-point	<b>B1</b>	<b>2.2a</b>	Ignore any incorrect negative value of $\det(\mathbf{H})$
				<b>[1]</b>		

Question		Answer	Marks	AO	Guidance
5	(a)	$3x \equiv 7 \equiv 20 \equiv 33 \dots$ $\Rightarrow x \equiv 11 \pmod{13}$	M1 A1	1.1 1.1	Method for finding suitable equivalents mod 13 Justification that $\text{hcf}(3, 13) = 1$ not required here <b>SCB1</b> for answer only with verification
		<b>ALT.</b> method: Multiply $3x \equiv 7 \pmod{13}$ by 9 to get $27x \equiv 63 \Rightarrow x \equiv 11 \pmod{13}$	M1 A1		
			[2]		
	(b)	$7(11 + 13m) \equiv 3 \pmod{31}$ their $m \equiv 6 \pmod{31}$ or $m = 31n + 6$ $\Rightarrow x = 13(31n + 6) + 11$ $x = 403n + 89$ or $x \equiv 89 \pmod{403}$ Since $\text{hcf}(13, 31) = 1$	M1 M1 A1 B1	2.1 1.1 2.2a 2.4	Set $x = 13m + 11$ in $7x \equiv 3 \pmod{31}$ <b>FT</b> their solution in part (a) Calculating $m$ in explicit form and substituting back for $x$
		<b>ALT.</b> method:  Solve $7x \equiv 3 \pmod{31} \Rightarrow x = 31n + 27$ or and equating the two forms: $13m + 11 = 31n + 27$ their $n \equiv 2 \pmod{13}$ or $n = 13k + 2$ or their $m \equiv 6 \pmod{31}$ or $m = 31k + 6 \Rightarrow$ $x = 31(13k + 2) + 27$ or $x = 13(31k + 6) + 11$ $x = 403n + 89$ or $x \equiv 89 \pmod{403}$ Since $\text{hcf}(13, 31) = 1$	M1 M1 A1 B1		Use of the Chinese Remainder theorem <b>may</b> be seen  Calculating either $n$ or $m$ in explicit form and substituting back for $x$
			[4]		

Question		Answer	Marks	AO	Guidance	
6	(a)	i	$\begin{array}{c cccc} \times_{10} & 2 & 4 & 6 & 8 \\ \hline 2 & 4 & 8 & 2 & 6 \\ 4 & 8 & 6 & 4 & 2 \\ 6 & 2 & 4 & 6 & 8 \\ 8 & 6 & 2 & 8 & 4 \end{array}$	B1	1.1	All correct
		ii	The identity is 6	B1	2.2a	
		iii	eg 4 has order 2 4 has order 2, 2 and 8 have order 4	B1 B1 [2]	1.1 2.2a	One correct order for 2 or 4 or 8 All correct (ignore consideration of 6)
	(b)	i	$(o(H) \text{ or }  H  =) 16$ since each of $a, b$ can take any one of 4 values <b>or</b> $4 \times 4$ (possibilities)	B1* B1dep [2]	1.1 2.4	<b>or</b> a list of all 16 matrices or any other convincing statement
		ii	The identity is $(e =) \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$	B1	2.2a	
		iii	$\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}, \begin{pmatrix} 4 & 0 \\ 0 & 6 \end{pmatrix}$ and $\begin{pmatrix} 6 & 0 \\ 0 & 4 \end{pmatrix}$	B1 B1 [2]	2.2a 2.2a	Any one correct All three and no extras

Question			Answer	Marks	AO	Guidance
6	(b)	iv	$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}^4 = \begin{pmatrix} a^4 & 0 \\ 0 & b^4 \end{pmatrix}$ or $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}^4 = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$ or $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}^4 = I$  Since $a, b$ have orders (1,) 2 or 4 (for all $a, b \in S$ ) <b>or</b> since the order of an element is a factor of 16  and order is not 1 or 2, ... <b>or</b> and order is not 1, 2, 8 or 16, ... it follows that $a^4 = b^4 = 6$ for all $a, b \in S$ <b>or</b> order is 4	<b>B1</b>	<b>3.1a</b>	Accept $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}^n = \begin{pmatrix} a^n & 0 \\ 0 & b^n \end{pmatrix}$ or $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}^2 = \begin{pmatrix} a^2 & 0 \\ 0 & b^2 \end{pmatrix}$
		v	NOT cyclic since no element of order 16	<b>M1</b>  <b>A1</b>  <b>[3]</b>	<b>2.1</b>  <b>2.4</b>	Or $a^2, b^2 = 6$ or 4 (from the Cayley table). For the elements for which $a^2$ and/or $b^2 \neq 6$ ...  ...it follows that $a^2 = b^2 = 4$ so $a^4 = b^4 = 4^2 = 6$
				<b>B1</b>  <b>[1]</b>	<b>2.4</b>	Conclusion <b>with</b> reason. Condone absence of 16 if appearing in part <b>(b)(i)</b> .

Question	Answer	Marks	AO	Guidance
7	<p><b>DR</b></p> $\frac{dx}{dt} = -3\cos^2 t \sin t \quad \text{and} \quad \frac{dy}{dt} = 3\sin^2 t \cos t$ $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 9\cos^2 t \sin^2 t (\cos^2 t + \sin^2 t) \text{ oe}$ $A = 2\pi \int_0^{\frac{1}{2}\pi} \sin^3 t \times \sqrt{\text{their} \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$ <p>Use of <math>\cos^2 t + \sin^2 t = 1</math></p> $= k \int \sin^4 t \cos t dt$ $\frac{1}{5} [\sin^5 t]$ $A = \frac{6}{5}\pi$	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>[8]</b></p>	<p><b>1.1</b></p> <p><b>1.1</b></p> <p><b>3.1a</b></p> <p><b>1.1</b></p> <p><b>3.1a</b></p> <p><b>1.1</b></p> <p><b>3.1a</b></p> <p><b>1.1</b></p>	<p>At least one attempted via chain rule. Allow incorrect coefficient(s)</p> <p>Both correct (penalise incorrect sign(s) here only and not again)</p> <p>Attempted (does not need to be factorised)</p> <p>Correct use of SA formula (Ignore lack of limits until the final line)</p> <p>Could be done at an earlier stage</p> <p>Integral of the form <math>k \int \sin^4 t \cos t dt</math></p> <p>From correct integration method (e.g. by substituting <math>s = \sin t</math>)</p> <p><b>cao</b> Correct limits must have appeared at some point to gain this mark.</p>

Question		Answer	Marks	AO	Guidance
8	(a)	<p><math>G_{n+1} + qG_n = 1.25</math> has a constant Particular Solution</p> $C = \frac{1.25}{1+q}$ <p>and Complementary Soln. <math>A(-q)^n</math></p> <p>so Gen. Soln. is <math>G_n = A(-q)^n + \frac{1.25}{1+q}</math> oe</p>	B1	1.2	Accept $A(q)^n$
		M1	1.1		
		A1	1.1		
			[3]		
		<p>Since <math>q</math> is small, <math>q^n \rightarrow 0</math> for large <math>n</math></p> <p><math>(1+q) \approx 1</math></p>	B1	2.1	Accept $\frac{1.25}{1+q} \rightarrow 1.25$ . Condone poor notation eg $0 + \frac{5}{4} \times \frac{1}{1+0}$ which scores <b>B1B1</b>
			B1	2.4	
			[2]		
		<p><math>(G_2 = 1.25 - qG_1)</math>  <math>1.2378 = 1.25 - 1.22q</math>  <math>q = 0.01</math></p> <p><math>(G_1 = 1.25 - qG_0)</math>  <math>1.22 = 1.25 - 0.01 \times G_0</math> so <math>G_0 = 3</math></p>	M1	3.1a	Or substituting $n = 1$ and $n = 2$ in their $G_n$ from part (a)(i) $1.22 = A(-q) + \frac{1.25}{1+q}$ and $1.2378 = Aq^2 + \frac{1.25}{1+q}$ $q = 0.01$ or $G_0 = 3$
			A1	1.1	
			A1	1.1	$q = 0.01$ and $G_0 = 3$ . If <b>A0A0</b> , substitution must be seen for <b>M1</b>
			[3]		

Question			Answer	Marks	AO	Guidance
8	b	i	$L = 1.25L - rL^2$ $L = \frac{1}{4r}$ oe	M1 A1 [2]	3.4 1.1	Setting all terms equal to $L$ (must be seen)  (condone $F_n = \frac{1}{4r}$ )
		ii	$(F_{n+2} <)F_{n+1} \left(1.25 - r \times \frac{5}{4r}\right) (= 0)$  $F_{n+2} = 0$ (or $F_{n+1} = 0$ ) or $F_{n+2} < 0$ (or $F_{n+1} < 0$ ) <b>or</b> number of fish becomes negative/zero/non-positive <b>and</b> 'population has become extinct'	M1 A1 [2]	3.3 3.5a	Substituting $F_n > \frac{5}{4r}$ or $F_n = \frac{5}{4r}$ into $(F_{n+2} =) 1.25F_{n+1} - rF_{n+1}F_n$  $F_{n+2} = 0$ or $F_{n+2} < 0$ can be demonstrated in the line above Or words to this effect. Must be in context. Not 'population becomes 0' or 'does not make sense'.
		iii	Use of the INT function preferred since the original model gives non-integer output values	B1 [1]	3.5b	<b>OR</b> words to this effect eg 'Cannot have half a fish'
		iv	2500  2497  <b>EITHER</b> the two outcomes are basically the same/similar <b>OR</b> that the similar outcome suggests that using INT is essentially redundant	B1* B1* B1dep [3]	2.2a 3.1b 3.2a	Limiting value without INT  Limiting value of $F_{n+2} = \text{INT}(1.25F_{n+1} - rF_{n+1}F_n)$ (at $F_5$ )  <b>Or</b> words to this effect Not just 'The INT model predicts a (slightly) smaller number of fish' or 'Model 1 (or 2) is more accurate'

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